## Efficient Global and Local Force Calculations Based on Continuum Sensitivity Analysis

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Abstract — Equations derived from continuum design sensitivity analysis are successfully applied to the calculation of both total force and force distributions. The proposed method has several advantages over the traditional approaches based on the Maxwell Stress Tensor or Virtual Work Principle.

## I. FORCE EXPRESSIONS

The knowledge of local force distributions, as well as of the global force, is essential in the design of electromagnetic systems. Traditional methods such as the Maxwell Stress Tensor (MST) and Virtual Work Principle (VWP) have important advantages but also significant drawbacks as documented in several publications (see, for example, [1]).

In the previous paper [2] we demonstrated that expressions (1) to (3) below – based on the continuum sensitivity analysis (CDSA), which itself stems from the VWP – used for calculating the global forces acting on magnetic materials, have advantages of easy implementation and improved accuracy. In this paper, the approach is extended to the evaluation of force distributions and its validity is tested using numerical models, including linear and nonlinear materials.

$$\mathbf{F}_{iron} = \int_{\gamma} [(\upsilon_1 - \upsilon_2) \mathbf{B}_1 \cdot \mathbf{B}_2] V_n d\gamma -$$

$$\int_{\gamma} [\int_0^{B_1} \mathbf{H} \cdot d\mathbf{B} - \int_0^{B_2} \mathbf{H} \cdot d\mathbf{B}] V_n d\gamma$$
(1)

$$\mathbf{F}_{magnet} = \int_{\gamma} [(\mathbf{M}_2 - \mathbf{M}_1) \cdot \mathbf{B}_2] V_n d\gamma$$
 (2)

$$\mathbf{F}_{conductor} = \int_{\gamma} [(\mathbf{J}_2 - \mathbf{J}_1) \cdot \mathbf{A}_2] V_n d\gamma$$
 (3)

Here,  $\gamma$  denotes the interface boundary between different materials and  $V_n$  means a unit vector normal to the interface.

## II. EXAMPLES

Expressions (1)-(3) give local force distributions on an interface between materials of different permeabilities, permanent magnetizations or current densities. It is important to note that in our approach there is no need for additional cumbersome manipulations, such as differentiating the jacobian matrix of a finite element formulation, as is the case in other methods based on the VWP [1]. The efficiency and accuracy of the proposed scheme are tested with two linear and nonlinear models shown in Fig. 1 against the magnetic charge method (MCM). For this comparison, a commercial software package MagNet 6 has been used. The force distributions on the surface of the ferromagnetic material show good agreement with those of the MCM, as presented in Fig. 2 and Fig. 3, for a linear and non-linear case, respectively.

Several interesting implications of the force expressions and comparisons of global and local forces between different methods for various numerical models will be presented in the extended version of the paper.

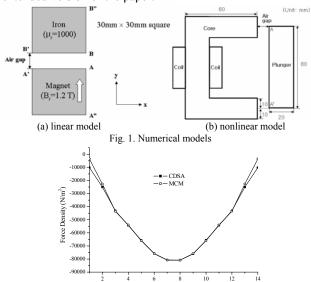


Fig. 2. Force distribution over the iron surface for a linear material.

Edge line number along B-B'

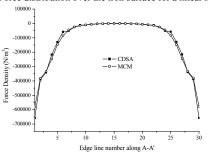


Fig. 3. Force distribution over the plunger surface for a nonlinear material.

The principal advantages of the CDSA method arise from the fact that the approach is derived from the VWP formulation but the expressions are similar to those of the MST, but with an important difference that no air-gap is needed to surround the region under consideration.

## III. REFERENCES

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