Exact BER Calculation of Asynchronous DS-CDMA Systems Communicating over Hoyt Channels

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***Abstract*— An asynchronous binary DS-CDMA system using random spreading sequences is considered in flat Hoyt fading channels. A new closed-form expression is derived for the conditional characteristic function of the multiple access interference. The exact average BER is expressed as a single numerical integration based on the characteristic function approach. The numerical results obtained from our exact BER analysis are verified by our simulation results and also compared to those obtained by the standard Gaussian approximation.**

I. Introduction

Code Division Multiple Access (CDMA) has been one of the most successful radio access techniques since the 1990s and Direct Sequence (DS) CDMA has been integrated into the third generation mobile systems. The Bit Error Ratio (BER) performance of various DS-CDMA systems has been extensively studied.

For the sake of computational simplicity, the most widely used approach of calculating the average BER of DS-CDMA systems is assuming that the multiple access interference (MAI) is Gaussian distributed or conditional Gaussian distributed based on the Central Limit Theorem (CLT). Various Gaussian approximation techniques have been proposed, such as the Standard Gaussian Approximation (SGA) [1]–[14], the Improved Gaussian Approximation (IGA) [1], [5], [8], [9], [11], [12], [14]–[16], the Simplified IGA (SIGA) [1], [5], [8], [10], [12], [14], as well as the Improved Holtzman Gaussian Approximation (IHGA) [10].

However, the accuracy of various Gaussian approximation techniques has long been criticized [11], especially when the number of users is low and short spreading sequences are used. Hence several exact BER evaluation techniques have also been developed without assuming a Gaussian MAI distribution, such as the series expansion of [11], [12], [17]–[19], or the employment of Moment Generating Functions (MGF) [20] and Characteristic Functions (CF) [1], [2], [10], [14], [17]. These techniques typically achieve more accurate BER evaluation at the cost of a high computation complexity.

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In the existing literature, most results are reported for the BERs of DS-CDMA systems communicating over Additive White Gaussian Noise (AWGN) channels [4]–[9], [11], [12], [14], [15], [17]–[19], [21]–[25], and a few studies also considered Rayleigh channels [1], [11] as well as Nakagami-m channels [2], [3], [10], [16], [20].

As far as the authors are aware, there are no studies on the exact BER analysis of asynchronous DS-CDMA systems using random spreading sequences communicating over Hoyt channels. The Hoyt distribution of Figure 1, also referred to as the Nakagami-q distribution [26], [27], was originally used for modeling radio channels subject to strong ionospheric scintillation, such as satellite links [28], [29]. Recently, it has been used more frequently as one of the important models for the statistical description of fading mobile radio channels [28], [30]. The Rayleigh distribution may be regarded as a special case of the Hoyt distribution [28], [30]. *The contribution of this paper is that we provide an exact BER expression, which requires only a single numerical integration with the aid of hypergeometric functions of several variables [31], [32].*

This paper is organized as follows. In Section II a general asynchronous DS-CDMA system using BPSK modulation communicating over Hoyt channels is presented. Then in Section III its exact BER performance using random spreading sequences is investigated based on the characteristic function approach. Our numerical results are presented in Section IV and finally our conclusions are provided in Section V.

II. System Model

We consider a general asynchronous BPSK modulated DS-CDMA system communicating over a Hoyt fading channel and assume that there are K simultaneously transmitting users. Binary random spreading sequences having L chips and a rectangular chip waveform are employed.

A. Hoyt (Nakagami-q) Distribution

If a complex random variable $\widetilde{h} = he^{j\varphi}$ is Hoyt distributed, then the Probability Density Function (PDF) of its modulus h

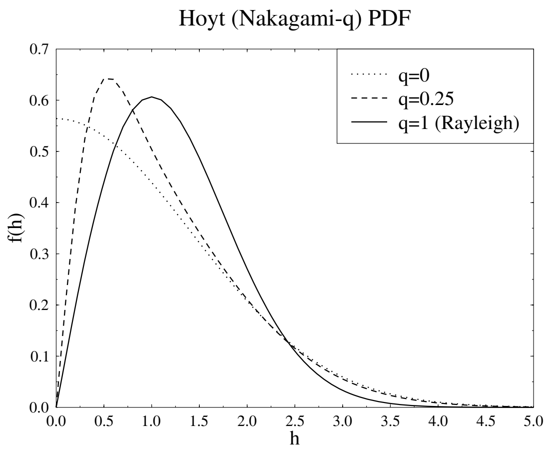

Fig. 1. The PDF of the Hoyt distribution. The Hoyt fading parameter is $q = 0$, 0.25 and 1 and the average power is $\Omega = 2$.

can be expressed as [26]–[28]:

$$f(h) = \frac{(1+q^2)h}{q\Omega} \exp\left[-\frac{(1+q^2)h^2}{4q^2\Omega}\right] \mathbb{I}_0\left[\frac{(1-q^4)h^2}{4q^2\Omega}\right], h \geq 0, \tag{1}$$

where $\Omega > 0$ is the average power, $0 \leq q \leq 1$ is the Hoyt fading parameter and $\mathbb{I}_0(x)$ is the zeroth-order modified Bessel function of the first kind [33]. The characteristic function of the modulus h can be shown to be:

$$\begin{aligned}\Phi_h(\omega) &= \frac{2q}{1+q^2}\mathbb{H}_7\left[1, 1, \frac{1}{2}, \frac{(1-q^2)^2}{4(1+q^2)^2}, -\frac{q^2\Omega}{(1+q^2)^2}\omega^2\right] \\ &+ j\frac{2q^2\sqrt{\pi\Omega}}{(1+q^2)^2}\omega\mathbb{H}_7\left[\frac{3}{2}, 1, \frac{3}{2}, \frac{(1-q^2)^2}{4(1+q^2)^2}, -\frac{q^2\Omega}{(1+q^2)^2}\omega^2\right],\end{aligned} \tag{2}$$

where $\mathbb{H}_7(\alpha, \gamma, \delta, x, y)$ is Horn's confluent hypergeometric function of two variables [31]. Equation 2 provides a closed-form expression for the characteristic function of the Hoyt distribution, which is equivalent to the formula given by Table II of [30], but here it is represented in its more compact form, rather than as a sum of infinite series.

The phase of the Hoyt random variable is by no means uniformly distributed [27]. For the sake of facilitating our analysis, we also use an alternative expression of the Hoyt random variable, which is $\widetilde{h} = h_x + jh_y$ [27], where h_x and h_y are independent zero-mean Gaussian random variables having a variance of σ_x^2 and σ_y^2, respectively. The relations of the various parameters are given by:

$$\Omega = \sigma_x^2 + \sigma_y^2, \tag{3}$$

$$q = \frac{\sigma_y}{\sigma_x}. \tag{4}$$

The Hoyt distribution becomes the Rayleigh distribution, when we have $q = 1$. Equations 1 and 2 reduce to the PDF and CF of the Rayleigh distribution given by Table II of [30], respectively.

B. Receiver

The received signal at the input of the coherent correlation receiver is given by:

$$r(t) = \Re\left\{\sum_{k=0}^{K} \widetilde{h}_k a_k(t-\tau_k) b_k(t-\tau_k) e^{j[\omega_c(t-\tau_k)+\theta_k]}\right\} + \eta(t), \tag{5}$$

where $\Re\{\widetilde{x}\}$ denotes the real part of the complex number $\widetilde{x}$. Furthermore, the received complex equivalent signals $\{\widetilde{h}_k\}_{k=0}^{K-1}$ are independent Hoyt distributed random variables having the parameters $\{\Omega_k,\ q_k\}_{k=0}^{K-1}$, or equivalently $\{\sigma_{kx}, \sigma_{ky}\}_{k=0}^{K-1}$. The carrier's angular frequency ω_c is common to all users, while the carrier phase shift $\{\theta_k\}_{k=0}^{K-1}$ and the time delay $\{\tau_k\}_{k=0}^{K-1}$ are independently and uniformly distributed in $[0, 2\pi)$ and $[0, T_s)$, respectively, where T_s is the bit duration. Finally, $\eta(t)$ is the zero-mean stationary Additive White Gaussian Noise (AWGN) having a double-sided power spectral density of $\frac{N_0}{2}$. The rectangular pulse having a duration of T is defined as:

$$p_T(t) = \begin{cases} 1, t \in [0, T), \\ 0, \text{otherwise}. \end{cases} \tag{6}$$

Hence the kth user's spreading signal $a_k(t)$ and data signal $b_k(t)$ can be expressed as:

$$a_k(t) = \sum_{m=-\infty}^{\infty} a_{k,m} p_{T_c}(t - mT_c), \tag{7}$$

$$b_k(t) = \sum_{m=-\infty}^{\infty} b_{k,m} p_{T_s}(t - mT_s), \tag{8}$$

where T_c is the chip duration satisfying $T_s = LT_c$. Both the spreading sequence $\{a_{k,m}\}_{m=0}^{L-1}$ and the data sequence $\{b_{k,m}\}_{m=-\infty}^{\infty}$ are mutually independent and symmetrically Bernoulli distributed [34], implying that we have $P\{a_{k,m} = \pm 1\} = P\{b_{k,m} = \pm 1\} = \frac{1}{2}$.

Without loss of generality, we assume that the 0th user's signal is the desired one. If the chip synchronization is perfect, the decision statistic at the output of the coherent correlation receiver is given by:

$$Z = h_0 L b_{0,0} + \sum_{k=1}^{K-1} \Re\left\{X_k \widetilde{h}_k e^{j\Delta_k}\right\} + \eta, \tag{9}$$

where the noise component η is a zero-mean Gaussian random variable having a variance of $\sigma_\eta^2 = \frac{N_0 L}{T_c}$, while the phase shift difference $\Delta_k = -\omega_c(\tau_k - \tau_0) + (\theta_k - \theta_0)$ between the kth and 0th user is uniformly distributed in $[0, 2\pi)$. The random variable X_k may be further expressed as [1], [6]:

$$\begin{aligned}X_k = &\sum_{m=0}^{L-2} Y_{k,m}\left[(1-\nu_k) + a_{0,m}a_{0,m+1}\nu_k\right] \\ &+ Y_{k,L-1}\nu_k + Y_{k,L}(1-\nu_k),\end{aligned} \tag{10}$$

where the $(L+1)$ random variables $\{Y_{k,m}\}_{m=0}^{L}$ are mutually independent and symmetric Bernoulli distributed, conditioned on the 0th user's spreading sequence $\{a_{0,m}\}_{m=0}^{L-1}$. Furthermore, the relative chip shifts $\{\nu_k\}_{k=1}^{K-1}$ between the kth and 0th user normalized by the chip duration are mutually independent and uniformly distributed in $[0, 1)$ [1], [6].

III. BER Analysis

Let B and A denote the number of chip boundaries both with and without chip-value transitions within the 0th user's spreading sequence, respectively, and define two sets $\mathcal{A}$ and $\mathcal{B}$ as follows [1], [6]:

$$\begin{aligned}\mathcal{A} &= \{-A, -(A-2), ..., A-2, A\}, \\ \mathcal{B} &= \{-B, -(B-2), ..., B-2, B\}.\end{aligned} \tag{11}$$

Then we have $A + B = L - 1$ and the Co-Channel Interference (CCI) $I_k = \Re\left\{X_k \widetilde{h}_k e^{j\Delta_k}\right\}$ imposed by the different interferers is mutually independent, conditioned on B [1], [6]. For the sake of simplicity, we will only consider the characteristic function range spanning over $\omega \geq 0$ in our later discussions in the context of Equation 18. Nevertheless, the characteristic function range spanning over $\omega \leq 0$ can be readily derived from the range spanning over $\omega \geq 0$ by exploiting the following property of the characteristic function [34] $\Phi(-\omega) = \Phi^*(\omega)$, where $\Phi^*(\omega)$ denotes the complex conjugate of $\Phi(\omega)$.

Upon exploiting the alternative expression of the Hoyt random variable $\widetilde{h}_k$ in Section II-A, it can be readily shown that I_k conditioned on X_k and Δ_k is a zero-mean Gaussian random variable having a variance of $\sigma_{kx}^2 \cos^2 \Delta_k + \sigma_{ky}^2 \sin^2 \Delta_k$. Hence we have the characteristic function of I_k conditioned on X_k and Δ_k in the following form:

$$\Phi_{I_k|X_k,\Delta_k}(\omega) = \exp\left[-\frac{1}{2}\left(\sigma_{kx}^2 \cos^2 \Delta_k + \sigma_{ky}^2 \sin^2 \Delta_k\right) X_k^2 \omega^2\right]. \tag{12}$$

Applying the integral identity of Equation 3.339 in [33], we arrive at the characteristic function of I_k conditioned on X_k by averaging $\Phi_{I_k|X_k,\Delta_k}(\omega)$ over $\Delta_k \in [0, 2\pi)$ in the form of:

$$\begin{aligned}\Phi_{I_k|X_k}(\omega) &= \exp\left[-\frac{1}{4}\left(\sigma_{kx}^2 + \sigma_{ky}^2\right) X_k^2 \omega^2\right] \\ &\quad \times \mathbb{I}_0\left[\frac{1}{4}\left(\sigma_{kx}^2 - \sigma_{ky}^2\right) X_k^2 \omega^2\right].\end{aligned} \tag{13}$$

Upon averaging $\Phi_{I_k|X_k}(\omega)$ over $\{Y_{k,m}\}_{m=0}^{L}$ and ν_k, we get the characteristic function of I_k conditioned on B [1] in the following form:

$$\begin{aligned}\Phi_{I_k|B}(\omega) &= 2^{-(L+1)} \sum_{d_1 \in \mathcal{A}} \sum_{d_2 \in \mathcal{B}} \binom{A}{\frac{d_1+A}{2}} \binom{B}{\frac{d_2+B}{2}} \\ &\quad \times \sum_{Y_{k,L-1}, Y_{k,L} \in \{\pm 1\}} \Phi_{I_k|\lambda_0,\lambda_1}(\omega),\end{aligned} \tag{14}$$

where the coefficients λ_0 and λ_1 are defined as:

$$\lambda_0 = d_1 + d_2 + Y_{k,L} \tag{15}$$

$$\lambda_1 = -2d_2 + Y_{k,L-1} - Y_{k,L}. \tag{16}$$

When invoking the series expansions of Equations 1.211-1 and 8.477-1 of [33], the conditional characteristic function $\Phi_{I_k|\lambda_0,\lambda_1}(\omega)$ may be expressed in the form of Equation 17 seen at the top of the next page. When $q = 1$, i.e. when we experience Rayleigh fading, Equation 17 reduces to the results of [1].

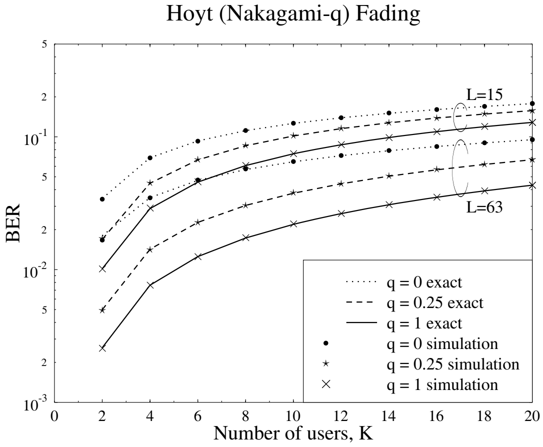

Fig. 2. BER versus the number of users K in an asynchronous DS-CDMA system using random spreading sequences and BPSK modulation communicating over Hoyt channels. The length of the random spreading sequences is $L = 15$ and 63. The Hoyt fading parameter is $q = 0$, 0.25 and 1, which is common to all users. The average power of all users at the receiver is equal and the background noise is ignored, i.e. we have $\gamma_{\text{SNR}} = \infty$.

Applying Parseval's theorem [30] and exploiting the fact that the CCI contribution $\{I_k\}$ conditioned on B is mutually independent [1], [6], we arrive at the 0th user's BER conditioned on B as follows:

$$P_{e|B} = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{1}{\omega}\Phi_\eta(\omega)\Im\left\{\Phi_{h_0}(\omega L)\right\}\prod_{k=1}^{K-1}\Phi_{I_k}(\omega)\mathrm{d}\omega, \tag{18}$$

where $\Phi_\eta(\omega)$ is the characteristic function of the noise component η and $\Im\{\Phi_{h_0}(\omega)\}$ is the imaginary part of the characteristic function of h_0, which was given by Equation 2.

Finally, we arrive at the overall average BER by averaging $P_{e|B}$ over all spreading sequences in the form of:

$$P_e = 2^{-(L-1)} \sum_{B=0}^{L-1} \binom{L-1}{B} P_{e|B}. \tag{19}$$

IV. Numerical Results

In this section we will verify the accuracy of our exact BER analysis provided in Section III and demonstrate the limited accuracy of the SGA method by simulations.

Figures 2 and 3 illustrate the average BER performance versus the number of users, when the effects of background noise are ignored. Figure 2 compares the results obtained from our exact BER analysis to our simulation results and shows that they match very well both for different spreading sequence lengths and for various Hoyt fading parameters. On the other hand, Figure 3 compares the results obtained using the SGA to our simulation results and demonstrates that the SGA overestimates the average BER, especially in the scenarios where

$$\Phi_{I_k|\lambda_0,\lambda_1}(\omega)=\begin{cases}\dfrac{x}{\lambda_1}\,\mathbb{F}^{1:0;0}_{1:0;1}\left(\begin{matrix}\left[\left(\frac{1}{2}\right):1,2\right]:-\,; & - & ; \\ \left[\left(\frac{3}{2}\right):1,2\right]:-\,; & [(1):1] & ;\end{matrix}\,-\dfrac{1}{4}(\sigma_{kx}^2+\sigma_{ky}^2)\omega^2x^2,\,\dfrac{1}{64}(\sigma_{kx}^2-\sigma_{ky}^2)^2\omega^4x^4\right)\Bigg|_{\lambda_0}^{\lambda_0+\lambda_1}, & \lambda_1\neq 0,\\ \exp\left[-\dfrac{1}{4}\left(\sigma_{kx}^2+\sigma_{ky}^2\right)\lambda_0^2\omega^2\right]\mathbb{I}_0\left[\dfrac{1}{4}\left(\sigma_{kx}^2-\sigma_{ky}^2\right)\lambda_0^2\omega^2\right], & \lambda_1=0,\end{cases} \quad (17)$$

where $\mathbb{F}^{A:B^{(1)};...;B^{(n)}}_{C:D^{(1)};...;D^{(n)}}\left(\begin{matrix}\left[(a):\theta^{(1)},...,\theta^{(n)}\right]:\left[(b^{(1)}):\phi^{(1)}\right];...;\left[(b^{(n)}):\phi^{(n)}\right];\\ \left[(c):\psi^{(1)},...,\psi^{(n)}\right]:\left[(d^{(1)}):\delta^{(1)}\right];...;\left[(d^{(n)}):\delta^{(n)}\right];\end{matrix}x_1,...,x_n\right)$ is the generalized Lauricella function of n variables defined as Equations 21 - 23 of [32] and $f(x)|_{x_1}^{x_2}=f(x_2)-f(x_1)$.

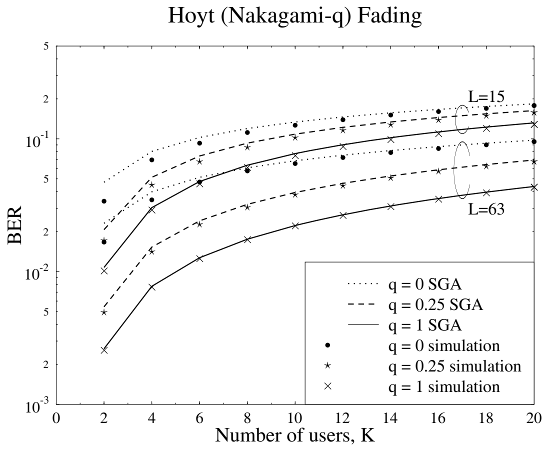

Fig. 3. BER versus the number of users K in an asynchronous DS-CDMA system using random spreading sequences and BPSK modulation communicating over Hoyt channels. The length of the random spreading sequences is $L=15$ and 63. The Hoyt fading parameter is $q=0$, 0.25 and 1. The average power of all users at the receiver is equal and the background noise is ignored, i.e. we have $\gamma_{\mathrm{SNR}}=\infty$.

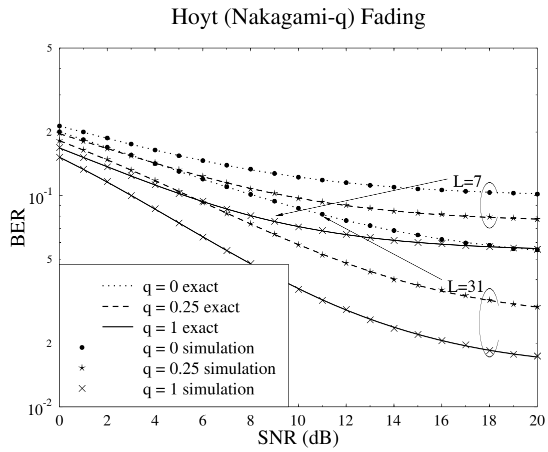

Fig. 4. BER versus per-bit SNR in an asynchronous DS-CDMA system using random spreading sequences and BPSK modulation communicating over Hoyt channels. The length of the random spreading sequences is $L=7$ and 31. The Hoyt fading parameter is $q=0$, 0.25 and 1. The average power of all users at the receiver is equal. The number of users is $K=4$.

either there is a limited number of interferers, or when the Hoyt fading parameter q is small or when short spreading sequences are used.

Figures 4 and 5 illustrate the average BER performance versus the per-bit SNR, when the number of users is $K=4$. Figure 4 compares the results obtained from our exact BER analysis to our simulation results and shows that they match well both for different spreading sequence lengths and for various Hoyt fading parameters. On the other hand, Figure 5 compares the results obtained by the SGA to our simulation results and demonstrates that the SGA still fails to accurately evaluate the average BER performance, particularly when the SNR is high, the Hoyt fading parameter q is low and when short spreading sequences are used.

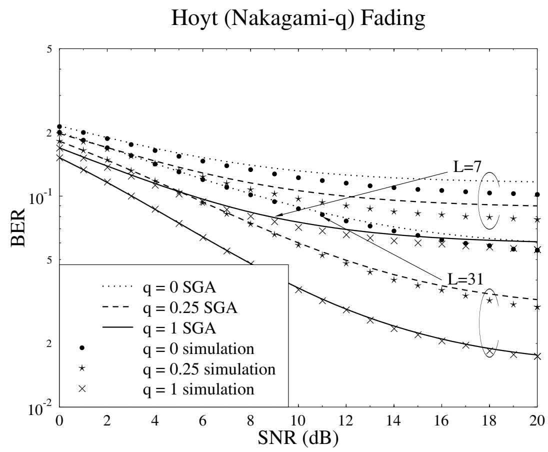

Fig. 5. BER versus per-bit SNR in an asynchronous DS-CDMA system using random spreading sequences and BPSK modulation communicating over Hoyt channels. The length of the random spreading sequences is $L=7$ and 31. The Hoyt fading parameter is $q=0$, 0.25 and 1. The average power of all users at the receiver is equal. The number of users is $K=4$.

V. Conclusion

An exact expression has been derived for calculating the average BER of an asynchronous DS-CDMA system using random spreading sequences and BPSK modulation for communicating over Hoyt fading channels. It is based on the characteristic function and requires only a single numerical integration. Furthermore, a new closed-form expression was provided for the conditional characteristic function of the interfering signal with the aid of the generalized Lauricella function in n variables [32]. Since the Rayleigh distribution is a special case of the Hoyt distribution, the results obtained by [1] may also be regarded as a special case of our results. Our simulation results verified the accuracy of our exact BER analysis for various combinations of the spreading sequence length and the Hoyt fading parameter. By contrast, the SGA over-estimates the average BER, especially when either there is a low number of interferers, or the SNR is high, the Hoyt fading parameter happens to be low or and short spreading sequences are used.

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