Generalized Low-Density Parity-Check Coding Aided Multilevel Codes

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Abstract – **Classic Low-Density Parity-Check (LDPC) codes have recently been used as component codes in Multilevel Coding (MLC) due to their impressive BER performance as well as owing to their flexible coding rates. In this paper, we proposed a Multilevel Coding invoking Generalized Low-Density Parity-Check (GLDPC) component codes, which is capable of outperforming the classic LDPC component codes at a reduced decoding latency, when communicating over AWGN and uncorrelated Rayleigh fading channels.**

1. INTRODUCTION

Multilevel Coding (MLC) was proposed by Imai and Hirawaki [1] as a bandwidth efficient coded modulation scheme designed for protecting each bit of a non-binary symbol with the aid of binary codes, while maintaining different target Bit Error Rates (BERs). Multistage Decoding (MSD) [2] was advocated for decoding MLCs, since its performance approaches that of the full maximum-likelihood decoding, while having the benefit of reduced decoding complexity. However, due to its high time delay and owing to the potential error propagation across the multiple decoding stages, it is not suitable for time-sensitive interactive audio/video applications. Alternatively, Parallel Independent Decoding (PID) [2] may also be employed as an efficient decoding strategy, where there is no information exchange across the different protection classes.

MLC schemes may be constructed using different component codes, for example convolutional codes, Bose-Chaudhuri-Hocquenghem (BCH) codes or turbo codes [3]. Recently, classic Low-Density Parity-Check (LDPC) codes [4] have also been commonly used as component codes [5] [6] owing to their flexible code rates and good BER performance. Belief Propagation (BP) [4] may be used for iterative soft decoding at each different BER protection level. In this paper, we propose a novel MLC design using Generalized LDPC (GLDPC) codes rather than classic LDPC codes [7] [8] as component codes, which has the benefit of an improved BER performance and an implementationally attractive parallel decoding structure.

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It is widely recognized that as a benefit of their block-based nature and random generator matrix construction, long block codes are capable of 'over-bridging' the channel fades and hence no channel interleaver is required for LDPC or GLDPC component codes. For our GLDPC codes, instead of using Gallager's single-error detecting parity check code [4], we employ binary BCH error-correcting codes [3] as the constituent codes. Simple iterative Soft-Input Soft-Output (SISO) decoders [3] are used for each constituent BCH code of the MLC scheme. We invoke both inner-iterations within the LDPC/GLDPC component codes and outer-iterations exchanging information between the LDPC/GLDPC block codes and the demapper as seen in Figure 2 and 3. Gray Mapping (GM) of the bits to modulated symbols is used for non-iterative decoding, while Set Partitioning (SP) based mapping is used for iterative decoding, because it provides improved iteration gains.

The rest of the paper is organized as follows. In Section 2 we describe the GLDPC codes adopted in our system, while our simulation results characterizing the proposed scheme are detailed in Section 3. Finally, our conclusions are presented in Section 4.

2. SYSTEM OVERVIEW

2.1. GLDPC Component Codes

We propose a MLC invoking GLDPC component codes [7] having a parity check matrix (PCM) illustrated in Figure 1. The PCM H was constructed by the single-error detecting parity check codes of a classic LDPC code [4] by the appropriately tessellated PCM H_0 of the binary BCH codes $C_0(n, k)$ used as constituent codes. The PCM was constructed with the aid of J so-called GLDPC superblocks. We opted for using J=2, since it results in a high minimum distance [7], despite its low decoding complexity. The J=2 superblocks are defined by two PCMs, which satisfy H^2=πH^1, where H^1 denotes the block diagonal matrix of the $first$ superblock, H^2 represents matrix of the $second$ superblock and π represents a pseudo-random column-wise permutation. This code construction produces L=N/n constituent codes, where N denotes the total coded block length.

Each BCH constituent code of the $first$ GLDPC superblock seen in the upper half of Figure 2 has an associated SISO de-

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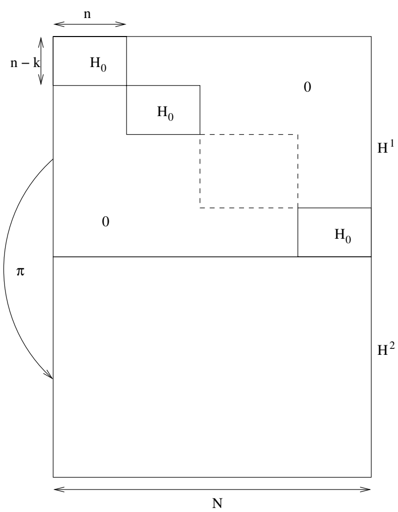

Figure 1: Parity Check Matrix (PCM) of a GLDPC Code, having J=2 superblocks.

coder and the BCH constituent codes are decoded in parallel, before the resultant $extrinsic$ information is fed into the $second$ interleaved GLDPC superblock portrayed at the bottom of Figure 2. This operation is repeated in an iterative inner GLDPC decoding loop. The substantial implementational benefit of this is that a number of cost-efficient, low-speed parallel SISO decoders maybe used instead of a single high-speed decoder.

Figure 2 portrays the L number of SISO decoders of the L constituent BCH codes. Since we have J=2 GLDPC superblocks, the channel's output information y is fed directly into the L number of parallel BCH SISO decoders of the $first$ GLDPC superblock, while after deinterleaving in the block π^{-1} into the $second$ GLDPC superblock of Figure 2. Following the last GLDPC iteration, the $aposteriori$ bit probabilities APP^2 generated at the output of the SISO decoder of Figure 2 will be used for obtaining the hard decision based GLDPC decoder output bits. The $extrinsic$ outputs Ext^1 of the $first$ superblock's SISO decoders are de-interleaved and used as $a\ priori$ information Apr^2 for each of the BCH constituent decoders of the $second$ GLDPC superblock in Figure 2. During the next inner iteration, the $extrinsic$ information Ext^2 arriving from the $second$ superblock is used as the $a\ priori$ information Apr^1 for the BCH constituent decoders of the $first$ GLDPC superblock of Figure 2, as in classic turbo detectors [3]. The processing block length of a constituent BCH SISO decoder is N/n, as opposed to N in a LDPC or turbo constituent decoder.

2.2. Modulation and Demodulation

Figure 3 shows the MLC/PID system model, employing the iterative GLDPC scheme of Figure 2 for each MLC protection class and having an additional outer decoding loop. A 3 bit/symbol encoded data is transmitted using 8-PSK modulation. We combine the three GLDPC component codes having different code rates, where each of the three source bits u_0, u_1 and u_2 is encoded into the associated coded bit v_0, v_1 and v_2 before the 8-PSK mapper, as seen in Figure 3. At the output of the channel, the received bits are demodulated and decoded with the aid of their corresponding GLDPC decoders for the sake of obtaining the decoded output bits of $\hat{u}_0$, $\hat{u}_1$ and $\hat{u}_2$, as seen in Figure 3. The 8PSK mapper employs Gray Mapping (GM) when non-iterative detection is used and Set Partitioning (SP), when iterative of detection is invoked, as outlined below.

Again, Gray mapping is employed in a non-iterative scheme, where the parallel decoding of the three bits is implemented without outer iterations. The absence of a long interleaver in Figure 2 has the potential of reducing the decoding delay imposed. However, for the sake of achieving a useful outer iteration gain in the decoder of our scheme seen in Figure 3, we also propose an iterative scheme employing SP based mapping, which is identical to that of Ungerböck's Trellis Coded Modulation (TCM) scheme, as described in Section 9.6.1 of [3]. Again, the outer iterations are illustrated in Figure 3, while the inner iterations are portrayed in Figure 2. The $extrinsic$ soft Log Likelihood Ratios (LLR) generated by the GLDPC decoders are converted into a stream of $a\ priori$ bit probabilities, which are then fed back to the input of the demapper seen in Figure 3 as the $a\ priori$ information $\hat{\mathbf{u}}$ used in the next outer iteration.

Since the $a\ priori$ information fed to the demapper of Figure 3 represents non-equiprobable bits after the first iteration, the achievable iteration gains maybe expected to increase by efficiently exploiting the $a\ priori$ probabilities, provided that an appropriate bit-to-symbol mapping scheme is used. In other words, after the first outer iteration, the channel output y seen in Figure 2 will be enhanced by the $a\ priori$ information P_a provided by the previous outer iteration. The extrinsic probability expression P_e of the MLC demapper of Figure 3 providing new information for enhancing our confidence in y was given by [9].

With the aid of the so-called equivalent capacity rule [2], we obtain the desired code rate of each component for 8-PSK modulation using Gray Mapping, yielding R_0/R_1/R_2 = 0.510/0.745/0.745. Given that the total number of uncoded input bits is k_i and the total number of channel coded output bits is n_i for the GLDPC encoder at the i^{th} MLC protection level, the coding rate of the ith GLDPC component code is [7]

$$R_i = 1 - J(1 - k_i/n_i). \tag{1}$$

Therefore, the overall effective throughput C of the proposed system is

$$C = P - J\sum_{i=0}^{i=L}(1 - k_i/n_i) \text{ bits/symbol}, \tag{2}$$

where P is the total number of modulation levels and we have $i \in \{0, 1, ..., P-1\}$. The total code rate of our system is

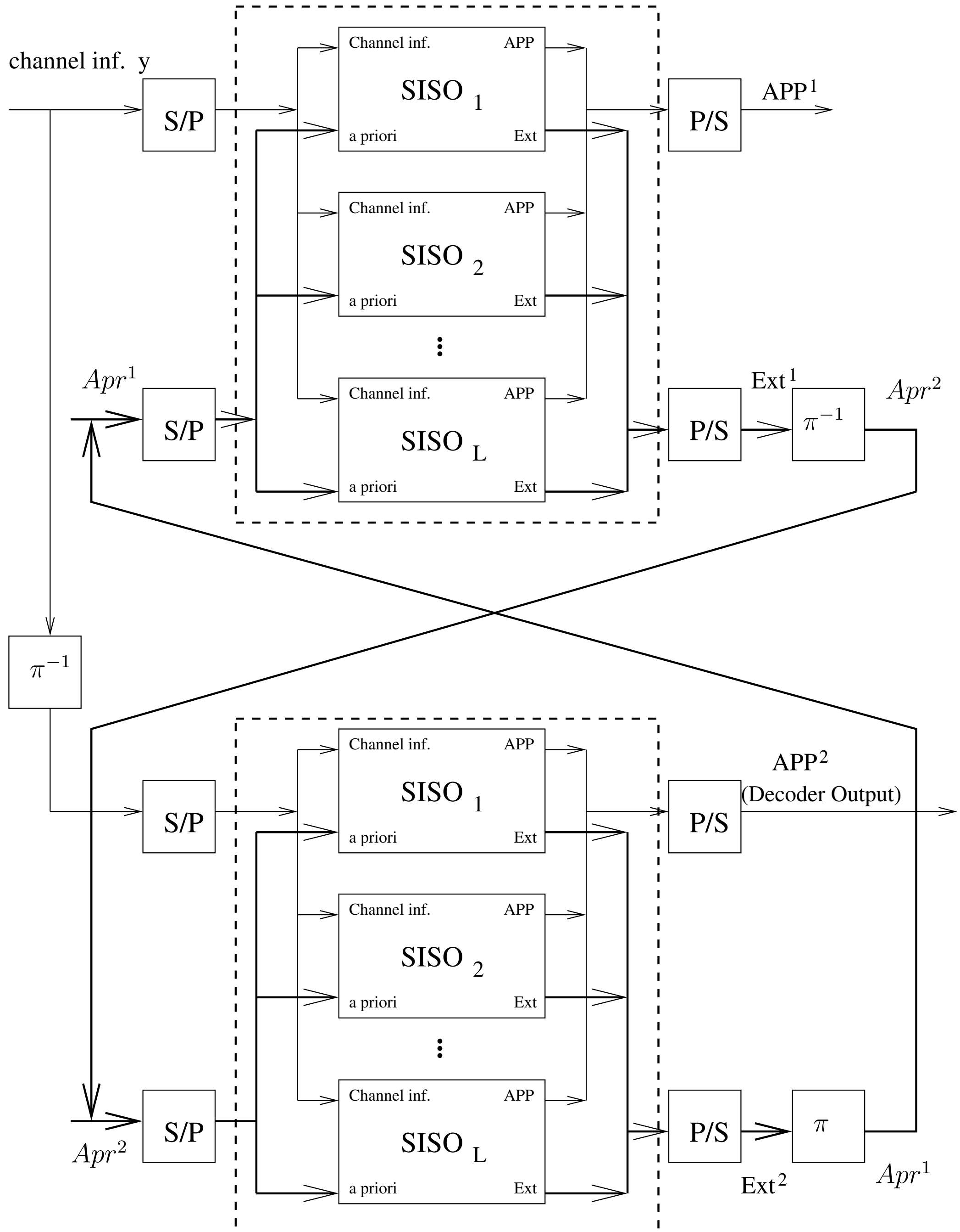

Figure 2: SISO BCH decoder of the GLDPC component codes.

R_t=$C/\log_2 M$, where M is the total number of modem constellation points obeying $M = 2^P$. The effective throughput of the system is therefore (0.51 + 0.745 + 0.745) = 2 bit/symbol. The BCH constituent codes employed in our scheme which approximate the R_i rates are the C_0(20,15), C_1(48,42) and C_2(48,42) codes, respectively. The system parameters are summarized in Table 1. Our benchmarker scheme employing classic LDPC codes uses the equivalent PID coding rates of R_0, R_1 and R_2.

3. SIMULATION RESULTS

The proposed MLC/PID GLDPC scheme using 8-PSK modulation was investigated, when communicating over both AWGN and uncorrelated Rayleigh fading channels. We employed ten GLDPC-BCH inner iterations in the spirit of Figure 2, a single outer iteration using Gray demapping and six outer iterations employing SP mapping in our scheme between the soft-decoded bits at the output of the three GLDPC decoders of Fig-

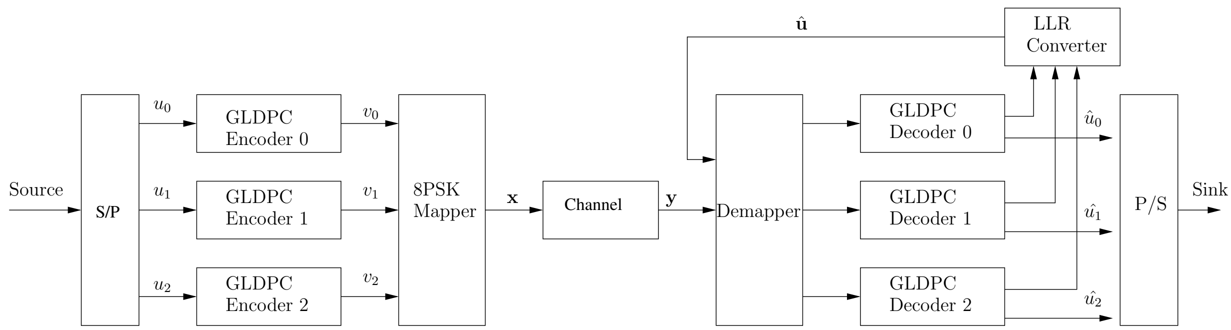

Figure 3: System model of MLC/PID using iteratively detected GLDPC inner codes as well as outer iterations. The iterative GLDPC decoder is seen in Figure 2.

PID Coding rate	R_0 0.51	R_1 0.745	R_2 0.745
BCH constituent codes	C_0 (20,15)	C_1 (48,42)	C_2 (48,42)

Modulation	8PSK
Non-iterative mapping scheme	Gray Mapping (GM)
Iterative mapping scheme	Set Partitioning (SP)
Number of symbols	2640

Table 1: System parameters table.

ure 3, in the spirit of Section 9.6.1 of [3]. Figure 4 shows that at BER=10^{-5}, the proposed scheme demonstrates an E_b/N_0 improvement of around 0.5dB in AWGN channels compared to our MLC-LDPC benchmarker system, while exhibiting a convenient parallel decoding structure. When employing SP based mapping and six outer iterations over AWGN channels, both systems achieve a further 2-2.5 dB performance improvement and the proposed MLC-GLDPC scheme retains its performance advantage. When communicating over uncorrelated Rayleigh fading channels, our MLC-GLDPC scheme outperforms the MLC-LDPC benchmarker by about 1dB in both the single outer-iteration Gray mapping and the six outer-iteration aided SP-based scenarios at BER=10^{-5}. This might appear to be a modest gain, but it is achieved with the aid of a more convenient parallel architecture.

We further investigate the effects of inner iterations in our MLC-GLDPC scheme with reference to our MLC-LDPC benchmarker in both AWGN and uncorrelated Rayleigh fading channels. The inner iterations are facilitated in the context of GLDPC component codes, since the information exchange can be carried out in the ”turbo-like” architecture shown in Figure 2 with the aid of a number of parallel, low complexity SISO decoders, each requiring a reduced block length in comparison to LDPC or turbo component codes. The number of inner iterations required for generating the most reliable extrinsic output for the outer iterations therefore also determines the delay imposed on the overall system.

We employ I_{outer}=6 outer iterations in our MLC-GLDPC

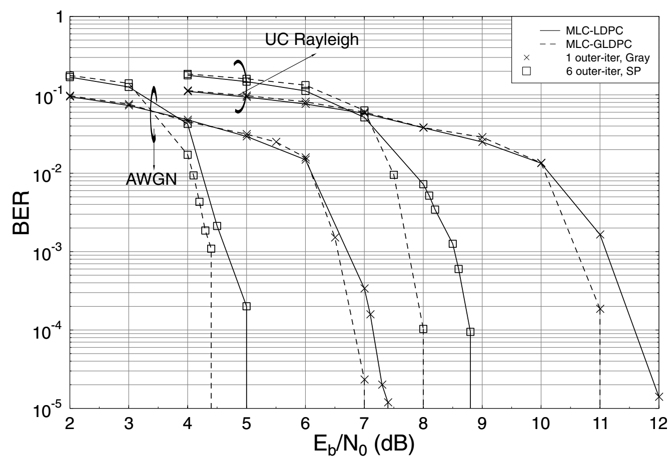

Figure 4: BER of both MLC-GLDPC and MLC-LDPC over an AWGN Channel and uncorrelated (UC) Rayleigh fading channel invoking I_{outer}=1 or 6 outer and I_{inner}=10 inner iterations. The effective throughput was 2 bits/symbol and the BCH codes were the (20,15), (48,42) and (48,42) schemes, respectively.

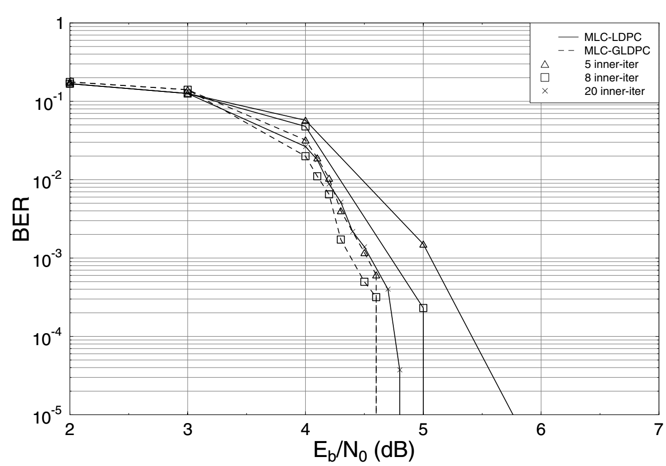

Figure 5: BER of both MLC-GLDPC and MLC-LDPC over an AWGN channel invoking I_{inner}=5, 8 or 20 inner and I_{outer}=6 outer iterations.

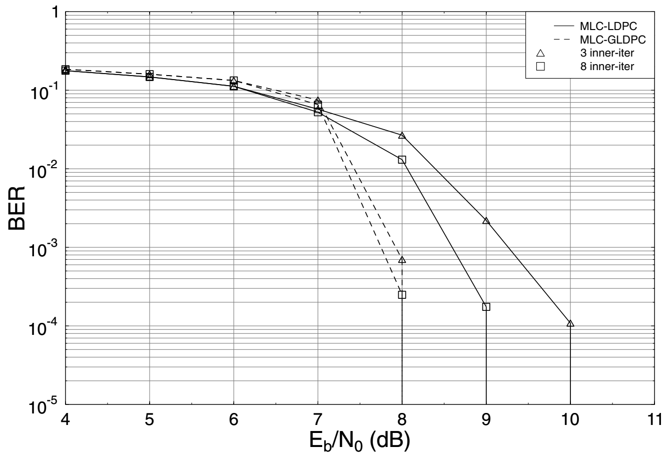

Figure 6: BER of both MLC-GLDPC and MLC-LDPC over uncorrelated Rayleigh fading channel invoking I_{inner}=5 or 8 inner and I_{outer}=6 outer iterations.

scheme, each invoking a different number of inner iterations (I_{inner}) using the SP mapping scheme. Figure 5 demonstrates that when transmitting over AWGN channels, our MLC-GLDPC scheme requires an E_b/N_0 value of around 4.6dB at BER=10^{-5}, in conjunction with I_{inner}=5 inner iterations. The MLC-LDPC benchmarker converges slowly at an E_b/N_0 close to 4.6dB, requiring up to I_{inner}=20 inner iterations for achieving BER=10^{-5}. In other words, the classic MLC-LDPC requires a quadrupled number of total iterations ($I_{outer}.I_{inner}$) compared to our MLC-GLDPC scheme for the sake of achieving a similar performance of BER=10^{-5}.

Let us now extend these investigations to the uncorrelated Rayleigh fading channel, where both schemes invoke the same number of $I_{outer} = 6$ outer iterations. The MLC-GLDPC scheme, achieves a coding advantage of 2dB compared to the MLC-LDPC scheme at BER=10^{-5}, when invoking $I_{inner} = 5$ inner iterations, as shown in Figure 6. This coding advantage is reduced to about 1dB, when $I_{inner} = 8$ inner iterations are employed. As observed from both Figures 5 and 6, our MLC-GLDPC scheme require $I_{inner} = 5$ inner iterations for achieving its best possible BER performance both in AWGN and uncorrelated Rayleigh fading channels.

4. CONCLUSIONS

In conclusion, this paper provided an insight into a range of MLC GLDPC schemes. Our simulations results suggested that the attainable SNR improvement compared to a classic random LDPC component code based MLC benchmarker ranged between 0.5dB and 2dB. This was achieved using the same number of iterations and an implementationally beneficial parallel architecture. We argued that multilevel coding using Gray mapping combined with parallel independent decoding is attractive in the context of supporting low-latency real time applications.

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