# Accurate BER Analysis of QPSK Modulated Asynchronous DS-CDMA Systems Communicating over Rayleigh Channels

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*Abstract*— **The accurate average BER calculation of an asynchronous DS-CDMA system using random spreading sequences is studied in Rayleigh fading channels. An accurate closed-form expression is derived for the conditional characteristic function of the multiple access interference. An accurate BER expression is provided, which only requires a single numerical integration. Our numerical simulation results verify its accuracy, and also demonstrate the relative inaccuracy of the Gaussian approximation.**

## I. Introduction

Spread spectrum Code Division Multiple Access (CDMA) was originally invented for counteracting jamming in military communications and has experienced a rapid evoluation since the launch of the first commercial CDMA system in 1995. The Bit Error Ratio (BER) performance of Direct-Sequence (DS) CDMA systems has been extensively studied, especially in the context of BPSK [1]–[19] and QPSK [4], [5], [7], [9], [16], [19]–[26] modulation.

The accurate average Bit Error Ratio (BER) performance of a system can be evaluated by typically integrating, i.e. averaging the generic BER formula over all random parameters of all users, but this requires the evaluation of multiple embedded integrals [19], which is irrealistic when the number of users is high. Several techniques have been developed for simplifying the accurate BER calculation of various modulation schemes, such as the series expansion of [4], [16]–[18], [25], or the employment of Moment Generating Functions (MGF) [13] and Characteristic Functions (CF) [1], [2], [4], [15], [26]. Since the computational complexity of the accurate analysis is usually excessive, various Gaussian approximation techniques based on the Central Limit Theorem (CLT) have also been proposed for simplifying the calculation of the average BER performance. A few examples are the Standard Gaussian Approximation (SGA) [1]–[3], [5], [6], [8]–[10], [15], [17]–[19], [22], [26], the Improved Gaussian Approximation (IGA) [1], [6], [10], [14], [17], [18], [22], [23], [26], the Simplified IGA (SIGA) [1], [6], [15], [18], [22], [26], as well as the Improved Holtzman's Gaussian Approximation (IHGA) [15]. In addition to the calculation of the average BER performance, BER bounds were also derived in [7], [8], [11], [22]. To avoid the complexity of the accurate BER analysis and the inaccuracy of various approximations, the SNR performance was investigated in [12], [20], [24].

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In the existing literature, the achievable BER performance over AWGN channels is the most extensively studied subject [4]–[12], [16]–[18], [20]–[23], [25], [26], but that over Rayleigh channels [1], [17] and Nakagami-$m$ channels [2], [3], [13]–[15] was also lavishly documented. The BER performance attainable using both deterministic spreading sequences [4], [7], [11], [13], [20], [21] and random spreading sequences [1]–[3], [5], [6], [8]–[10], [14]–[19], [22], [23], [25], [26] was also investigated.

However, to our knowledge, the accurate BER analysis of asynchronous DS-CDMA systems using random spreading sequences and QPSK modulation for transmission over Rayleigh channels is an open problem. The BER performance of a similar system using BPSK modulation has been studied in [1]. Since QPSK has been favoured in existing systems [27], we will extend the results of [1] to the QPSK scenario in this paper. Due to the multiple access interference (MAI) induced crosstalk between the in-phase and quadrature-phase branches of QPSK systems, their BER performance is different from that of BPSK systems, although they are identical in the absence of MAI [28].

The organization of this paper is as follows. In Section II an asynchronous DS-CDMA system using I/Q modulation is considered in the context of a Rayleigh channel. Then in Section III an accurate BER expression based on the CF approach is derived for the BER calculation of the system using random spreading sequences. In Section IV our numerical results are presented and finally, in Section V our conclusions are provided.

## II. System Model

We consider an asynchronous DS-CDMA system over a Rayleigh fading channel using the I/Q modulation, random spreading sequences and the rectangular chip waveform. Assume that there are $K$ simultaneously transmitting users. Each user is assigned two random binary spreading sequences,

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$\{a_{2k,m}\}_{m=0}^{L-1}$ and $\{a_{2k+1,m}\}_{m=0}^{L-1}$, having $L$ chips for its in-phase and quadrature-phase branch, respectively. The $k$th user's spreading sequences along with his/her data sequences, $\{b_{2k,m}\}_{m=-\infty}^{\infty}$ and $\{b_{2k+1,m}\}_{m=-\infty}^{\infty}$, modulate the phase of the in-phase and quadrature-phase branch, respectively.

The rectangular pulse $p_T(t)$ having a duration of $T$ is defined as:

$$p_T(t) = \begin{cases} 1, t \in [0, T), \\ 0, \text{otherwise}. \end{cases} \quad (1)$$

Hence the spreading signals $\{a_i(t)\}_{i=2k,2k+1}$ and the data signals $\{b_i(t)\}_{i=2k,2k+1}$ are expressed as follows, respectively:

$$a_i(t) = \sum_{m=-\infty}^{\infty} a_{i,m} p_{T_c}(t - mT_c), \quad (2)$$

$$b_i(t) = \sum_{m=-\infty}^{\infty} b_{i,m} p_{T_s}(t - mT_s), \quad (3)$$

where $T_c$ and $T_s$ are the chip duration and the symbol duration, respectively, and satisfying $T_s = LT_c$.

The received signal $r(t)$ at the input of the coherent matched filter receiver is given by:

$$\begin{aligned} r(t) = \sum_{k=0}^{K-1} h_k \{&a_{2k}(t-\tau_k) b_{2k}(t-\tau_k)\cos[\omega_c(t-\tau_k)+\theta_k] \\ &+ a_{2k+1}(t-\tau_k) b_{2k+1}(t-\tau_k)\sin[\omega_c(t-\tau_k)+\theta_k]\} \\ &+ \eta(t), \end{aligned} \quad (4)$$

where $\eta(t)$ is the zero-mean stationary Additive White Gaussian Noise (AWGN) having a double-sided power spectral density of $\frac{N_0}{2}$, $\omega_c$ is the common carrier frequency, $\{\theta_k\}_{k=0}^{K-1}$ and $\{\tau_k\}_{k=0}^{K-1}$ are the phase shift and the time delay, respectively, both of which are independently and uniformly distributed in $[0, 2\pi)$ and $[0, T_s)$, respectively. The amplitude $\{h_k\}_{k=0}^{K-1}$ is mutually independent and Rayleigh distributed having a Probability Distribution Function (PDF) of $f_{h_k}(x)$ expressed as:

$$f_{h_k}(x) = \begin{cases} \frac{x}{\sigma_k^2} e^{-\frac{x^2}{2\sigma_k^2}}, x \geq 0, \\ 0, \qquad x < 0. \end{cases} \quad (5)$$

Without loss of generality, we assume that the 0th user's signal is the desired one. If the phase and chip synchronization are perfect, the decision statistic of $\widetilde{Z}$ at the output of the correlation receiver matched to the 0th user's signal is given by:

$$\begin{aligned} \widetilde{Z} &= \frac{2}{T_c}\int_0^{T_s} r(t) a_0(t)\Psi(t)\cos(\omega_c t)\mathrm{d}t \\ &\quad - j\frac{2}{T_c}\int_0^{T_s} r(t) a_1(t)\Psi(t)\sin(\omega_c t)\mathrm{d}t \\ &= \widetilde{D} + \sum_{k=1}^{K-1}\widetilde{I}_k + \widetilde{\eta}, \end{aligned} \quad (6)$$

where $\widetilde{D}$ is the desired signal component, $\widetilde{I}_k$ is the co-channel interference component incurred by the $k$th user and $\widetilde{\eta}$ is the noise component.

The desired signal $\widetilde{D}$ can be expressed as:

$$\widetilde{D} = D^I + jD^Q = h_0 L(b_{0,0} + jb_{1,0}), \quad (7)$$

where $D^I$ and $D^Q$ are the in-phase and quadrature-phase components of the desired signal, respectively.

The noise component $\widetilde{\eta}$ can be shown to be a zero-mean complex-valued Gaussian variable having a variance of $\sigma_{\widetilde{\eta}}^2 = \frac{2N_0 L}{T_c}$. Hence its in-phase and quadrature-phase components, $\eta^I$ and $\eta^Q$, are mutually independent, zero-mean real-valued Gaussian variables having a variance of $\sigma_{\eta^I}^2 = \sigma_{\eta^Q}^2 = \frac{N_0 L}{T_c}$.

The co-channel interference $\widetilde{I}_k$ incurred by the $k$th user can be expressed as:

$$\begin{aligned} \widetilde{I}_k = I_k^I + jI_k^Q = h_k(&J_{2k,0}\cos\Delta_k - J_{2k+1,0}\sin\Delta_k \\ &+ jJ_{2k,1}\sin\Delta_k + jJ_{2k+1,1}\cos\Delta_k), \end{aligned} \quad (8)$$

where $I_k^I$ and $I_k^Q$ are the in-phase and quadrature-phase co-channel interference components, respectively and $\Delta_k = -\omega_c(\tau_k - \tau_0) + (\theta_k - \theta_0)$ is the phase shift difference between the $k$th user and the 0th user. The interference terms $J_{i,i'}$, $i \in \{2k, 2k+1\}$ and $i' \in \{0, 1\}$, are given by [29]:

$$\begin{aligned} J_{i,i'} = b_{i,-1}&[c_{i,i'}(\xi_k - L)(1-\nu_k) + c_{i,i'}(\xi_k + 1 - L)\nu_k] \\ &+ b_{i,0}[c_{i,i'}(\xi_k)(1-\nu_k) + c_{i,i'}(\xi_k+1)\nu_k], \end{aligned} \quad (9)$$

where $\xi_k = \lfloor\frac{\tau_k - \tau_0}{T_c} \mod L\rfloor$, $\nu_k = (\frac{\tau_k - \tau_0}{T_c} \mod L) - \xi_k$. The aperiodic cross-correlation function between the pair of spreading sequences $\{a_{km}\}_{m=0}^{L-1}$ and $\{a_{im}\}_{m=0}^{L-1}$ both having a length of $L$ chips is defined as:

$$c_{ki}(\xi) = \begin{cases} \sum_{m=0}^{L-1-\xi} a_{k,m} a_{i,m+\xi}, 0 \leq \xi \leq L-1, \\ \sum_{m=0}^{L-1+\xi} a_{k,m-\xi} a_{i,m}, -(L-1) \leq \xi < 0, \\ 0, \qquad |\xi| \geq L. \end{cases} \quad (10)$$

Equation 10 becomes the aperiodic auto-correlation function $c_{kk}(\xi)$ when $k = i$.

## III. BER Analysis

In this section, we investigate the BER performance of an asynchronous DS-CDMA system conditioned on the 0th user's I/Q spreading sequences, $\{a_{0,m}\}_{m=0}^{L-1}$ and $\{a_{1,m}\}_{m=0}^{L-1}$. Hence we have that $\{\Delta_k\}_{k=0}^{K-1}$ and $\{\nu_k\}_{k=0}^{K-1}$ are independently and uniformly distributed over $[0, 2\pi)$ and $[0, T_c)$, respectively, for the asynchronous system, $\{a_{i,m}\}_{m=0}^{L-1}, i = 2k, 2k+1$ are mutually independent and symmetrically Bernoulli distributed [30], i.e. we have $P\{a_{i,m} = \pm 1\} = \frac{1}{2}$ for random spreading sequences. Furthermore, the data bits $\{b_{i,m}\}_{m=-\infty}^{\infty}, i = 2k, 2k+1$ are also assumed to be mutually independent and symmetrically Bernoulli distributed.

In contrast to the performance analysis of [31], we investigate the average BER rather than the average Symbol Error Ratio (SER), since the average BER over of the in-phase and quadrature-phase branches can always be expressed as the average of the in-phase BER and the quadrature-phase BER without the assumption of independence between these two branches.

### A. Accurate Analysis

It has been shown in the context of BPSK modulation in [8] that the interference imposed by the $(K-1)$ interfering users would be mutually independent, if and only if it was conditioned on the spreading sequence of the 0th user. It is readily to show that this is also valid for our system using I/Q modulation.

We will analyze the average in-phase BER, $P_e^I$, of the 0th user. The average quadrature-phase BER, $P_e^Q$, of the 0th user may be derived in the same way. For the sake of simplifying the expression of $I_k^I$ in Equation 8, we define a set of $(2L+2)$ random variables $\{Y_{i,m}\}_{m=0}^{L}, i = 2k, 2k+1$ by:

$$Y_{i,m} = \begin{cases} b_{i,-1}a_{i,m-\xi_k+L}a_{0,m}, & m = 0, ..., \xi_k - 1, \\ b_{i,0}a_{i,m-\xi_k}a_{0,m}, & m = \xi_k, ..., L-2, \\ b_{i,-1}a_{i,L-1-\xi_k}a_{0,0}, & m = L-1, \\ b_{i,0}a_{i,L-1-\xi_k}a_{o,L-1}, & m = L. \end{cases} \quad (11)$$

Similar to [8], these random variables can be shown to be mutually independent and symmetrically Bernoulli distributed, if conditioned on $\{a_{0,m}\}_{m=0}^{L-1}$. Hence the in-phase interference component $I_k^I$ in Equation 8 can be rewritten as:

$$I_k^I = X_{2k}h_k\cos\Delta_k - X_{2k+1}h_k\sin\Delta_k, \quad (12)$$

where the random variables $\{X_i\}_{i=2k,2k+1}$ are defined as:

$$X_i = \sum_{m=0}^{L-2} Y_{i,m}\left[(1-\nu_k) + a_{0,m}a_{0,m+1}\nu_k\right] + Y_{i,L-1}\nu_k + Y_{i,L}(1-\nu_k), \quad (13)$$

and where the $(L-1)$ possible chip combinations of $\{a_{0,m}a_{0,m+1}\}_{m=0}^{L-2}$ can be categorized into two sets according to whether there is a chip value change or not. Let $B$ and $A$ denote the number of chip boundaries both with and without chip-value transitions within the 0th user's spreading sequence, respectively [8]. Then we have $A+B = L-1$ and $A-B = \sum_{m=0}^{L-2} a_{0,m}a_{0,m+1} = c_{0,0}(1)$.

Since $h_k$ is a Rayleigh distributed random variable as defined in Equation 5 and $\Delta_k$ is uniformly distributed in $[0, 2\pi]$, the random variables, $h_k\cos\Delta_k$ and $-h_k\sin\Delta_k$, are independently and identically Gaussian distributed with zero-mean and variance of $\sigma_k^2$ [30]. Hence $I_k^I$ is also Gaussian distributed conditioned on $X_{2k}$ and $X_{2k+1}$ and its conditional CF is given as follows:

$$\Phi_{I_k^I|X_{2k},X_{2k+1}}(\omega) = \exp\left[-\frac{1}{2}\left(X_{2k}^2 + X_{2k+1}^2\right)\sigma_k^2\omega^2\right] \quad (14)$$

Following a similar derivation to that in [1], the CF of $I_k^I$ conditioned on $B$ can be expressed as:

$$\Phi_{I_k^I|B}(\omega) = 2^{-2(L+1)} \sum_{d_1\in\mathcal{A}}\sum_{d_2\in\mathcal{B}}\sum_{d_3\in\mathcal{A}}\sum_{d_4\in\mathcal{B}} \binom{A}{\frac{d_1+A}{2}}\binom{B}{\frac{d_2+B}{2}} \times \binom{A}{\frac{d_3+A}{2}}\binom{B}{\frac{d_4+B}{2}} \sum_{l=0}^{15} \Phi_{I_k^I|\lambda_0,\lambda_1,\lambda_2}(\omega), \quad (15)$$

where the sets $\mathcal{A}$, $\mathcal{B}$ are defined as:

$$\begin{aligned} \mathcal{A} &= \{-A, -(A-2), ..., A-2, A\}, \\ \mathcal{B} &= \{-B, -(B-2), ..., B-2, B\}. \end{aligned} \quad (16)$$

The conditional CF $\Phi_{I_k^I|\lambda_0,\lambda_1,\lambda_2}(\omega)$ is defined as:

$$\Phi_{I_k^I|\lambda_0,\lambda_1,\lambda_2}(\omega) = \begin{cases} \exp\left(-\frac{1}{2}\lambda_0\sigma_k^2\omega^2\right), & \text{if } \lambda_1 = \lambda_2 = 0, \\ \frac{1}{\lambda_1\sigma_k^2\omega^2}\left[1 - \exp\left(-\lambda_1\sigma_k^2\omega^2\right)\right]\exp\left(-\frac{1}{2}\lambda_0\sigma_k^2\omega^2\right), & \\ & \text{if } \lambda_1 \neq 0, \lambda_2 = 0, \\ \frac{\sqrt{\pi}}{\sigma_k\omega\sqrt{2\lambda_2}}\exp\left[\frac{1}{2}\sigma_k^2\omega^2\left(\frac{\lambda_1^2}{\lambda_2} - \lambda_0\right)\right] & \\ \times\left\{\operatorname{erfc}\left(\frac{\lambda_1\sigma_k\omega}{\sqrt{2\lambda_2}}\right) - \operatorname{erfc}\left[\sigma_k\omega\sqrt{\frac{\lambda_2}{2}}\left(1 + \frac{\lambda_1}{\lambda_2}\right)\right]\right\}, & \\ & \text{if } \lambda_2 \neq 0, \end{cases} \quad (17)$$

where the complementary function $\operatorname{erfc}(x)$ is defined as in [28] and the coefficients $\lambda_0$, $\lambda_1$ and $\lambda_2$ are given by:

$$\begin{aligned} \lambda_0 &= (d_1 + d_2 + Y_{2k,L})^2 + (d_3 + d_4 + Y_{2k+1,L})^2, \\ \lambda_1 &= (d_1 + d_2 + Y_{2k,L})(-2d_2 + Y_{2k,L-1} - Y_{2k,L}) \\ &\quad + (d_3 + d_4 + Y_{2k+1,L})(-2d_4 + Y_{2k+1,L-1} - Y_{2k+1,L}), \\ \lambda_2 &= (-2d_2 + Y_{2k,L-1} - Y_{2k,L})^2 \\ &\quad + (-2d_4 + Y_{2k+1,L-1} - Y_{2k+1,L})^2. \end{aligned} \quad (18)$$

Since the co-channel interference contributions $\{I_k^I\}_{k=1}^{K-1}$ conditioned on $B$ are mutually independent [1], [8], the CF of the total in-phase interference $I^I = \sum_{k=1}^{K-1} I_k^I$ conditioned on $B$ is given by:

$$\Phi_{I^I|B}(\omega) = \prod_{k=1}^{K-1} \Phi_{I_k^I|B}(\omega). \quad (19)$$

Hence the in-phase BER of the 0th user conditioned on $B$ can be shown to be [1]:

$$P_{e|B}^I = \frac{1}{2} - \frac{\sigma_0 L}{\sqrt{2\pi}}\int_0^\infty \Phi_{I|B}(\omega)\Phi_{\eta^I}(\omega)\exp\left(-\frac{1}{2}\sigma_0^2L^2\omega^2\right)\mathrm{d}\omega, \quad (20)$$

where $\Phi_{\eta^I}(\omega)$ is the CF of the in-phase noise $\eta^I$:

$$\Phi_{\eta^I}(\omega) = \exp\left(-\frac{1}{2}\sigma_{\eta^I}^2\omega^2\right). \quad (21)$$

Then the average in-phase BER of the 0th user is obtained by averaging $P_{e|B}^I$ over all spreading sequences:

$$P_e^I = 2^{-(L-1)}\sum_{B=0}^{L-1}\binom{L-1}{B}P_{e|B}^I. \quad (22)$$

Following the same approach, we may conclude that the average quadrature-phase BER, $P_e^Q$, of the 0th user has the same value as the average in-phase BER, $P_e^I$. Finally, we arrive

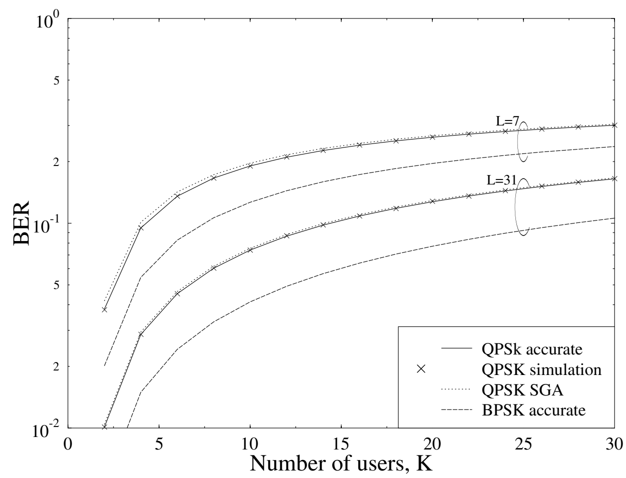


Fig. 1. The BER versus the number of users $K$ in an asynchronous DS-CDMA system using random spreading sequences and QPSK modulation. The length of the random spreading sequences is $L = 7$ and 31, respectively, the average power of all users at the receiver is equal and the background noise is ignored, i.e. when $\gamma_{\mathrm{SNR}} = \infty$.

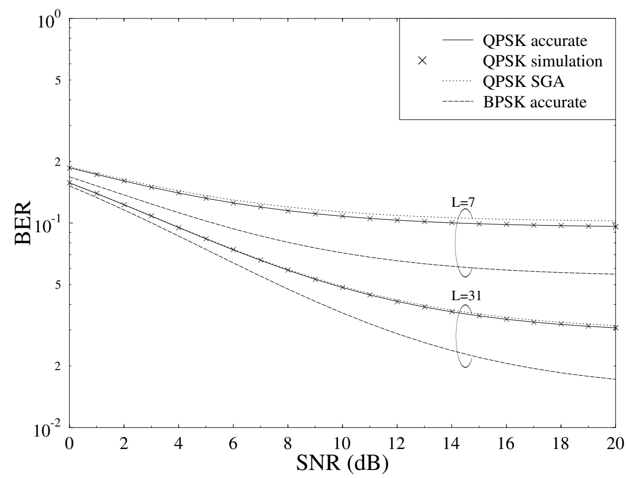


Fig. 2. The BER versus per-bit SNR in an asynchronous DS-CDMA system using random spreading sequences and QPSK modulation. The length of the random spreading sequences is $L = 7$ and 31, respectively, the average power of all users at the receiver is equal. The number of users is $K = 4$.

at the overall average BER, $P_e$, averaged over both the in-phase and quadrature-phase branches of the 0th user, yielding:

$$P_e = \frac{1}{2}(P_e^I + P_e^Q) = P_e^I. \tag{23}$$

### B. Standard Gaussian Approximation

Owing to its simplicity, the SGA is widely used for performance analysis, when the number of interferers is sufficiently high, where the MAI is assumed to be Gaussian owing to the CLT [30].

The variance of the interference $I_k^I$ in Equation 12 can be obtained by averaging $I_k^I$ over $\nu_k$, $h_k$, $\Delta_k$ and all spreading sequences, yielding:

$$\sigma_{I_k^I}^2 = \frac{4}{3} L \sigma_k^2 \tag{24}$$

Hence the average BER approximated by SGA can be shown to be:

$$P_e \approx \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + \frac{\sigma_{\eta^I}^2}{\sigma_0^2 L^2} + \frac{4}{3L}\sum_{k=1}^{K-1}\frac{\sigma_k^2}{\sigma_0^2}}}\right). \tag{25}$$

## IV. Numerical Results

We will compare the results obtained by our accurate analysis provided in Section III-A, by the SGA of Section III-B, by the BPSK system of [1] and those of our simulations described in this section.

Figure 1 shows that the results obtained by our accurate analysis exactly match those obtained by simulations for two different-length random spreading sequences, when using $L = 7$ and 31. However, the SGA over-estimates the BER, especially in the scenario, where there is a low number of interfering users and when short spreading sequences are used. The BER of QPSK system is higher than that of BPSK systems due to the cross-talk between the in-phase and quadrature-phase branches.

Similar to Figure 1, Figure 2 also shows that the results obtained by our accurate analysis match those obtained by simulation for both different-length random spreading sequences, i.e. for $L = 7$ and 31. By contrast, the SGA slightly over-estimates BER, particularly, when the SNR is high and where short spreading sequences are used. The BER of QPSK systems is higher than that of BPSK systems due to the cross-talk between the in-phase and quadrature-phase branches.

## V. Conclusion

In this paper we investigated the average BER performance of an asynchronous DS-CDMA system using I/Q modulation and random spreading sequences, when communicating over Rayleigh channels. A new closed-form expression was derived for the conditional CF of the MAI. Furthermore, an accurate expression based on the CF approach was provided for calculating the average BER of the system, which requires only a single integration. The accuracy of our accurate BER expression was confirmed by our simulation results for various spreading sequence lengths. By contrast, the limited accuracy of the SGA was also demonstrated, which becomes more prevalent when a low number of interferers is encountered and short spreading sequences are used.


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