Semantic barbs: what's in an observation?

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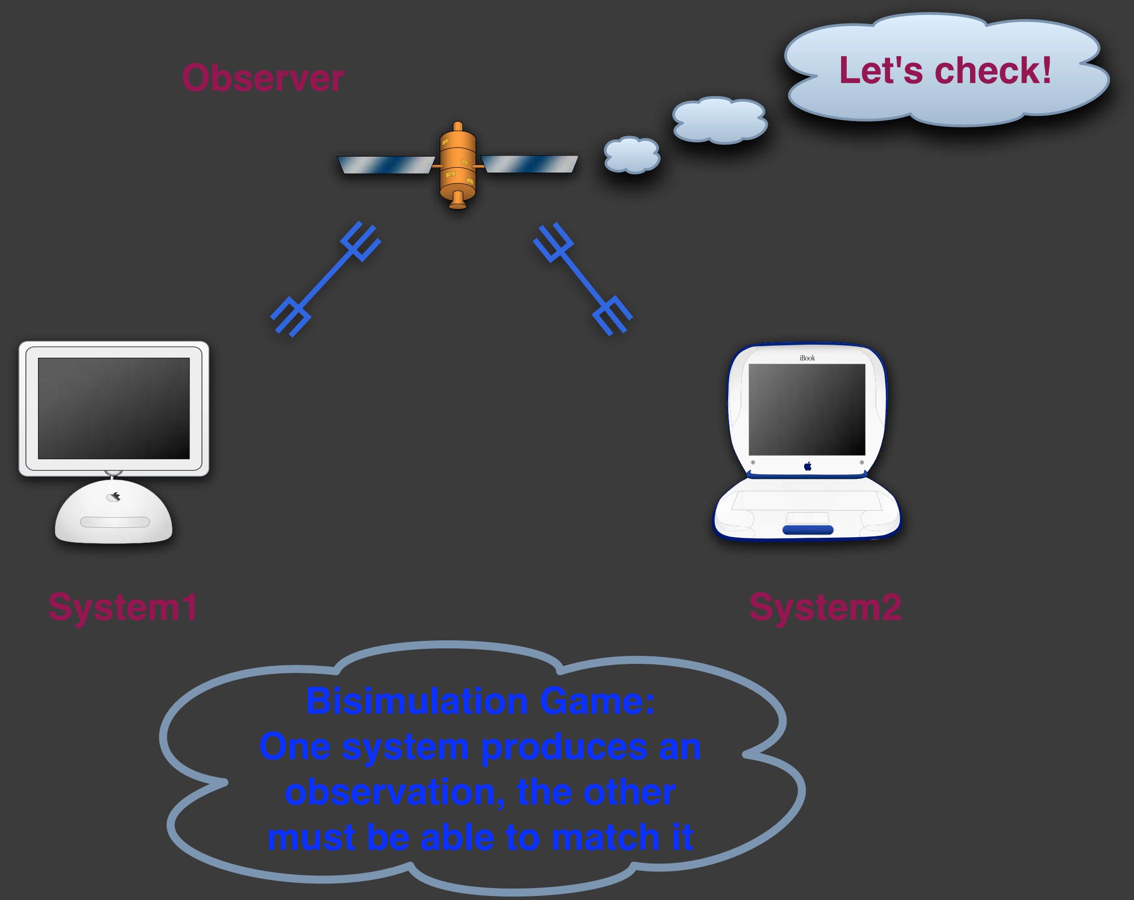
Blackbox testing / bisimulation



if $Sys_1 - obs \rightarrow Sys_1'$ **then** $Sys_2 - obs \rightarrow Sys_2'$

with Sys_1' and Sys_2' equivalent

vice versa with Sys_1 and Sys_2 exchanged

The holy grail

most current calculi have an underlying reduction semantics

Suppose we have syntax + reduction semantics:

Goal 1: Obtain a canonical contextual equivalence $\cong$
= derive barbs

Goal 2: Obtain a bisimulation proof method for $\cong$
= derive labels

both stories start with seminal papers by Robin Milner

Goal 1: Barbs

- **basic observable**
- normally only **immediate** observations
- introduced by Milner & Sangiorgi (1992) for CCS
  - reduction congruence is coarser than bisimilarity in CCS
- **barbs come with no explanation**
  - calculus-specific choices of barbs - often the “natural” choice forced by an a priori labelled semantics & labelled equivalence

basic immediate observables

Observable properties

$$@ : \mathbf{T} \times \mathbf{\Gamma} \to \mathbf{\Pi}$$

terms contexts processes

$$\bot \subseteq \mathbf{\Pi}$$ "successful processes"

$$(-)^{\bot} : \mathcal{P}(\mathbf{T}) \to \mathcal{P}(\mathbf{\Gamma})$$

$$T \mapsto \{\gamma \in \mathbf{\Gamma} \mid \forall t \in T.\ t \perp \gamma\}$$

contexts successful for all terms in T.

$$(-)^{\bot} : \mathcal{P}(\mathbf{\Gamma}) \to \mathcal{P}(\mathbf{T})$$

$$\Gamma \mapsto \{t \in \mathbf{T} \mid \forall \gamma \in \Gamma.\ t \perp \gamma\}$$

terms successful for all contexts in Γ.

means $t@\gamma \in \bot$

... the usual properties follow

$$X \subseteq Y \Rightarrow Y^\perp \subseteq X^\perp$$

$$X \subseteq X^{\perp\perp} \quad \text{and} \quad X^\perp = X^{\perp\perp\perp}$$

$$X^\perp \cap Y^\perp = (X \cup Y)^\perp \quad \text{but} \quad X^\perp \cup Y^\perp \subseteq (X \cap Y)^\perp$$

$\{t_1\}^\perp = \{t_2\}^\perp$ — t_1 and t_2 have the same observations

Biorthogonal: a set V such that $V = V^{\perp\perp}$

Fact: biorthogonals are closed under arbitrary intersections but not in general under (even binary) unions

$$\textstyle\bigcap_i V_i = \bigcap_i V_i^{\perp\perp} = \bigcap_i (V_i^\perp)^\perp = (\bigcup_i V_i^\perp)^\perp$$

$$V_1 \cup V_2 = V_1^{\perp\perp} \cup V_2^{\perp\perp} \subseteq (V_1^\perp \cap V_2^\perp)^\perp = (V_1 \cup V_2)^{\perp\perp}$$

basic immediate observables

Idealised calculi

T

$$P ::= \epsilon \mid P \parallel P \mid M.P$$
$$M ::= a? \mid a! \qquad (a \in A)$$

Synchrony

$$a!P \parallel a?Q \rightarrow P \parallel Q \quad (a \in A)$$

Asynchrony

$$a! \parallel a?P \rightarrow P \qquad (a \in A)$$

$$(a! \stackrel{\text{def}}{=} a!\epsilon)$$

Broadcast

$$a!P \parallel \prod_i a?.Q_i \rightarrow P \parallel \prod_i Q_i$$

Immediate observations

$$\Gamma \qquad C ::= \epsilon \mid C \parallel C \mid M_{\checkmark}$$
$$M_{\checkmark} ::= M.\checkmark$$

$$\Pi \qquad P_{\checkmark} ::= P_{\checkmark} \parallel P_{\checkmark} \mid P \mid C \mid \checkmark$$

$$\begin{aligned} @ : P \times C &\to P_{\checkmark} \\ (t, \gamma) &\mapsto t \parallel \gamma \end{aligned}$$

π is spent $\stackrel{\text{def}}{=}$ it has precisely one $\checkmark$ as a component

$$\pi \in \bot \quad \text{iff} \quad \exists \pi' \in P_{\checkmark}. \quad \pi' \text{ spent} \ \wedge \ \pi \to \pi'$$

Immediate observations: examples

$$a!P \parallel a?Q \;\rightarrow\; P \parallel Q \quad (a \in A)$$

$$\{a!\}^{\perp\perp} = [a?\checkmark]^{\perp} = [a!P]$$

$$\{a?\}^{\perp\perp} = [a?P]$$

||-ideal generated by t

$$\{a? \parallel b!c?\}^{\perp\perp} = [a!\checkmark, b?\checkmark]^{\perp} = [a?P \parallel b!Q]$$

$$\{a?, b!c?\}^{\perp\perp} = [a!\checkmark \parallel b?\checkmark] = [a?P, b!Q]$$

$$a! \parallel a?P \quad\rightarrow\quad P \qquad (a \in A)$$

$$\{a!\}^{\perp\perp} = [a?\checkmark]^{\perp} = [a!.P]$$

$\{a?\}^{\perp\perp} = \mathbf{T}$, in particular $\{a?\}^{\perp} = \{\epsilon\}^{\perp}$

basic immediate observables

Basic observations

$$[a!P, b!Q] = \{a?\checkmark \parallel b?\checkmark\}^{\perp} = \{a!\}^{\perp\perp} \cup \{b?\}^{\perp\perp}$$

$$[a!P \parallel b!Q] = \{a?\checkmark, b?\checkmark\}^{\perp} = \{a! \parallel b!\}^{\perp\perp}$$

For V, W biorthogonals $V + W = (V \cup W)^{\perp\perp} = (V^{\perp} \cap W^{\perp})^{\perp}$

Biorthogonal V is *irreducible* when
$V = W + W' \Rightarrow V = W \vee V = W'$

$[a!P, b!Q]$ is reducible

$[a!P \parallel b!Q]$ is irreducible, but

$[a!P \parallel b!Q]^{\perp} = [a?\checkmark, b?\checkmark] = \{a?\checkmark\}^{\perp\perp} \cup \{b?\checkmark\}^{\perp\perp}$ is reducible

basic immediate observables

Barbs

A barb is a **proper biorthogonal** V st:

1. V is **irreducible**;
2. $V^\perp$ is **irreducible**.

$$T{\downarrow_B} \stackrel{\text{def}}{=} T^{\perp\perp} \subseteq B$$

$$t{\Downarrow_B} \stackrel{\text{def}}{=} \exists t'.t \rightarrow^* t' \wedge t'{\downarrow_B}$$

Thm 1

the synchronous barbs are:

$\{a!\}^{\perp\perp}$ & $\{a?\}^{\perp\perp}$

Thm 2

the asynchronous barbs are:

$\{a!\}^{\perp\perp}$

Proof relies on:

1. $V + W = V \cup W$
2. Irreducibles are generated by a single element

Barbs for real calculi

- Since only immediate observations are needed:
  - full calculi such as CCS or Pi can be translated to idealised calculi in order to find barbs

Join-like features

$$\begin{array}{rcl} a?P \parallel a!P' & \rightarrow & P \parallel P' \\ ab?P \parallel a!P' \parallel b!P'' & \rightarrow & P \parallel P' \parallel P'' \end{array}$$

$$\{a?\checkmark\}^{\perp\perp} = [a!P]^{\perp} = [a?\checkmark]$$

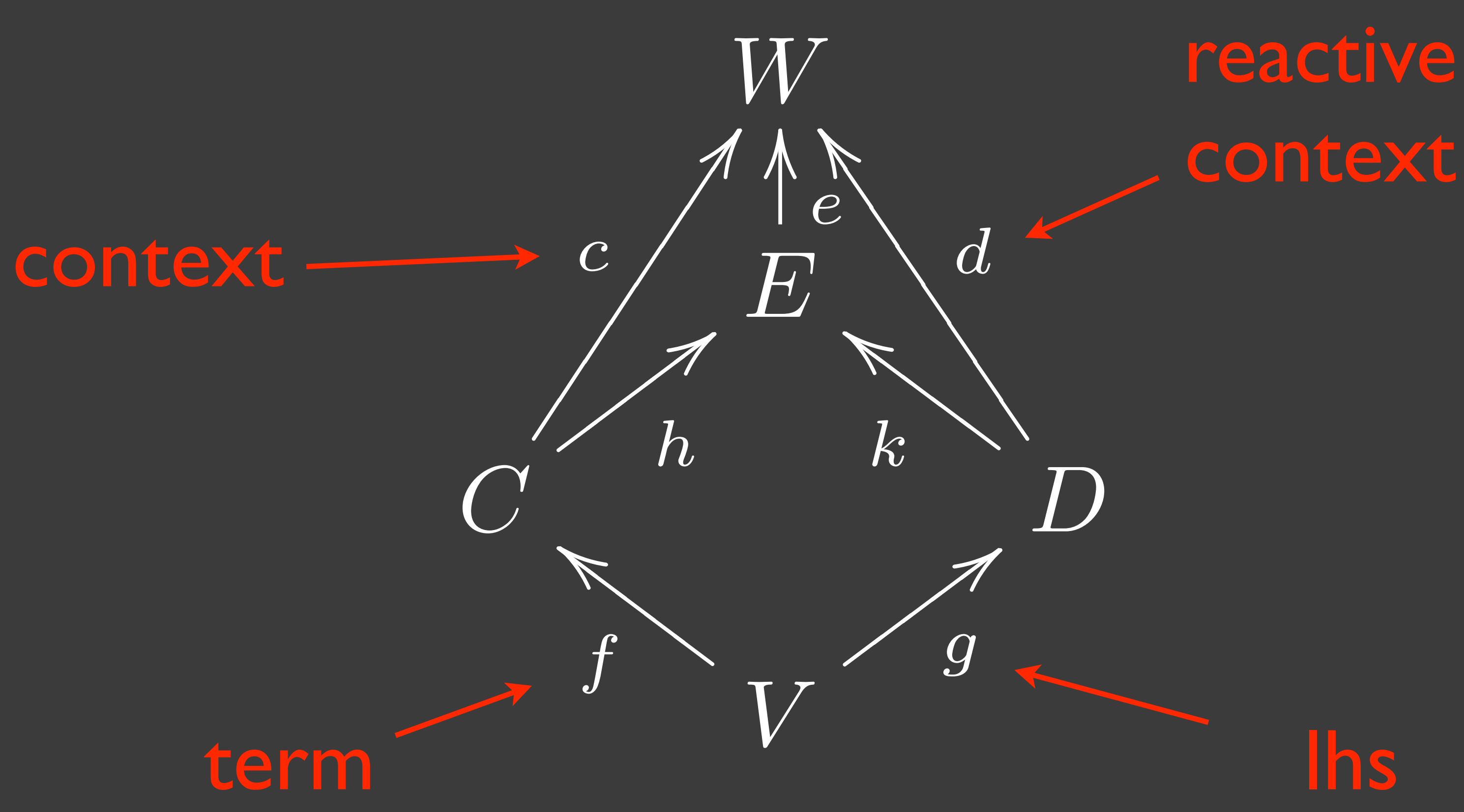
$$\{b?\checkmark\}^{\perp\perp} = [b?\checkmark]$$

$$[a?\checkmark, b?\checkmark]^{\perp\perp} = [a!P \parallel b!Q]^{\perp} = [a?\checkmark, b?\checkmark, ab?\checkmark]$$

This calculus' biorthogonals are not closed under union! Hard to characterise barbs.

Goal 2: Labels

- Leifer and Milner 2000 - relative pushouts
  - labels are "smallest contexts which allow reduction"



Problems

$$a!P \parallel a?Q \;\rightarrow\; P \parallel Q \quad (a \in A)$$

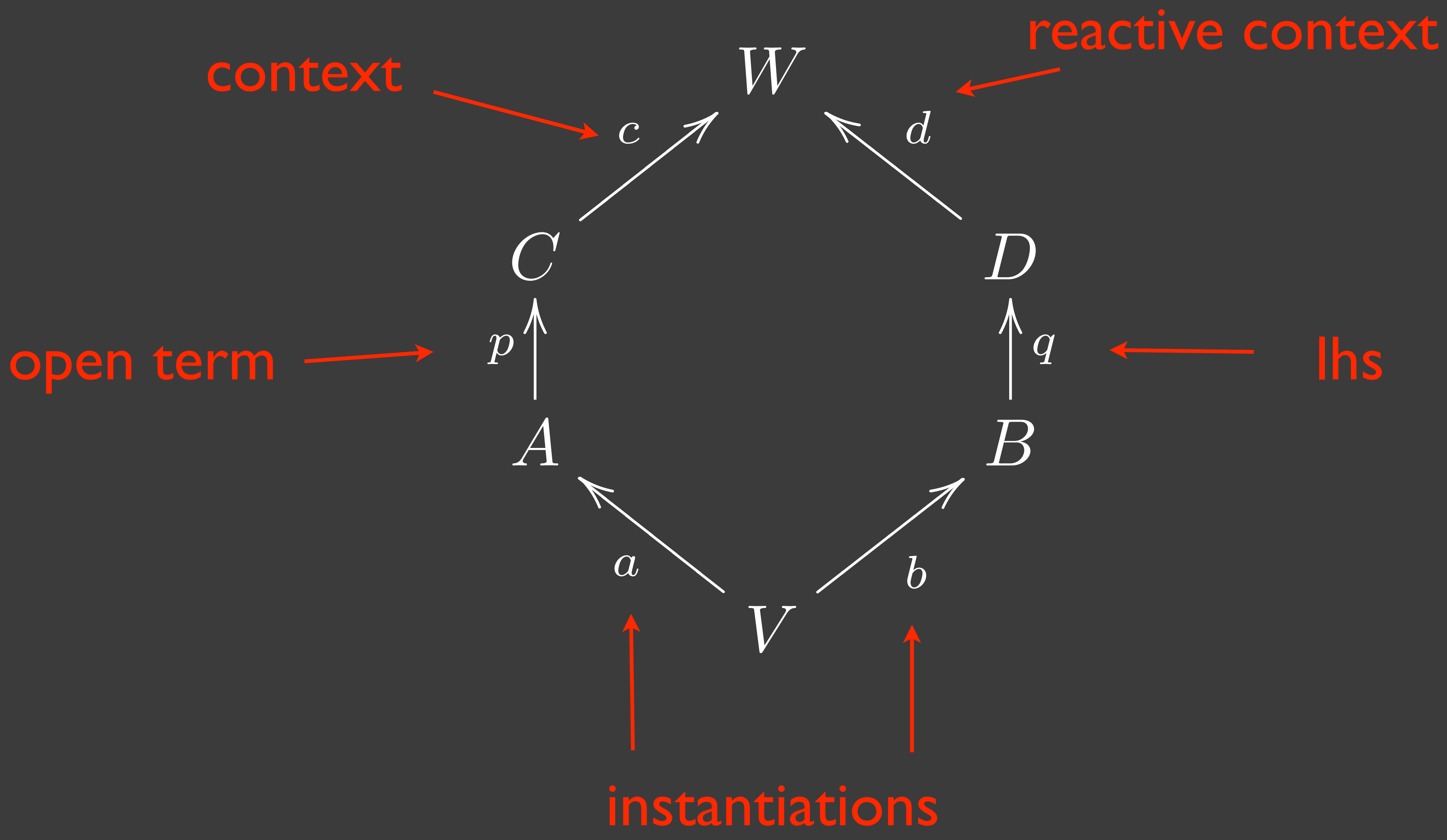
instantiating P and Q leads to infinitely many ground rules

... and so to infinitely branching rpo lts with infinitely many useless labels

$$a!1 \parallel a?2 \quad \rightarrow \quad 1 \parallel 2 \qquad (a \in A)$$

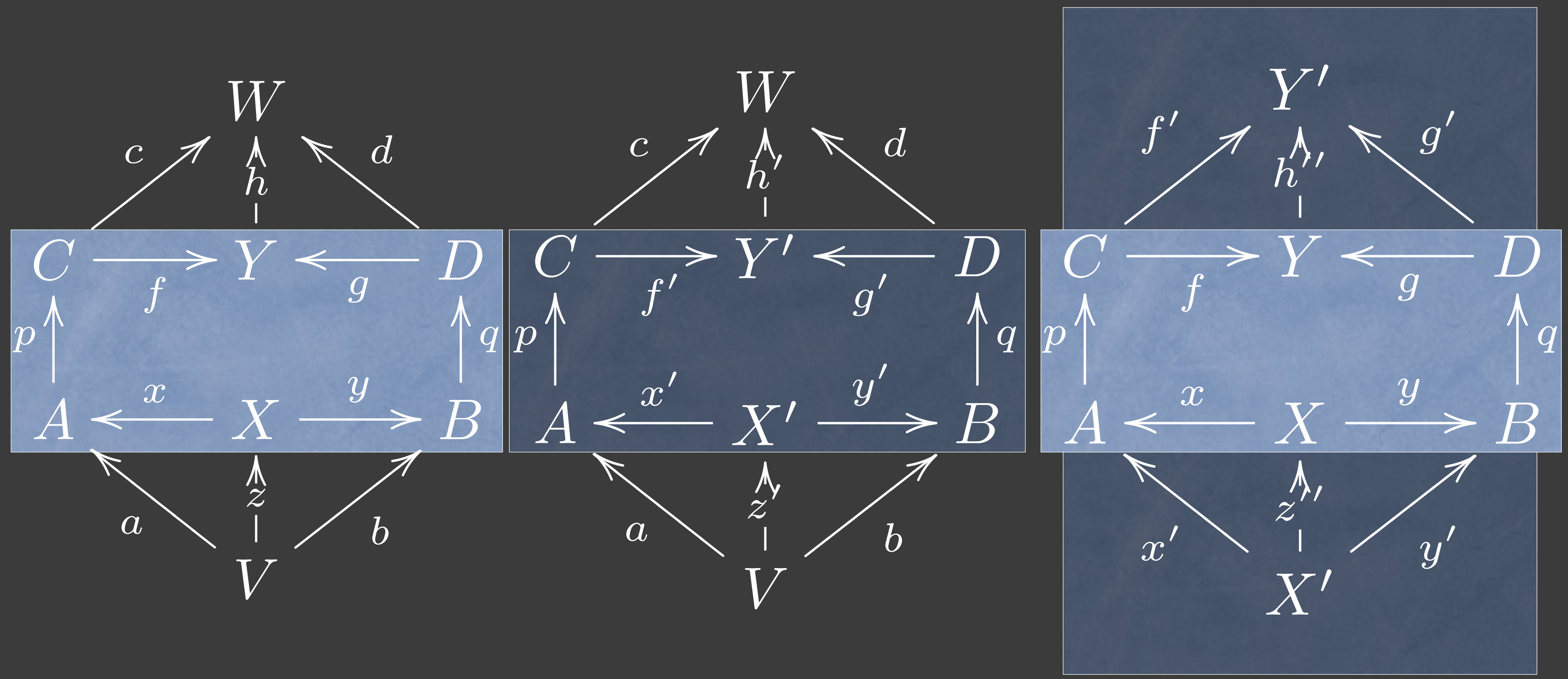
Hexagons

Semantics for parametric terms



Luxes

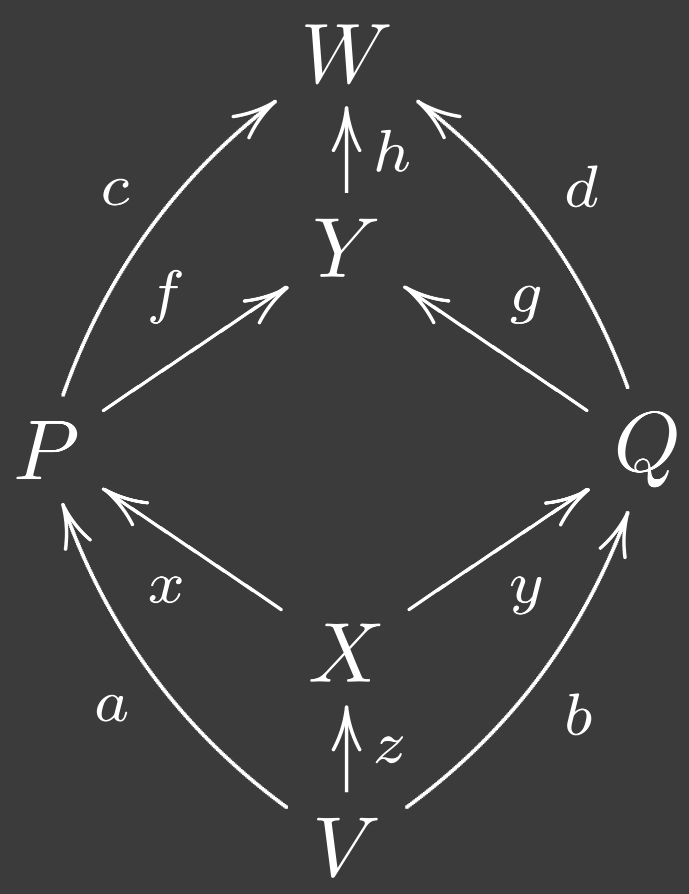
locally universal hexagons



or simply a coproduct in a twisted arrow category...

Theorem

A category has luxes when it
- has relative pushouts
- has relative pullbacks
- rpo's and rpb's "commute"



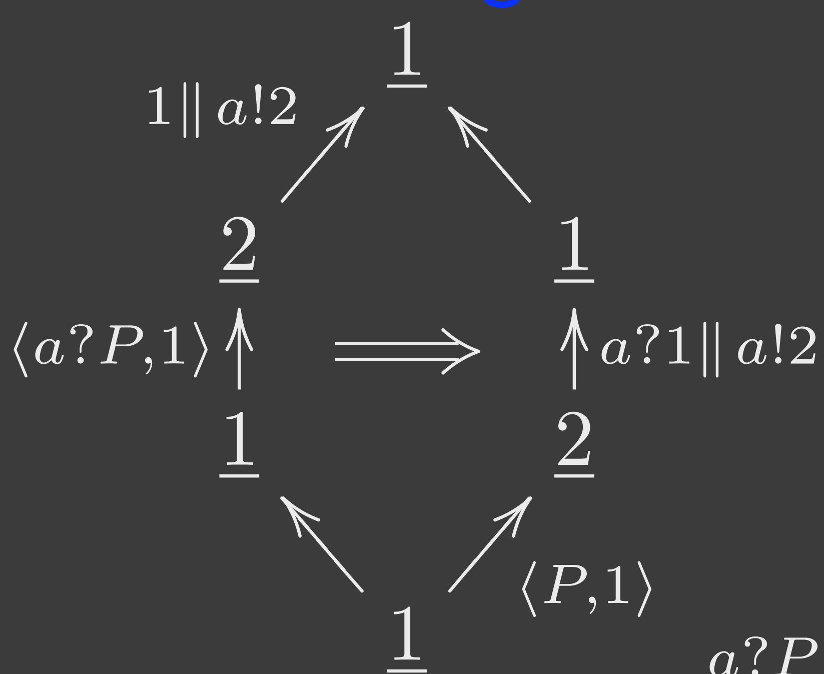
Set doesn't have luxes, but many "syntactic" categories do.

Examples

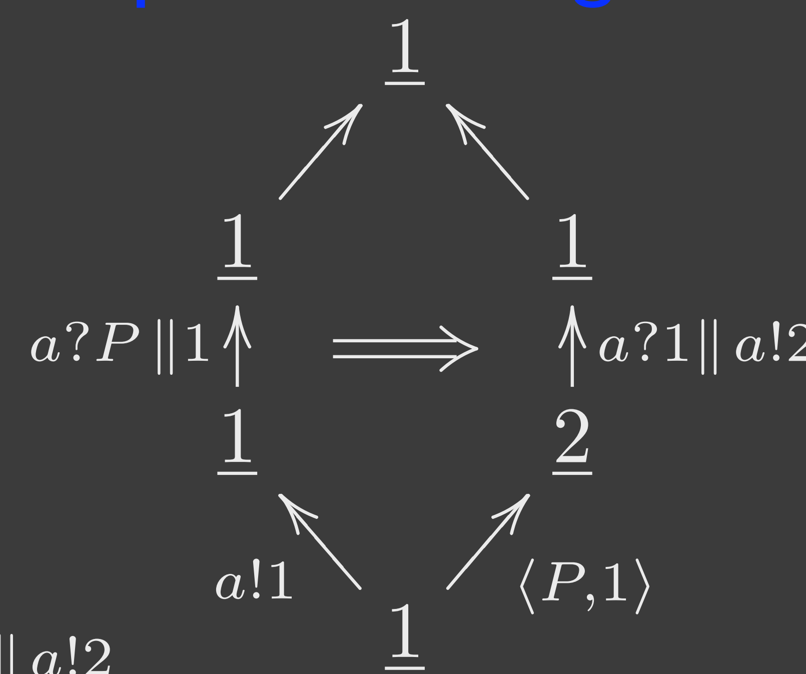
$$P \ ::= \ \epsilon \ | \ a?P \ | \ a!P \ | \ P \parallel P$$

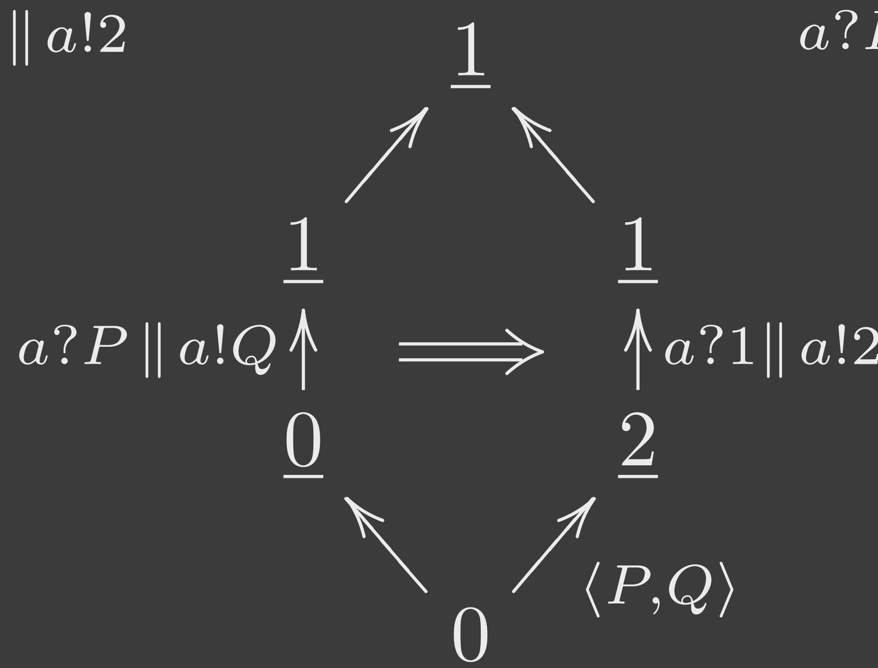
let P & Q be some terms

context gives



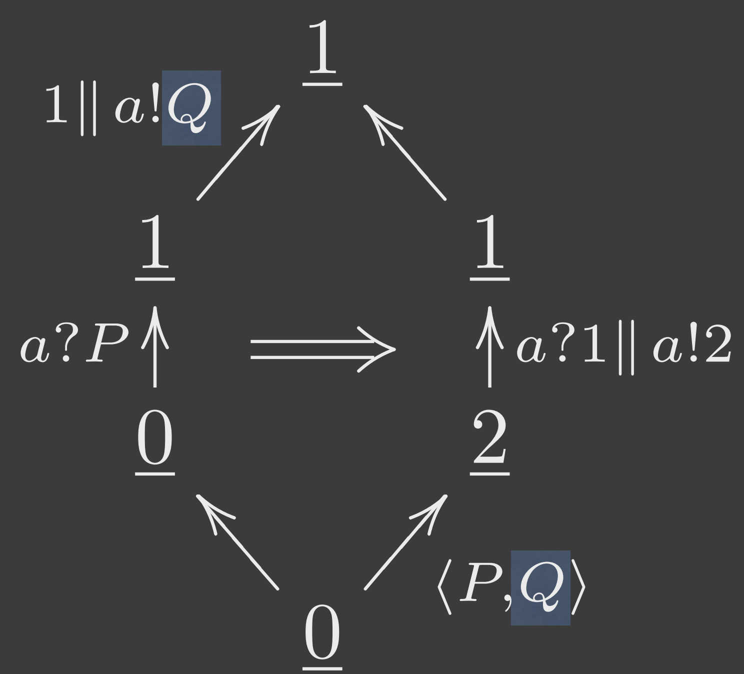
parameters give

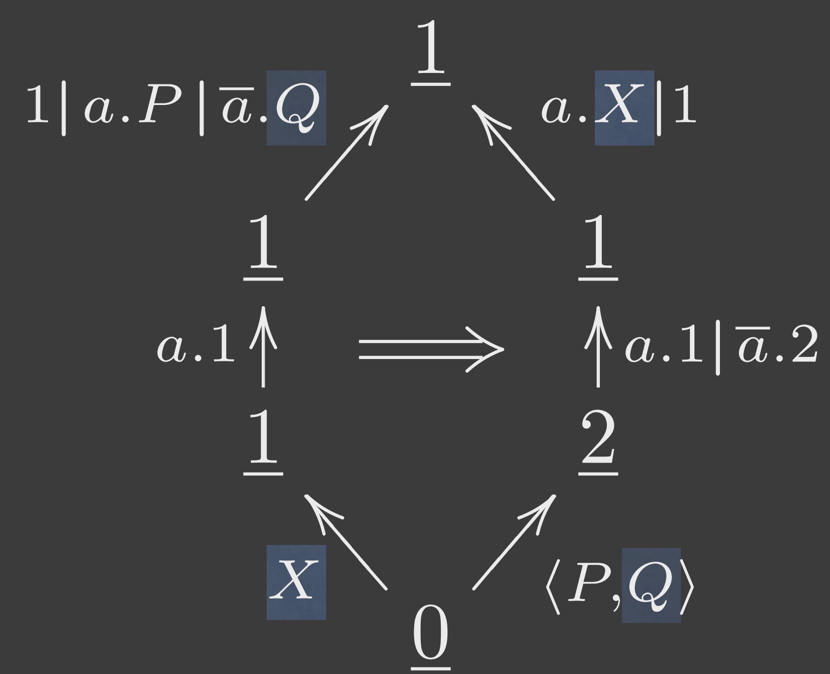




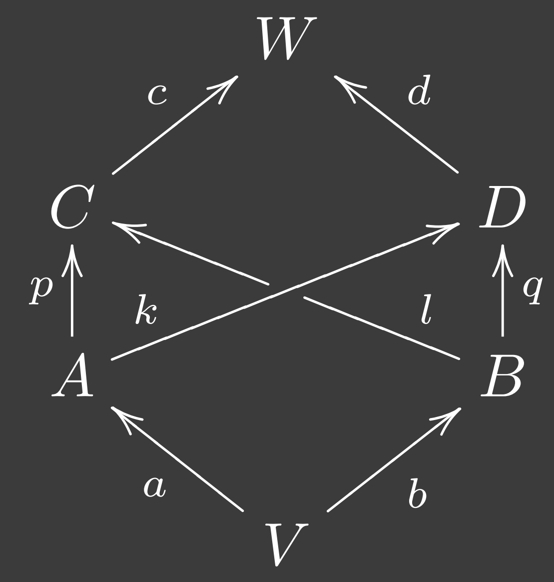
nothing gives

Problems





possible solution:



... but as yet no lts or congruence theorem

Related work

- Barbs
  - basic biorthogonality framework: Girard's phase semantics for linear logic, Pitt's toptop-closed relations, Krivine's realisability, P.-A. Mellies and J. Voullion LICS '05
  - irreducibility: basic algebraic geometry
- Labels
  - F. Bonchi, B. Koenig, U. Montanari. *Saturated semantics for reactive systems*. Proceedings of LICS '05;
  - F. Bonchi, F. Gadducci, B. Koenig. Process bisimulation via a graphical encoding. Proceedings of ICGT '06.
  - O. Jensen. PhD thesis, Cambridge '06.
  - Robin Milner's work on bigraphs

Conclusions

- Barbs
  - study interesting reduction rules
- Labels
  - understand relationship between the contribution of contexts and parameters
  - derive asynchronous labels (Honda-Tokoro)
- J. Rathke, V. Sassone and P. Sobocinski. *Semantic barbs and biorthogonality.* Submitted, 2006.
- B. Klin, V. Sassone and P. Sobocinski. *Labels from reductions: towards a general theory.* Proceedings of Calco'05, 2005.