A VHDL-AMS BASED TIME-DOMAIN SKIN DEPTH MODEL FOR EDGE COUPLED LOSSY TRANSMISSION STRIPLINE

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Abstract

This contribution presents a time-domain model of the skin depth effect in a lossy transmission line. The model was developed and implemented in VHDL-AMS for a two-wire edge coupled line but it is general and can be used for other types of lossy transmission line and high-frequency applications with skin depth effect. The salient feature of the model is the use of signal variation rate instead of frequency in the signal-dependent resistance that models the skin depth losses. VHDL-AMS simulation experiments are presented to validate the model.

1   Introduction

A novel, signal-dependent time-domain model of buried stripline has been developed and implemented in VHDL-AMS. The model successfully captures the frequency dependent skin effect as observed at frequencies into the GHz region. Also, efficient simulations were achieved with our time-domain model applied to a frequency-domain problem. Digital (pulses) and analogue (sinusoidal) excitations were examined. Many well known circuit simulators have their own built in models of transmission lines of various types. It is in common agreement in industrial and academic worlds that these models become inaccurate as interconnect speeds are pushed into the GHz region. One of the major frequency dependent effects these models fail to include is the skin effect [6]. The skin effect is most easily described as an observed increase in resistance of the transmission line with the square root of frequency. It is caused by the self-inductance of the conductor, which at high rates of current change causes an increase in the inductive reactance, thus forcing the current to the outer regions of the conductor. This is effectively a reduction in the area of the line to a skin depth and hence an increase in resistance. This paper describes a VHDL-AMS [10] implementation of the time-domain hierarchical model which hopes to accurately model coupled transmission lines buried in dielectric. Experiments were carried out using various stimuli through the SystemVision2004 simulator from Mentor Graphics.
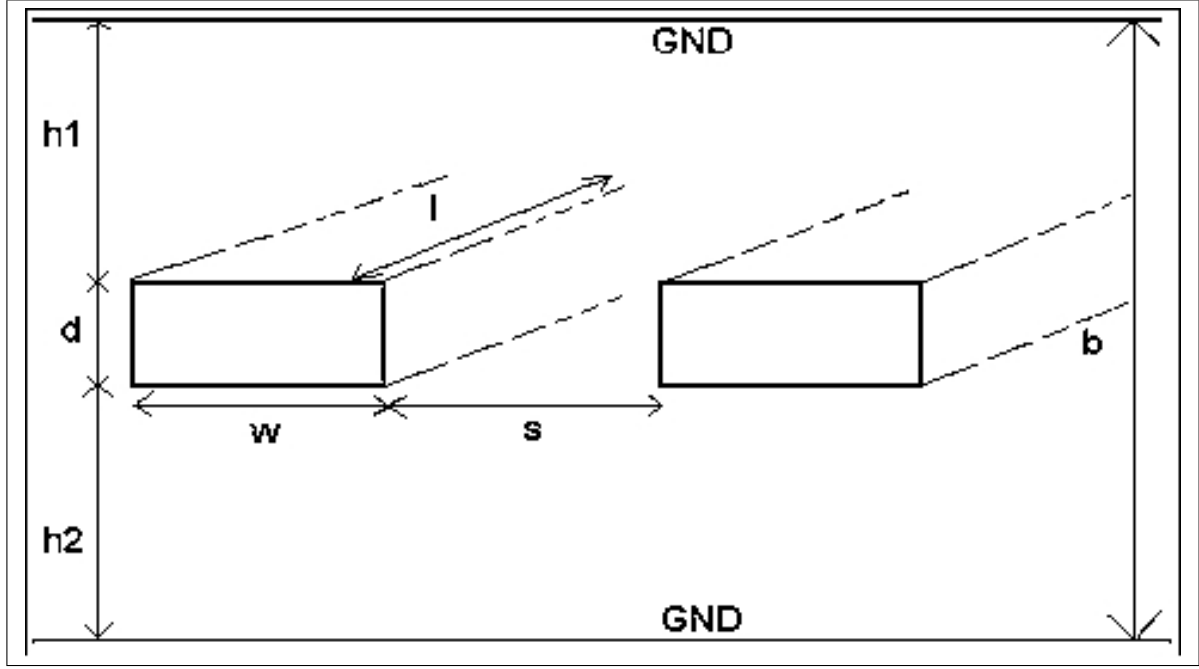
FIGURE 1: Edge coupled stripline

2   Skin Depth Effect Modelling

The edge-coupled buried transmission line under investigation is illustrated in Figure 1. At low frequencies, typically up to a few kHz, the current is distributed nearly uniformly throughout the conductor. The changing current causes a time varying magnetic field according to Ampere's law which, according to Faraday's law will induce an EMF in the conductor such that a displacement current is generated which opposes the current flow in the centre of the conductor and reinforces it near the outside of the conductor. At high frequencies this results in the current bunching into the outer regions of the conductor as this is the path of least inductance. The region in which the current travels at high frequency is called the skin depth and is commonly given [7] by :

$$\delta = \sqrt{\frac{2\rho}{2\pi f U_0 U_r}} \tag{1}$$

Where δ is the skin depth, ρ is the resistivity of copper and f is the maximum frequency at which the line will be operated. Equation 1 is combined with a resistance formula to give the resistance of a line at a certain frequency with a certain skin depth for a rectangular stripline:

$$R_{AC} = \frac{\rho L}{A} \tag{2}$$

Where L is the length of the line and A is the cross sectional area of the line.
The skin depth takes hold for the type of material used under consideration here at [7]:

$$f_\delta = \frac{1}{2\pi} \frac{8\rho}{U_0 U_r} \left(\frac{w+t}{wt}\right)^2 \tag{3}$$

Where f_δ is the skin effect onset frequency, w is the track width and t is the track thickness. A number of suggestions for skin effect modelling were made[12, 9] but mostly in frequency domain. In a recent work [9] skin and proximity effects are modelled in time-domain using the volume filament method and extending it by using a rule based R L modelling and reduction method. However the work only claims the model works up to dimensions of on chip wiring and not necessarily board level wiring possibly due to computational expense. Also, there does not seem to be as yet an efficient VHDL-AMS implementation of a lossy transmission line which would include essential effects such as skin effect, coupling and work from zero frequency to multi-GHz.

3 Edge Coupled Lossy Transmission Line Model

It is accepted by many authorities on signal integrity that in order to model high frequency transmission lines with sharp transitions the lumped components need distributing along the line into sections comprising RCLG. The rule is that the delay of the signal through each section should be no more than 1/10 the signal rise time. The required number of sections is therefore estimated by the following equation:

$$Sections = \frac{Delay\ of\ line}{Delay\ of\ Section} = \frac{\left(\frac{L}{C/\epsilon_r}\right)}{0.1t_r} \tag{4}$$

Where C is the speed of light, ϵ_r is the dielectric constant and t_r is the rise time of the signal (given here as 1/8 the period).

3.1 Lumped Sections

Figure 2 shows a two-wire coupled transmission line where the coupling is capacitive and inductive. Also shown are two sources in antiphase which is how the lines under investigation are driven, in a very similar manner to Low Voltage Differential Signalling (LVDS) [11].

Having ascertained that each section comprises RCLG elements and a mutual inductance M between the lines, methods are required for calculating their values. Methods and equations used for this purpose follow. In the absence of a buried stripline for test, a 2D field solver was used to generate matrices of RCLG with which the answers could be verified against. The DC resistive component can be found for each section using:

$$R_{dc} = \frac{\rho L}{wd} \tag{5}$$

As previously stated, the current travels within a skin depth as frequency increases, this skin depth is given by 1. DC resistance is used when there is slow or no change in the signal value. It should be pointed out that for a sine wave as it assumes a constant rate of change over a period, the rate of change is slower than that of real data being sent with steeper edges being launched into the line hence higher values of AC resistance are expected for real data. The self inductance of each line is calculated by multiplying the delay of the line in seconds by the odd
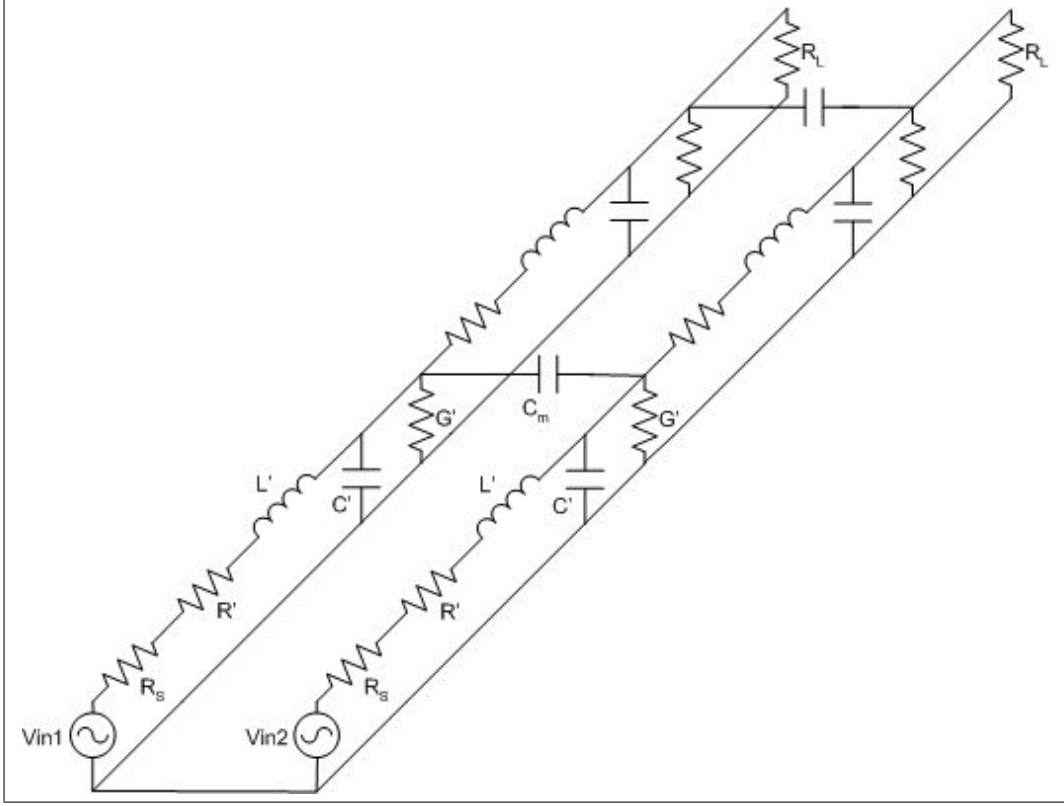
FIGURE 2: Lumped element sections of a coupled transmission line.

mode characteristic impedance where $Z0_{odd\_edge}$ is the odd mode characteristic impedance for edge coupled lines [8]:

$$Z0_{odd\_edge} = \frac{60}{\sqrt{\epsilon_r}} \ln \left(\frac{1.9b}{0.8w + t} \right) \tag{6}$$

Mutual inductance can be found when any two of differential impedance, mutual capacitance, differential capacitance, sum of capacitances, differential inductance or sum of inductances are known about the lines of interest using Equation 7.

$$Z0_{odd} = \sqrt{\frac{L}{C}} \tag{7}$$

The mutual inductance is usually calculated by considering the 2x2 matrices of inductance and capacitance and taking the differences of the 1st rows of each to calculate the odd-mode differential characteristic impedance [3, 4, 5, 1].

$$Z0_{odd\_diff} = \frac{3.30894 \times 10^{-9} \sqrt{\epsilon_r}}{C_{total\_per\_line}} \tag{8}$$

Where ϵ_r is dielectric constant and $C_{total\_per\_line}$ is the total capacitance of a single line. Equation (7) was rearranged for C as Z and L could be accurately determined by other means. Mutual capacitance is given [2] as:

$$a = \left(\tanh \left(0.5\pi \frac{s}{b} \right) \right)^{-1}$$

$$C_{mut} = \frac{2\epsilon_0 \epsilon r}{\pi} \left(1 + 2.66 \frac{t}{b} \right) \ln a \qquad (9)$$

It should be noted at this point that the mutual capacitances have a negative sign. For edge coupled lines, with the odd mode impedance and inductance known, the line capacitance to ground can be found. The sum of capacitances to ground and mutual capacitance is found and used with the odd mode impedance to find the difference in inductances [1]. With the line inductance and difference in inductances known, the mutual inductance is a simple subtraction of line difference in inductances from line inductance.

4 VHDL-AMS Time-Domain Model of Skin Depth Effect

The model is a longitudinally distributed lumped section edge coupled stripline between dual ground planes in FR4. Effects modelled include skin effect, capacitive and inductive coupling along with proximity effects, Ohmic losses, reflections and dielectric leakage. The model consists of the generation module (top-level module) and the netlist module (section module) as explained below.

4.1 Generation Module

The top-level module uses a VHDL-AMS *generate* statement with the VHDL-AMS nature *electrical_vector*(*n downto* 0) to create the transmission line sections made from the lowest netlist level which includes the Signal Dependent Resistance (SDR). This is done by declaring a vector of terminals and connectng them up using the generate statement. Figure 3 shows code from the generation module which creates and links the number of sections (netlists) required as passed from the testbench to the ports (terminals) in the electrical vector.

4.2 Netllist Module

The netlist module contains all lumped components for a single section of coupled transmission line. These components are: two Signal Dependent Resistors (SDR) in series, two conductances to ground, two capacitors to ground, one mutual coupling capacitor and two inductors in series which also contain mutual inductances. This is seen in the centre section of Figure 2.

In the time domain modelling of real data there is no concept of frequency in that there are no cyclic continuous patterns that a period can be ascertained from. This frequency would be that used for calculating values for frequency dependent components such as the resistive elements

here. This is difficult in time domain unless the signal is a pure tone sine wave with constant frequency conveying zero information. For long periods of 1's or 0's, the line is effectively at DC so allocating resistances to all sections for the maximum frequency of AC signal would lead to a large over estimate of loss on the line, unless data was always $101010\ldots$. This model is capable of simulating long series of zeros or ones i.e. non sinusoidal stimulous.

Using a method of trigonometric identities and back substitution the frequency term was removed from the skin effect equation as detailed below:

$$V = A\sin(2\pi f\, t)$$

$$\frac{dV}{dt} = A2\pi f\cos(2\pi ft) \tag{10}$$

Where A is the signal amplitude, f is frequency and t is time.

Using identity:

$$\cos^2\alpha + \sin^2\alpha = 1$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha} \tag{11}$$

And:

$$V = A\sin\alpha$$

$$\sin\alpha = \frac{V}{A} \tag{12}$$

The following estimate for the signal rate is obtained:

$$\frac{dV}{dt} = 2\pi f\sqrt{A^2-V^2} \tag{13}$$

Leaving the final equation for the skin depth with frequency components eliminated:

$$\delta = \sqrt{\frac{2\rho\sqrt{A^2-V^2}}{U_0 U_r dV/dt}} \tag{14}$$

```
sections: for i in 1 to (total_sections) generate
    s_rest: entity work.netlist
        generic map(
            generics => values
            ...
            )
        port map(
            ports => ports
            ...
            );
    end generate;
```

FIGURE 3: VHDL-AMS generate statement in the generation module.

When modelling, if such terms as $\sqrt{A^2 - V^2}$ occur in the numerator, they could effectively cause the whole equation to go to zero if V = A, techniques such as adding a very small number to the result of such terms are used to stop things either tending to zero or infinity. This is done as a skin depth of zero means infinite resistance and will clearly never happen at the datarates in question. With the effective skin depth now calculable in a time-domain simulation, it could be incorporated into the resistance equation where the area term is replaced by the skin depth area for a rectangular cross section line:

$$R_{AC} = \frac{\rho L}{2\delta(W + D - 2\delta)} \tag{15}$$

These equations exist in the SDR component of the model where a VHDL-AMS process is run which is sensitive to significant changes in rate of change of signal across the resistor and to the differences between signal values currently and previously evaluated every time the process runs. This sensitivity to significant changes in signal rate protects the model against $\frac{dV}{dt} = 0$ which would effectively give a DC condition but equation (14) for skin depth would tend to infinity and cause the model to fail. The process calculates a skin depth and associated resistance for the resistor which is passed out of the process and incorporated into an Ohm's law expression for voltage dropped across the resistor. Faster rates of change give smaller skin depths which give higher resistance. Figure 4 shows the algorithm used in the SDR. The skin depth is given an initial value calculated by using the trace depth to give a near DC value of resistance and avoid large discontinuities in resistance.

Two thresholds are defined which are measures of the voltage now compared with the last voltage at re-evaluation and the present rate of change of voltage compared with the last rate of change of voltage at the last re-evaluation. This allows an efficient simulation to run because the thresholds can be used as a resolution control. Figure 5 contains VHDL-AMS code which shows both thresholds.

As can be seen from Figure 5, the boolean threshold signals v_thresh and vdot_thresh receive either TRUE or FALSE dependent on the evaluation of the condition following them. The X'ABOVE(Y) attribute becomes true whenever X-Y > Zero. The X'DOT attribute returns the current rate of change X. Figure 5 combines these attributes. A process is sensitive to v_thresh and vdot_thresh. Each time a threshold is breached, the value of resistance for that section is calculated in a VHDL-AMS process. The present value of voltage is recorded, as is the instantaneous rate of change of voltage. These are used as comparisons after the process is finished with the new voltage and new rate of change of voltage to determine if a threshold has been breached and the value of resistance needs recalculating again.

A new SDR is generated for each section, it was required to add a multiplicative factor of 2.78 to Equation (15) which will now be explained. The only unknown in determining the skin effect resistance was the amplitude, A of the signal due to the attenuation in each section. The problem here is that if it is assumed in each section that A is the same magnitude as when launched into the line, each section's resistance would be calculated as if the maximum amplitude of the signal had reached it, and there had been no losses previous to the section in question. Hence according to the skin depth equation, a larger skin depth and hence an underestimate of the resistance would be given. In the model, A is given an initial estimated value calculated from the DC offset and amplitude of the signal (dependant on signalling type such as infiniband for example), a relationship was found linking zero DC offset of the signal
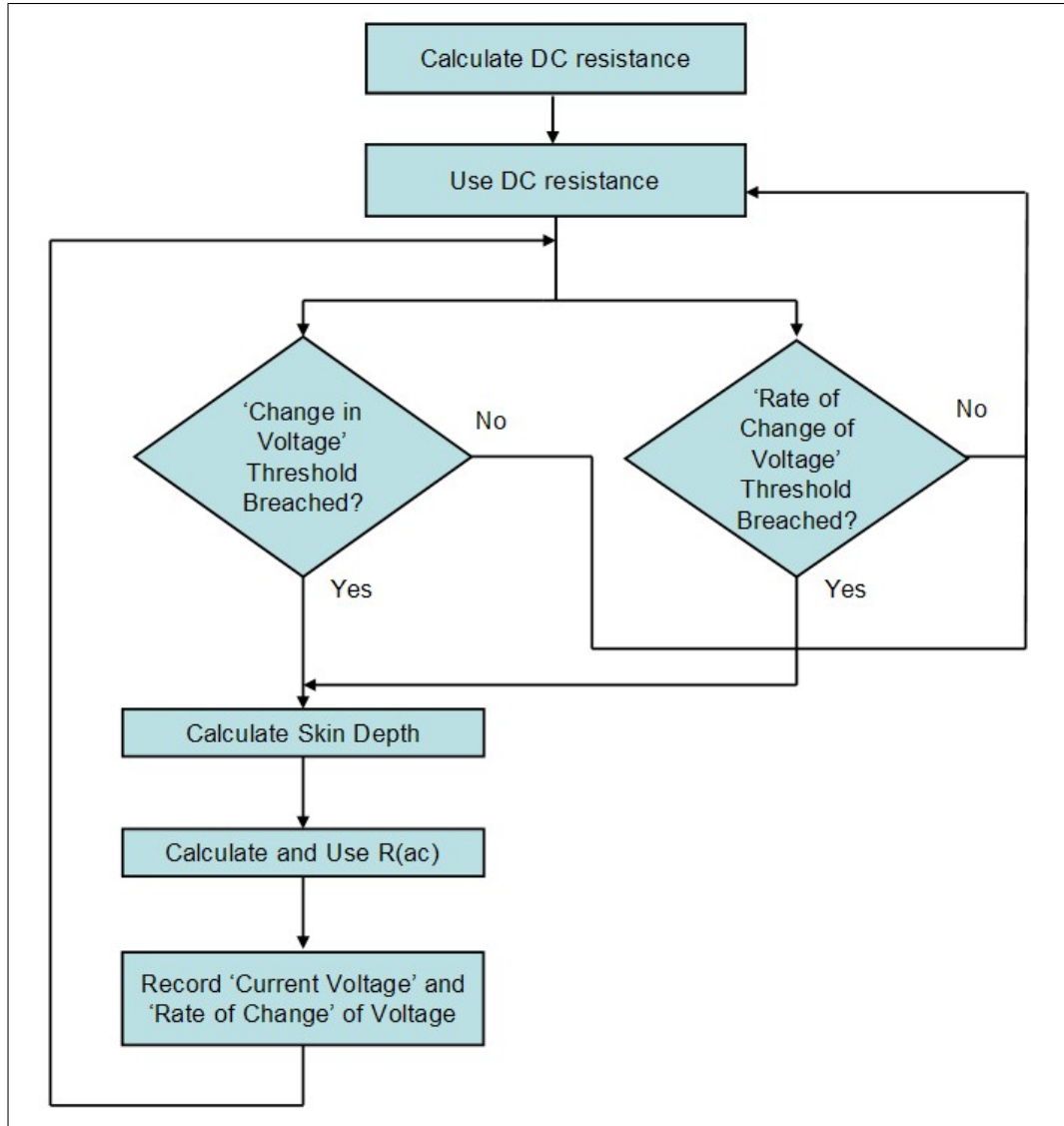
FIGURE 4: Algorithm for SDR calculation.

v_thresh	<=	not v_sense'above(v_last-dv) or v_sense'above(v_last+dv);
vdot_thresh	<=	not v_sense'dot'above(vdot_last-d_vdot) or
		v_sense'dot'above(vdot_last+d_vdot);

FIGURE 5: Threshold control statement.

and associated AC resistance to an arbitrary DC offset and was found to be approximately a multiplication factor of 2.78. This restored the near true value of R_{AC}. Any minor errors in this adjustment would be spread across 168 sections and be of the order of thousandths of an Ohm in each section hence would be of negligible effect. It was decided that a fairly coarse estimate (over time) of resistance would be sufficient. This would save CPU time in simulation and the new values of resistance would remain a constant until the next evaluation so the line would still have a raised (lowered) resistance which would follow rates of change of signal along the line. Minor incremental adjustments of resistance would have negligible effect on final output of the line. There are high frequency effects such as proximity and ground plane return effects not considered in this model. The proximity effect is where in odd mode the current bunches on the sides of the lines nearest each other and ground plane return is where the returning current bunches under the line. Both effects slightly raise the line resistance. Another effect not considered is the frequency dependent dielectric absorption. Losses due to this effect are typically negligible compared with the skin-depth effect losses.

5   Experiments and Results

The VHDL-AMS testbench takes in transmission line dimensions and material properties. It also takes in the frequency of operation (for a sine wave stimulus and for calculation of sections) and the amplitude and offset of the stimulus (again for sine wave stimulation). The testbench calculates the number of sections, conductance, capacitances and inductances per metre using methods discussed before passing the values down to an intermediate generation module. The testbench also passes frequency, amplitude and offset information to two sine wave sources in antiphase to mimic odd mode Low Voltage Differential Signalling (LVDS) [8].

All sources in the tests were LVDS-like but with a 1.2V offset. A configurable piecewise linear (PWL) voltage supply was made in VHDL-AMS with a 0-20, 20-80 and 80-100 percent of amplitude description over time. The waveform is specified as having a rise and fall time of 1/8 the period. The purpose of the sinusoidal simulation is to ascertain whether the simulation gave the same results for an assumed constant frequency, hence a constant skin depth could be calculated to be applied throughout the simulation to each section. The simulation was run twice, once with all frequency dependent effects modelled and again with a constant value of resistance based on a constant frequency. From analysing Figure 6 with waveform measurement software could be seen that there was on average less than 1mV difference between an assumed fixed frequency and hence fixed skin depth and the signal dependent parameter model for a sinewave. There are large differences in steady state voltages because for a fixed frequency, the model calculates the initial voltages based on a fixed high frequency and so the line appears with higher impedance. For the signal dependent case, the line is initially at DC and so the potential divider action is not so prominent. For the pulsed simulation a 1010 pattern was used and the results are shown in Figure 7 and 7. Figure 7 shows a piecewise linear input and its resulting output in the top section. Clear attenuation and rounding of the signal can be seen in the output. The lower section of the figure shows the resistance with skin effect, proximity and ground return effects are roughly estimated and also taken into account. Both parts show data for the first and last sections indicating there is increased resistance (in series in each section) on transitions. A transition is a high rate of change and so gives smaller skin
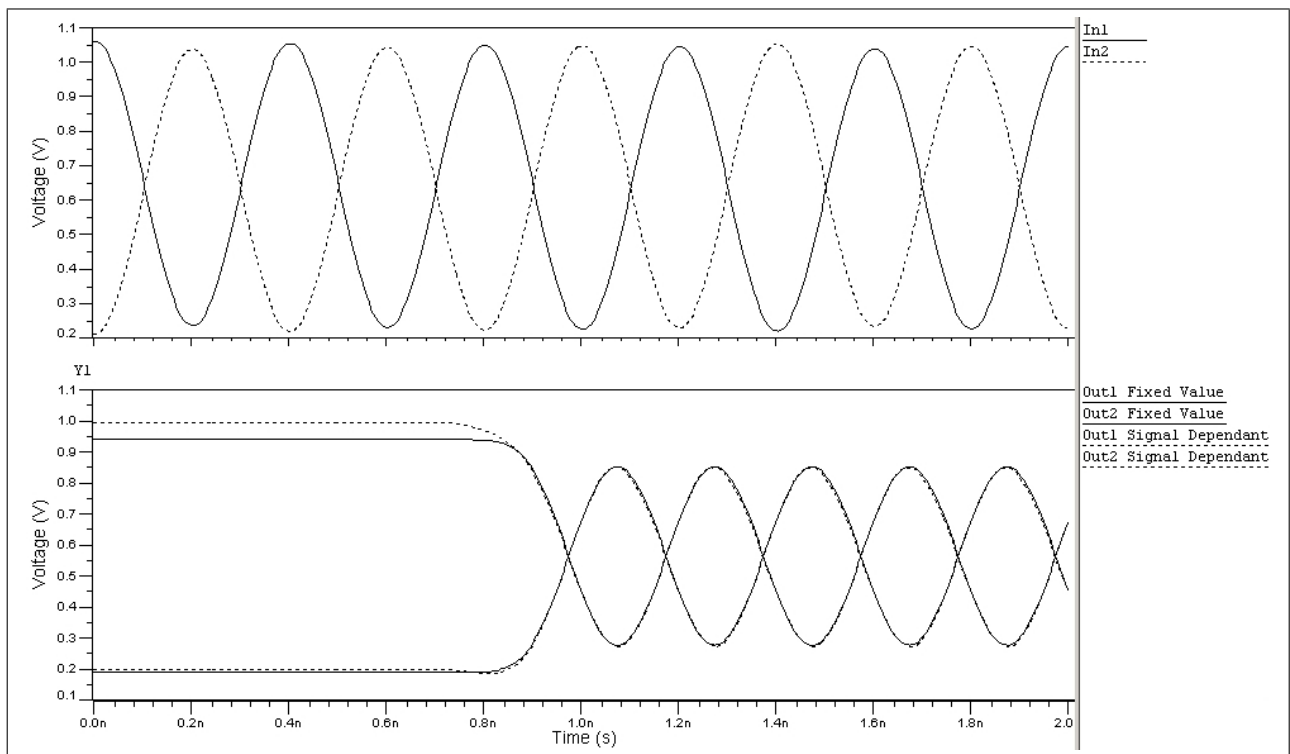
FIGURE 6: Fixed frequency vs. signal dependant parameter effects on signal in edge coupled line.
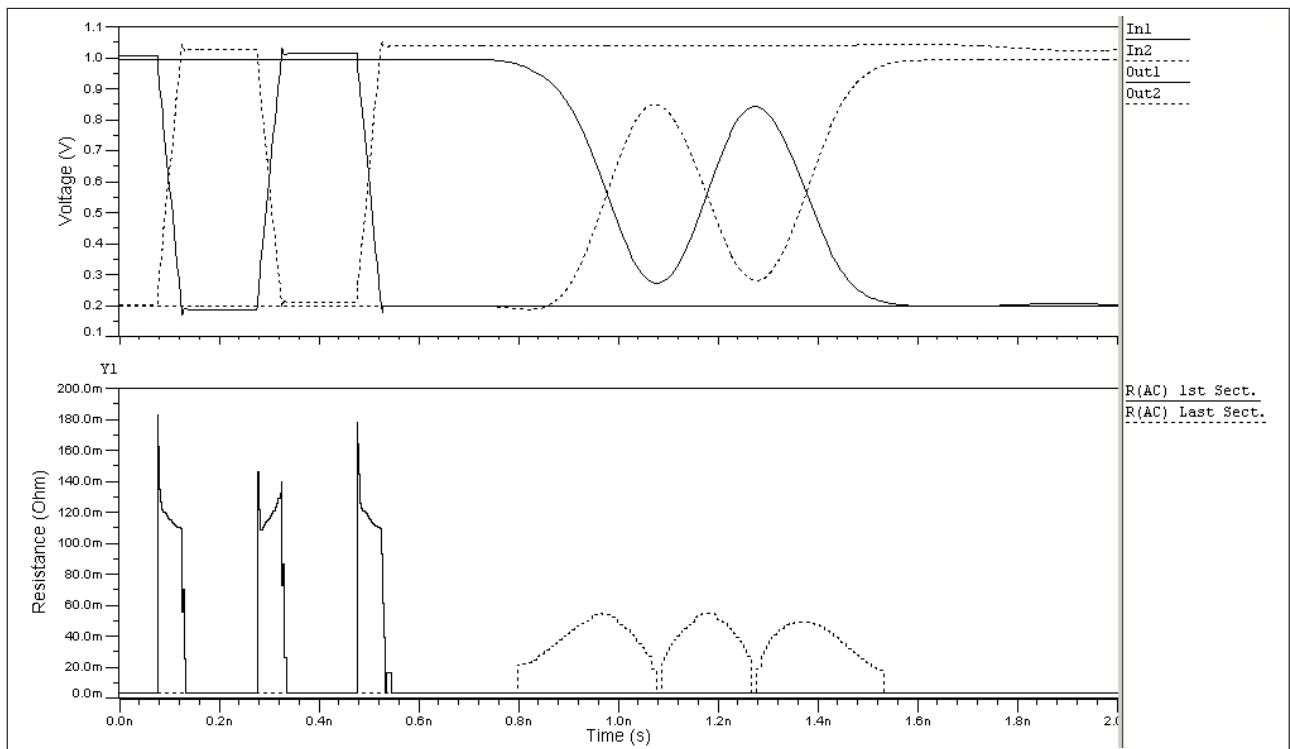

FIGURE 7: Piece Wise Linear input and output with associated skin effect resistances for edge coupled line.

depth. In the first sections the shape of the value of resistance and conductance are such that they follow the PWL discretisation of the input signal. The values for resistance are lower at the output because the signal has been attenuated so there will be a slower rate of change in signal across each section. There is an obvious increase in rise time of the signal also. Figure 8 shows results for the same stimulus but for fixed frequency and signal dependent models. The outputs for both are shown in the lower section.

There is an obvious difference in initial and end voltages for reasons discussed above. For realistic data containing many patterns of 1's and 0's losses may traditionally be overestimated or worse still the changing impedance of the line due to skin effect may cause unwanted crosstalk or reflections thus giving unforeseen signal integrity issues. This frequency dependent model should allow any data pattern to be accurately and efficiently modelled.

6   Conclusion

A novel time-domain signal dependent skin effect time domain model of edge coupled transmission line was proposed and successfully implemented in VHDL-AMS. The pulsed simulation results showed the model to simulate signal dependent behaviour, with frequency dependent resistance changing with frequency. The sinusoidal results showed that when a sine wave was used, the models matched closely. Work has been conducted and similar results have been found for broadside coupled lines. The work conducted here is in conjunction with Xyratex Technology Ltd, a network and storage solutions company based in the UK associated with a fast switching product.
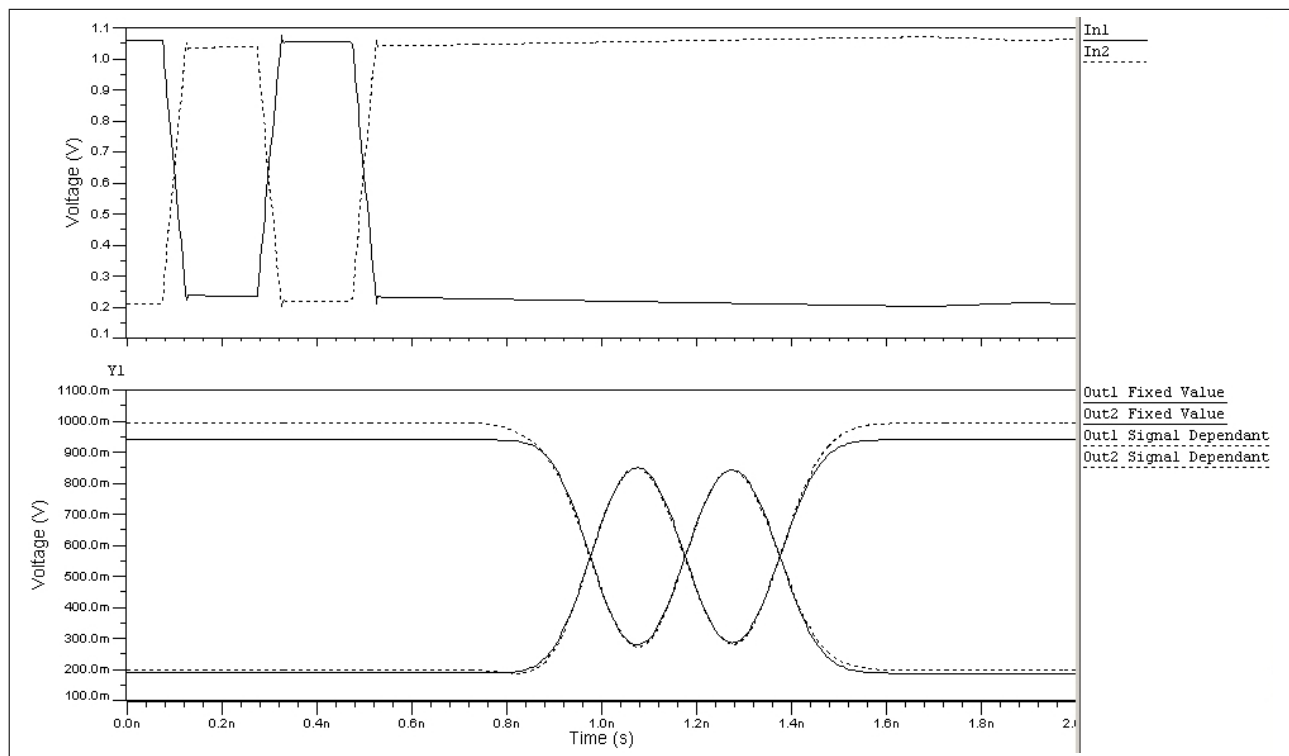

FIGURE 8: Fixed frequency vs. sig. dependant effects on signal in edge coupled line.

References

[1] P. Clayton. *Analysis of Multiconductor Transmission Lines.* John Wiley and Sons, Inc., 1994.

[2] S.B. Cohn. Shielded Coupled-Strip Transmission Line. *Microwave Theory and Techniques, IEEE Transactions*, 3:29, 1955.

[3] S.B. Cohn. Characteristic Impedances of Broadside-Coupled Strip Transmission Lines. *Microwave Theory and Techniques, IEEE Transactions*, 8:633, 1960.

[4] S.B. Cohn. Thickness Corrections for Capacitive Obstacles and Strip Conductors. *Microwave Theory and Techniques, IEEE Transactions*, 8:638–644, 1960.

[5] S. Farleigh. Broadside Coupled Strip Transmission Line. http://www.mathcad.com/Library/LibraryContent/MathML/broad_reference.htm. cited 07-Mar-2005.

[6] J.D. Jackson. *Classical Electrodynamics. 3rd ed.* John Wiley and Sons, Inc., 1999.

[7] H.W. Johnson. 2-d quasistatic Field Solvers. EDN.com. http://www.edn.com/contents/images/159699.pdf. cited 27-Sept-2001.

[8] National Semiconductor's LVDS Group. LVDS Owner's Manual. LVDS Owner's Manual. http://www.lvds.national.com. cited 24-Oct-2004.

[9] M. Shizhong and Y.I. Ismail. Modeling skin and proximity effects with reduced realizable rl circuits. *Very Large Scale Integration (VLSI) Systems, IEEE Transactions*, 12:437, 2004.

[10] IEEE Computer Society. IEEE draft standard VHDL-AMS. Language Reference Manual.

[11] TIA subcommittee. Tr30.2. Electrical Characteristics of Low Voltage Differential Signalling (LVDS) Interface Circuits. http://www.t10.org/ftp/t10/document.95/95-268r0.pdf. cited 20-Feb-2005.

[12] R.L. Wigington and N.S. Nahman. Transient analysis of coaxial cables considering skin effect. *IRE*, 1957.