

# Elements towards a foundation of computational trust

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joint work K. Krukow and M. Nielsen

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# Computational trust

Trust is an ineffable notion that permeates very many things.

## What trust are we going to have in this talk?

Computer idealisation of “trust” to support decision-making in open networks. No human emotion, nor philosophical/sociological concept.

Gathering prominence in open applications involving safety guarantees in a wide sense

- **credential-based trust**: e.g., public-key infrastructures, authentication and resource access control, network security.
- **reputation-based trust**: e.g., social networks, P2P, trust metrics, probabilistic approaches.
- **trust models**: e.g., security policies, languages, game theory.
- **trust in information sources**: e.g., information filtering and provenance, content trust, user interaction, social concerns.

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# Trust and reputation systems

## Reputation

- **behavioural**: perception that an agent creates through past actions about its intentions and norms of behaviour.
- **social**: calculated on the basis of observations made by others.

An agent's reputation may affect the trust that others have toward it.

## Trust

- **subjective**: a level of the subjective expectation an agent has about another's future behaviour based on based on the history of their encounters and of hearsay.

**Confidence** in the trust assessment is also a parameter of importance.

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# Trust and security

## E.g.: Reputation-based access control

$p$ 's 'trust' in  $q$ 's actions at time  $t$ , is determined by  $p$ 's observations of  $q$ 's behaviour up *until* time  $t$  according to a given policy  $\psi$ .

## Example

You download what claims to be a new cool browser from some unknown site. Your trust policy may be:

- *allow the program to connect to a remote site if and only if it has neither tried to **open a local file that it has not created**, nor to **modify a file it has created**, nor to **create a sub-process**.*

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# Outline

- 1 Some computational trust systems
- 2 Towards model comparison
- 3 Modelling behavioural information
  - Event structures as a trust model
- 4 Probabilistic event structures
- 5 A Bayesian event model

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# EigenTrust (Kamvar et al)

- Some novel ideas well established by now.
- A set  $\mathcal{P}$  of  $n$  peers who interact pairwise and mutually rate the interaction either **sat** or **unsat**.
  - ▶ Peer  $i$  computes a local ‘trust value’ in peer  $j$ :

$$s_{ij} = \text{sat}(i, j) - \text{unsat}(i, j) \sqcup 0.$$

- ▶ Peer  $i$  then defines a normalised measure of its local trust in  $j$ :

$$c_{ij} = \frac{s_{ij}}{\sum_j s_{ij}}$$

- $[c_{ij}]$  defines a Markov chain (i.e.,  $\sum_j c_{ij} = 1$ ), with stationary distribution  $(t_j)_{j \in \mathcal{P}}$ . The **global trust** value for principal  $j$  is  $t_j$ .
- Simulations prove that **EigenTrust** is a smart system. Yet, no much is said formally about properties, e.g. safety guarantees and the incidence of values like  $t_j$ .

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# Simple Probabilistic Systems

The model  $\lambda_\theta$ :

- Each principal  $p$  behaves in each interaction according to a fixed and independent probability  $\theta_p$  of ‘success’ (and therefore  $1 - \theta_p$  of ‘failure’).

The framework:

- **Interface** (Trust computation algorithm,  $\mathcal{A}$ ):
  - ▶ **Input**: A sequence  $h = x_1 x_2 \cdots x_n$  for  $n \geq 0$  and  $x_i \in \{\mathbf{s}, \mathbf{f}\}$ .
  - ▶ **Output**: A probability distribution  $\pi : \{\mathbf{s}, \mathbf{f}\} \rightarrow [0, 1]$ .
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# Maximum likelihood (Despotovic and Aberer)

## Trust computation $\mathcal{A}_0$

$$\mathcal{A}_0(\mathbf{s} | h) = \frac{N_{\mathbf{s}}(h)}{|h|} \qquad \mathcal{A}_0(\mathbf{f} | h) = \frac{N_{\mathbf{f}}(h)}{|h|}$$

$N_x(h)$  = “number of  $x$ 's in  $h$ ”

Bayesian analysis inspired by  $\lambda_\beta$  model:  $f(\theta | \alpha \beta) \propto \theta^{\alpha-1} (1 - \theta)^{\beta-1}$

### Properties:

- Well defined semantics:  $\mathcal{A}_0(\mathbf{s} | h)$  is interpreted as a *probability* of success in the next interaction.
- Solidly based on probability theory and Bayesian analysis.
- Formal result:  $\mathcal{A}_0(\mathbf{s} | h) \rightarrow \theta_p$  as  $|h| \rightarrow \infty$ .

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## Beta models (Mui et al)

Even more tightly inspired by Bayesian analysis and by  $\lambda_\beta$

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# TRAVOS (Teacy et al)

## Trust computation $\mathcal{A}_2$

Based on  $\lambda_\beta$ , like  $\mathcal{A}_1$ , but with serious approach to reputation. One of the few systems to also accounts for “malicious” reports.

# Our elements of foundation

## Recall the framework

- **Interface** (Trust computation algorithm,  $\mathcal{A}$ ):
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- 1 model comparison
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# Cross entropy

An information-theoretic “distance” on distributions

Cross entropy of distributions  $\mathbf{p}, \mathbf{q} : \{o_1, \dots, o_m\} \rightarrow [0, 1]$ .

$$D(\mathbf{p} \parallel \mathbf{q}) = \sum_{i=1}^m \mathbf{p}(o_i) \cdot \log(\mathbf{p}(o_i)/\mathbf{q}(o_i))$$

It holds  $0 \leq D(\mathbf{p} \parallel \mathbf{q}) \leq \infty$ , and  $D(\mathbf{p} \parallel \mathbf{q}) = 0$  iff  $\mathbf{p} = \mathbf{q}$ .

- Established measure in statistics for comparing distributions.
- Information-theoretic: the average amount of information discriminating  $\mathbf{p}$  from  $\mathbf{q}$ .

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# Expected cross entropy

A measure on probabilistic trust algorithms

- Goal of a probabilistic trust algorithm  $\mathcal{A}$ : given a history  $\mathbf{X}$ , approximate a distribution on the outcomes  $O = \{o_1, \dots, o_m\}$ .
- Different histories  $\mathbf{X}$  result in different output distributions  $\mathcal{A}(\cdot | \mathbf{X})$ .

Expected cross entropy from  $\lambda$  to  $\mathcal{A}$

$$ED^n(\lambda \parallel \mathcal{A}) = \sum_{\mathbf{X} \in O^n} \text{Prob}(\mathbf{X} | \lambda) \cdot D(\text{Prob}(\cdot | \mathbf{X} \lambda) \parallel \mathcal{A}(\cdot | \mathbf{X}))$$

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# An application of cross entropy (1/2)

Consider the beta model  $\lambda_\beta$  and the algorithms  $\mathcal{A}_0$  of maximum likelihood (Despotovic et al.) and  $\mathcal{A}_1$  beta (Mui et al.).

## Theorem

If  $\theta = 0$  or  $\theta = 1$  then  $\mathcal{A}_0$  computes the exact distribution, whereas  $\mathcal{A}_1$  does not. That is, for all  $n > 0$  we have:

$$ED^n(\lambda_\beta \parallel \mathcal{A}_0) = 0 < ED^n(\lambda_\beta \parallel \mathcal{A}_1)$$

If  $0 < \theta < 1$ , then  $ED^n(\lambda_\beta \parallel \mathcal{A}_0) = \infty$ , and  $\mathcal{A}_1$  is always better.

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# An application of cross entropy (2/2)

## A parametric algorithm $\mathcal{A}_\epsilon$

$$\mathcal{A}_\epsilon(\mathbf{s} \mid h) = \frac{N_{\mathbf{s}}(h) + \epsilon}{|h| + 2\epsilon}, \quad \mathcal{A}_\epsilon(\mathbf{f} \mid h) = \frac{N_{\mathbf{f}}(h) + \epsilon}{|h| + 2\epsilon}$$

## Theorem

*For any  $\theta \in [0, 1]$ ,  $\theta \neq 1/2$  there exists  $\bar{\epsilon} \in [0, \infty)$  that minimises  $\text{ED}^n(\lambda_\theta \parallel \mathcal{A}_\epsilon)$ , simultaneously for all  $n$ .*

*Furthermore,  $\text{ED}^n(\lambda_\theta \parallel \mathcal{A}_\epsilon)$  is a decreasing function of  $\epsilon$  on the interval  $(0, \bar{\epsilon})$ , and increasing on  $(\bar{\epsilon}, \infty)$ .*

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Furthermore,  $\text{ED}^n(\lambda_\beta \parallel \mathcal{A}_\epsilon)$  is a decreasing function of  $\epsilon$  on the interval  $(0, \bar{\epsilon})$ , and increasing on  $(\bar{\epsilon}, \infty)$ .

That is, unless behaviour is completely unbiased, there exists a unique best  $\mathcal{A}_\epsilon$  algorithm that for all  $n$  outperforms all the others.

If  $\theta = 1/2$ , the larger the  $\epsilon$ , the better.

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- Algorithm  $\mathcal{A}_0$  is optimal for  $\theta = 0$  and for  $\theta = 1$ .
- Algorithm  $\mathcal{A}_1$  is optimal for  $\theta = \frac{1}{2} \pm \frac{1}{\sqrt{12}}$ .

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# A trust model based on event structures

Move from  $O = \{\mathbf{s}, \mathbf{f}\}$  to complex outcomes

## Interactions and protocols

- At an abstract level, entities in a distributed system interact according to protocols;
- Information about an external entity is just information about (the outcome of) a number of (past) protocol runs with that entity.

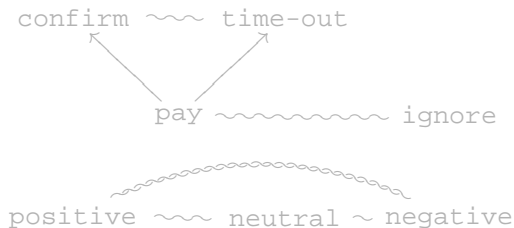
## Events as model of information

- A protocol can be specified as a **concurrent process**, at different levels of abstractions.
- Event structures were invented to give formal semantics to truly concurrent processes, expressing “**causation**” and “**conflict**.”

# A model for behavioural information

- $ES = (E, \leq, \#)$ , with  $E$  a set of events,  $\leq$  and  $\#$  relations on  $E$ .
- Information about a session is a finite set of events  $x \subseteq E$ , called a **configuration** (which is ‘conflict-free’ and ‘causally-closed’).
- Information about several interactions is a sequence of outcomes  $h = x_1 x_2 \cdots x_n \in \mathcal{C}_{ES}^*$ , called a **history**.

eBay (simplified) example:

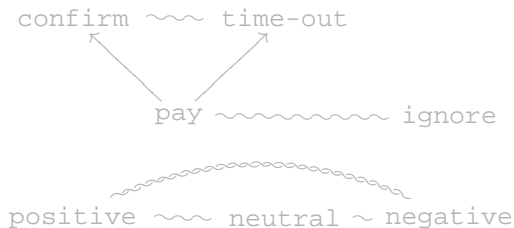


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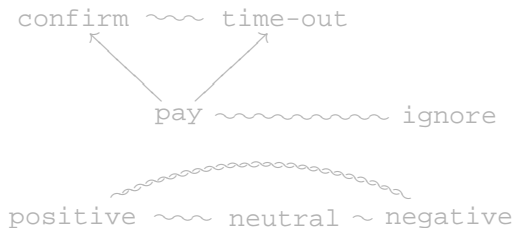
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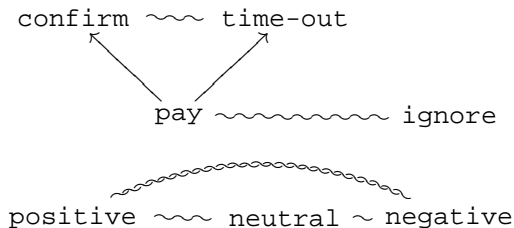


e.g.,  $h = \{\text{pay, confirm, pos}\} \{\text{pay, confirm, neu}\} \{\text{pay}\}$

# A model for behavioural information

- $ES = (E, \leq, \#)$ , with  $E$  a set of events,  $\leq$  and  $\#$  relations on  $E$ .
- Information about a session is a finite set of events  $x \subseteq E$ , called a **configuration** (which is 'conflict-free' and 'causally-closed').
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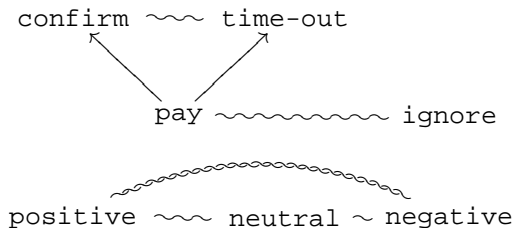


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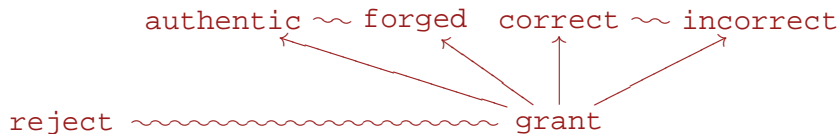
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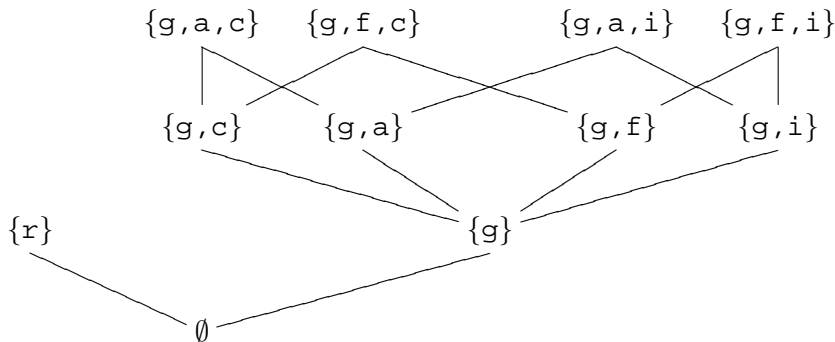
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# Running example: interactions over an e-purse



# Modelling outcomes and behaviour

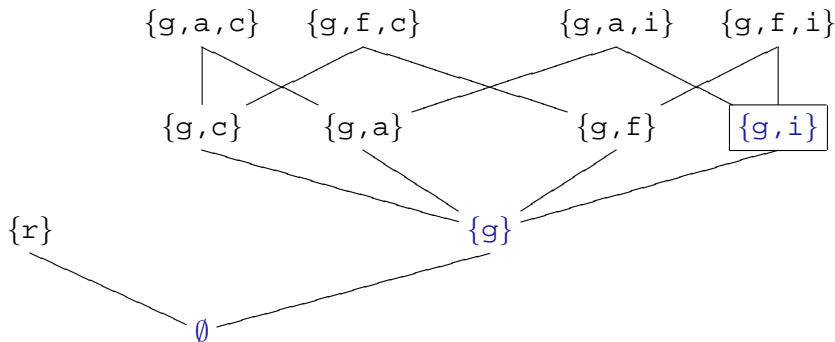
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- **Behaviour** is a sequence of outcomes

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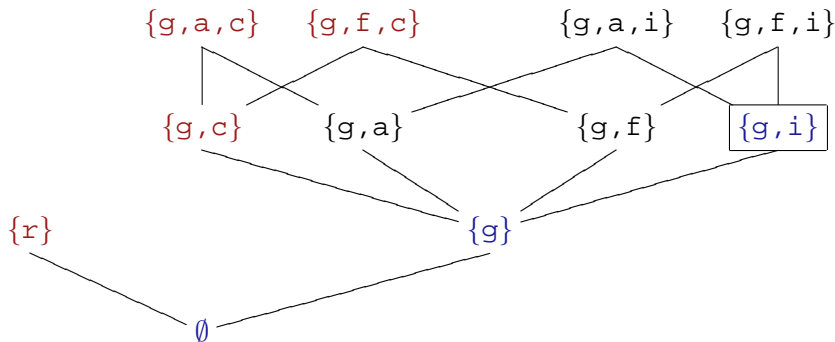
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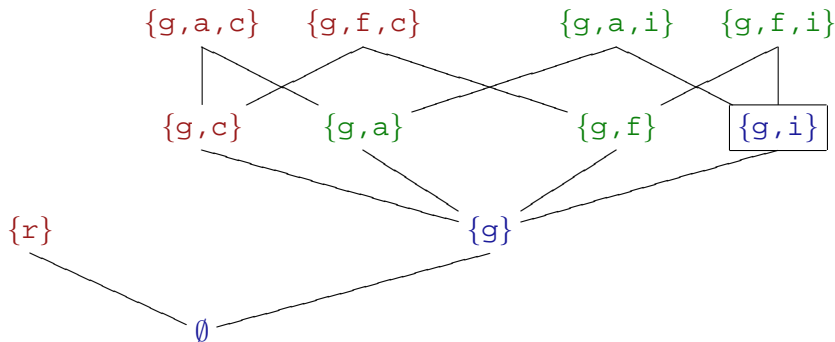
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# Outline

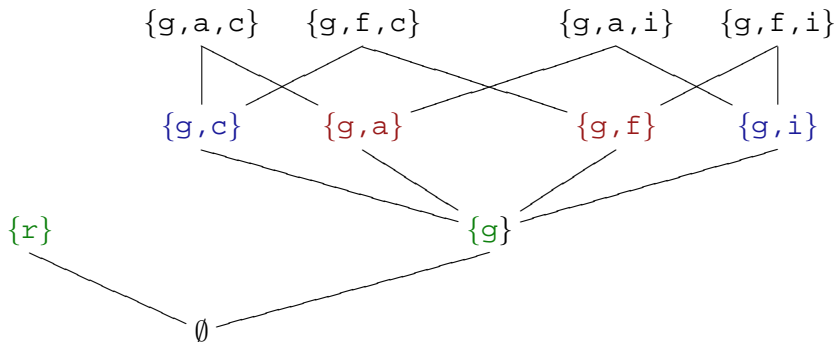
- 1 Some computational trust systems
- 2 Towards model comparison
- 3 Modelling behavioural information
  - Event structures as a trust model
- 4 Probabilistic event structures**
- 5 A Bayesian event model

# Confusion-free event structures (Varacca et al)

- Immediate conflict  $\#_\mu$ :  $e \# e'$  and there is  $x$  that enables both.
- Confusion free:  $\#_\mu$  is transitive and  $e \#_\mu e'$  implies  $[e] = [e']$ .
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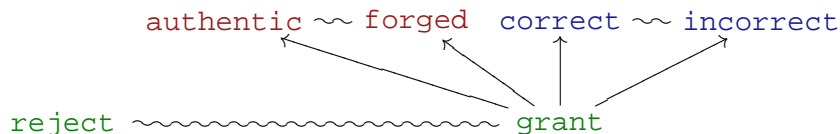
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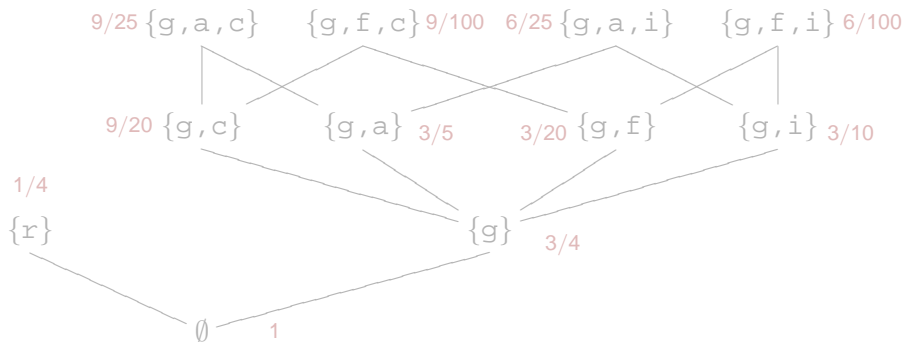
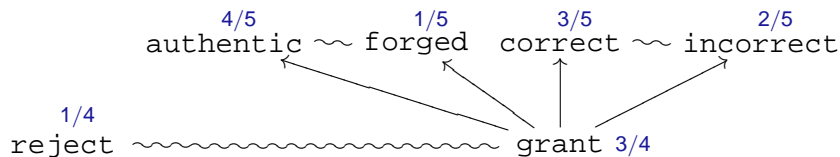
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So, there are three cells in the e-purse event structure

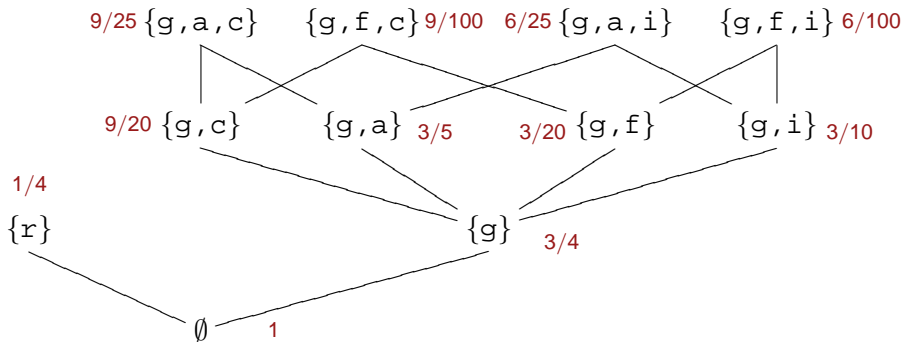
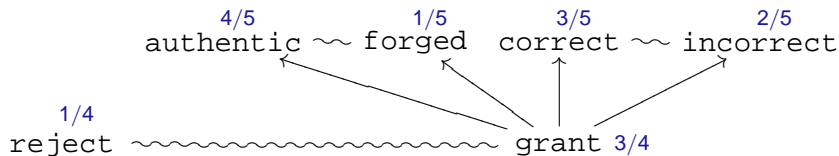


- Cell valuation: a function  $p : E \rightarrow [0, 1]$  such that  $p[c] = 1$ , for all  $c$ .

# Cell valuation



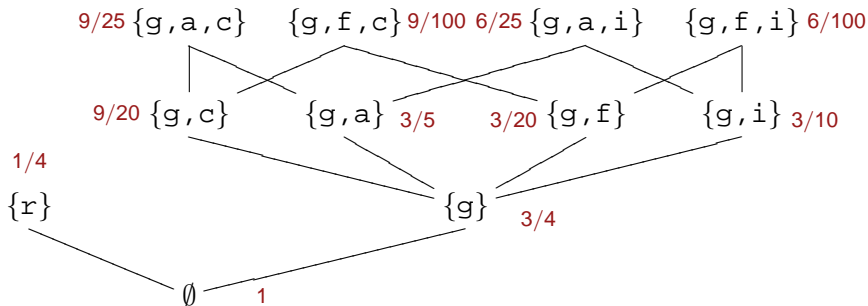
# Cell valuation



# Properties of cell valuations

Define  $p(x) = \prod_{e \in x} p(e)$ . Then

- $p[\emptyset] = 1$ ;
- $p[x] \geq p[x']$  if  $x \subseteq x'$ ;
- $p$  is a probability distribution on maximal configurations.



So,  $p(x)$  is the probability that  $x$  is contained in the final outcome.

# Outline

- 1 Some computational trust systems
- 2 Towards model comparison
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# Estimating cell valuations

How to assign valuations to cells? They are the model's unknowns.

Theorem (Bayes)

$$Prob[\Theta | \mathbf{X} \lambda] \propto Prob[\mathbf{X} | \Theta \lambda] \cdot Prob[\Theta | \lambda]$$

A second-order notion: we not are interested in  $\mathbf{X}$  or its probability, but in the expected value of  $\Theta$ ! So, we will:

- start with a prior hypothesis  $\Theta$ ; this will be a cell valuation;
- record the events  $\mathbf{X}$  as they happen during the interactions;
- compute the posterior; this is a new model fitting better with the evidence and allowing us better predictions (in a precise sense).

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# Cells vs eventless outcomes

Let  $c_1, \dots, c_M$  be the set of cells of  $E$ , with  $c_i = \{e_1^i, \dots, e_{K_i}^i\}$ .

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- The occurrence of an  $x$  from  $\{\mathbf{s}, \mathbf{f}\}$  is a random process with two outcomes, a **binomial (Bernoulli) trial** on  $\theta$ .
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# A bit of magic: the Dirichlet probability distribution



The Dirichlet family  $\mathcal{D}(\Theta | \alpha) \propto \prod \Theta_1^{\alpha_1-1} \dots \Theta_K^{\alpha_K-1}$

## Theorem

The Dirichlet family is a *conjugate prior* for multinomial trials. That is, if

- $\text{Prob}[\Theta | \lambda]$  is  $\mathcal{D}(\Theta | \alpha_1, \dots, \alpha_K)$  and
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So, we start with a family  $\mathcal{D}(\Theta_{c_i} | \alpha_{c_i})$ , and then use multinomial trials  $\mathbf{X} : E \rightarrow \omega$  to keep updating the valuation as  $\mathcal{D}(\Theta_{c_i} | \alpha_{c_i} + \mathbf{X}_{c_i})$ .

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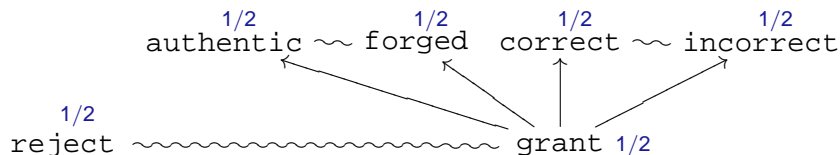
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# The Bayesian process

Start with a uniform distribution for each cell.



## Theorem

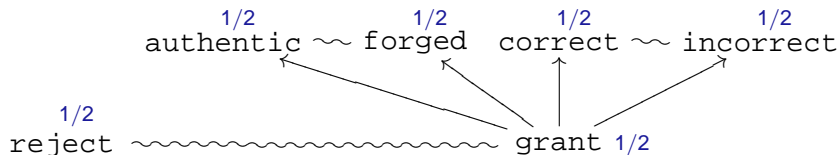
$$E[\Theta_{e_j} | \mathbf{X} \lambda] = \frac{\alpha_{e_j} + \mathbf{X}(e_j)}{\sum_{k=1}^{K_i} (\alpha_{e_k} + \mathbf{X}(e_k))}$$

## Corollary

$$E[\text{next outcome is } x | \mathbf{X} \lambda] = \prod_{e \in X} E[\Theta_e | \mathbf{X} \lambda]$$

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Suppose that  $\mathbf{X} = \{r \mapsto 2, g \mapsto 8, a \mapsto 7, f \mapsto 1, c \mapsto 3, i \mapsto 5\}$ . Then



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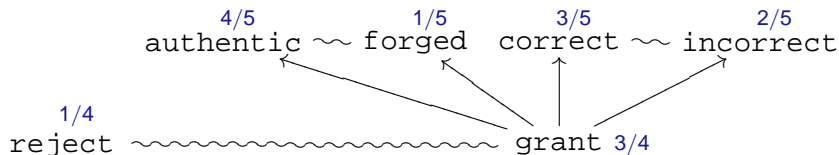
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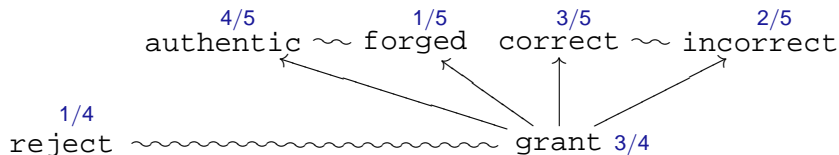
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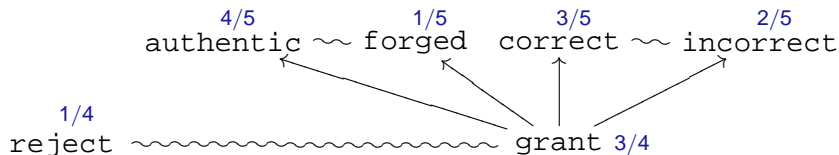
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# Interpretation of results

As a result, we have lifted the trust computational algorithms based on  $\lambda_\beta$  to our event-based models by replacing

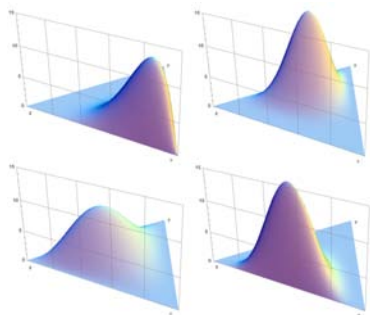
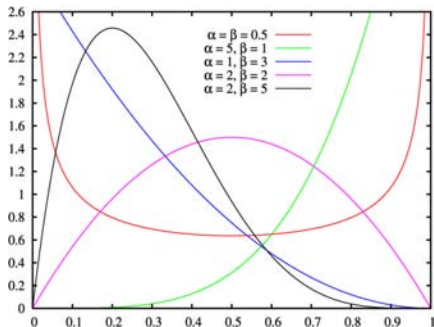
Binomials (Bernoulli) trials  
 $\beta$ -distribution



multinomial trials;



Dirichlet distribution.

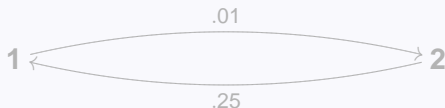


# Future directions (1/2)

## Hidden Markov Models

Probability parameters can change as the internal state change, probabilistically. HMM is  $\lambda = (A, B, \pi)$ , where

- $A$  is a Markov chain, describing state transitions;
- $B$  is family of distributions  $B_s : O \rightarrow [0, 1]$ ;
- $\pi$  is the initial state distribution.



$$\pi_1 = 1$$

$$B_1(a) = .95$$

$$B_1(b) = .05$$

$$O = \{a, b\}$$

$$\pi_2 = 0$$

$$B_2(a) = .05$$

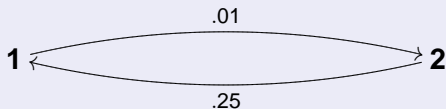
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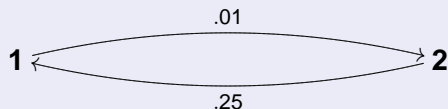
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# Future directions (2/2)

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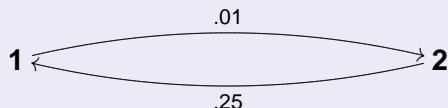
### Bayesian analysis:

- What models best explain (and thus predict) observations?
- How to approximate a HMM from a sequence of observations?

History  $h = a^{10}b^2$ . A counting algorithm would then assign high probability to  $a$  occurring next. But the last two  $b$ 's suggest a state change might have occurred, which would in reality make that probability very low.

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### Bayesian analysis:

- What models best explain (and thus predict) observations?
- How to approximate a HMM from a sequence of observations?

**History**  $h = a^{10}b^2$ . A counting algorithm would then assign high probability to  $a$  occurring next. But the last two  $b$ 's suggest a state change might have occurred, which would in reality make that probability very low.

# Summary

- A framework for “trust and reputation systems”
  - ▶ applications to security and history-based access control.
- Basic policies can be specified declaratively and verified efficiently. Quantified policies are expressive, and quantified model checking is decidable (though hard with many quantifiers).
- Bayesian approach to observations and approximations, formal results based on probability theory. Towards model comparison and complex-outcomes Bayesian model.
- Future work
  - ▶ Probabilistic logic.
  - ▶ Dynamic models with variable structure.
  - ▶ Better integration of reputation in the model.
  - ▶ Relationships with game-theoretic models.



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