Semantic barbs:
what’s in an observation?

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Blackbox testing / bisimulation

Let's check!

Observer
System1 System2

Bisimulation Game:
One system produces an observation, the other must be able to match it

if $Sys_1 - obs \rightarrow Sys'_1$ then $Sys_2 - obs \rightarrow Sys'_2$

with $Sys'_1$ and $Sys'_2$ equivalent vice versa with $Sys_1$ and $Sys_2$ exchanged
The holy grail

most current calculi have an underlying reduction semantics

Suppose we have syntax + reduction semantics:

Goal 1: Obtain a canonical contextual equivalence \( \approx \)

= derive barbs

Goal 2: Obtain a bisimulation proof method for \( \equiv \)

= derive labels

both stories start with seminal papers by Robin Milner
Goal 1: Barbs

- **basic observable**
- normally only **immediate** observations
- introduced by Milner & Sangiorgi (1992) for CCS
- reduction congruence is coarser than bisimilarity in CCS
- **barbs come with no explanation**
  - calculus-specific choices of barbs - often the “natural” choice forced by an a priori labelled semantics & labelled equivalence
Observable properties

\[ \mathcal{O} : T \times \Gamma \to \Pi \]

- \text{terms}
- \text{contexts}
- \text{processes}

\[
\bot \subseteq \Pi \quad \text{"successful processes"}
\]

\[
(\_ \downarrow) : \mathcal{P}(T) \to \mathcal{P}(\Gamma)
\]

\[
T \mapsto \{ \gamma \in \Gamma \mid \forall t \in T. \ t \perp \gamma \}
\]

- contexts successful for all terms in \( T \).

\[
(\_ \downarrow) : \mathcal{P}(\Gamma) \to \mathcal{P}(T)
\]

\[
\Gamma \mapsto \{ t \in T \mid \forall \gamma \in \Gamma. \ t \perp \gamma \}
\]

- terms successful for all contexts in \( \Gamma \).

- means \( t \Gamma \in \bot \)
... the usual properties follow

\[ X \subseteq Y \Rightarrow Y^\perp \subseteq X^\perp \]
\[ X \subseteq X^{\perp \perp} \quad \text{and} \quad X^\perp = X^{\perp \perp \perp} \]
\[ X^\perp \cap Y^\perp = (X \cup Y)^\perp \quad \text{but} \quad X^\perp \cup Y^\perp \subseteq (X \cap Y)^\perp \]

\[ \{t_1\}^\perp = \{t_2\}^\perp \quad \text{— } t_1 \text{ and } t_2 \text{ have the same observations} \]

**Biorthogonal:** a set \( V \) such that \( V = V^{\perp \perp} \)

**Fact:** biorthogonals are closed under arbitrary intersections but not in general under (even binary) unions

\[ \bigcap_i V_i = \bigcap_i V_i^{\perp \perp} = \bigcap_i (V_i^\perp)^\perp = (\bigcup_i V_i^\perp)^\perp \]
\[ V_1 \cup V_2 = V_1^{\perp \perp} \cup V_2^{\perp \perp} \subseteq (V_1^\perp \cap V_2^\perp)^\perp = (V_1 \cup V_2)^{\perp \perp} \]
Idealised calculi

\[
\begin{align*}
T & \quad P ::= & \epsilon & | & P \parallel P & | & M.P \\
M & ::= & a? & | & a! & (a \in A)
\end{align*}
\]

Synchrony

\[a!P \parallel a?Q \rightarrow P \parallel Q \quad (a \in A)\]

Asynchrony

\[a! \parallel a?P \rightarrow P \quad (a \in A)\]

Broadcast

\[a!P \parallel \prod_i a?.Q_i \rightarrow P \parallel \prod_i Q_i\]
Immediate observations

\[ @: P \times C \to P^\check \]
\[ (t, \gamma) \mapsto t \parallel \gamma \]

\[ \pi \text{ is spent } \overset{\text{def}}{=} \text{it has precisely one } \check \text{ as a component} \]

\[ \pi \in \bot \text{ iff } \exists \pi' \in P^\check. \ \pi' \text{ spent } \land \ \pi \to \pi' \]
Immediate observations: examples

\[ a! P \parallel a? Q \rightarrow P \parallel Q \quad (a \in A) \]

\[ \{ a! \} \downarrow \downarrow = [a? \checkmark] \downarrow = [a! P] \]

\[ \{ a? \} \downarrow \downarrow = [a? P] \]

\[ \{ a? \parallel b! c? \} \downarrow \downarrow = [a! \checkmark, b? \checkmark] \downarrow = [a? P \parallel b! Q] \]

\[ \{ a?, b! c? \} \downarrow \downarrow = [a! \checkmark \parallel b? \checkmark] = [a? P, b! Q] \]

\[ a! \parallel a? P \rightarrow P \quad (a \in A) \]

\[ \{ a! \} \downarrow \downarrow = [a? \checkmark] \downarrow = [a!..P] \]

\[ \{ a? \} \downarrow \downarrow = T, \text{ in particular } \{ a? \} \downarrow = \{ \epsilon \} \downarrow \]
Basic observations

\[[a!P, b!Q] = \{ a?\checkmark \parallel b?\checkmark \}\perp = \{ a! \}\perp\perp \cup \{ b? \}\perp\perp\]

\[[a!P \parallel b!Q] = \{ a?\checkmark , b?\checkmark \}\perp = \{ a! \parallel b! \}\perp\perp\]

For \( V, W \) biorthogonals \( V + W = (V \cup W)\perp\perp = (V\perp \cap W\perp)\perp\)

\[\text{Biorthogonal } V \text{ is irreducible when } V = W + W' \Rightarrow V = W \lor V = W'\]

\[[a!P, b!Q] \text{ is reducible}\]

\[[a!P \parallel b!Q] \text{ is irreducible, but}\]

\[[a!P \parallel b!Q]^\perp = [a?\checkmark , b?\checkmark] = \{ a?\checkmark \}\perp\perp \cup \{ b?\checkmark \}\perp\perp \text{ is reducible}\]
Barbs

A barb is a **proper biorthogonal** $V$ st:

1. $V$ is **irreducible**;
2. $V^\bot$ is **irreducible**.

**Thm 1**

the synchronous barbs are:

\[
\{a!\} \quad \& \quad \{a?\}
\]

**Thm 2**

the asynchronous barbs are:

\[
\{a!\}
\]

Proof relies on:

1. $V + W = V \cup W$
2. Irreducibles are generated by a single element
Barbs for real calculi

• Since only immediate observations are needed:
  
  • full calculi such as CCS or Pi can be translated to idealised calculi in order to find barbs
Join-like features

\[
\begin{align*}
\text{a?P} \parallel \text{a!P'} & \rightarrow \text{P} \parallel \text{P'} \\
\text{ab?P} \parallel \text{a!P'} \parallel \text{b!P''} & \rightarrow \text{P} \parallel \text{P'} \parallel \text{P''}
\end{align*}
\]

\[
\begin{align*}
\{ \text{a?✓} \} & \perp\perp = [\text{a!P}] \perp = [\text{a?✓}] \\
\{ \text{b?✓} \} & \perp\perp = [\text{b?✓}] \\
[\text{a?✓}, \text{b?✓}] & \perp\perp = [\text{a!P} \parallel \text{b!Q}] \perp = [\text{a?✓}, \text{b?✓}, \text{ab?✓}]
\end{align*}
\]

This calculus’ biorthogonals are not closed under union! Hard to characterise barbs.
Goal 2: Labels

- Leifer and Milner 2000 - relative pushouts
  - labels are “smallest contexts which allow reduction”
Problems

\[ \begin{align*}
    a!P \parallel a?Q & \rightarrow P \parallel Q \quad (a \in A) \\
\end{align*} \]

instantiating \( P \) and \( Q \) leads to infinitely many ground rules

... and so to infinitely branching rpo

its with infinitely many useless labels

\[ \begin{align*}
    a!1 \parallel a?2 & \rightarrow 1 \parallel 2 \quad (a \in A) \\
\end{align*} \]
Hexagons

Semantics for parametric terms

context → open term

c → W → d

reactive context → lhs

instantiations → V

A → p → C

B → q → D

V → a → A

B → b → V
or simply a coproduct in a twisted arrow category...
A category has luxes when it
- has relative pushouts
- has relative pullbacks
- rpo’s and rpb’s “commute”

**Set** doesn’t have luxes, but many “syntactic” categories do.
Examples

\[ P ::= \epsilon \mid a?P \mid a!P \mid P \parallel P \]

let \( P \) & \( Q \) be some terms

context gives

parameters give

nothing gives
Problems

possible solution:

... but as yet no lts or congruence theorem
Related work

- Barbs
  - **basic biorthogonality framework**: Girard’s phase semantics for linear logic, Pitt’s toptop-closed relations, Krivine’s realisability, P.-A. Mellies and J. Voullion LICS ’05
  - **irreducibility**: basic algebraic geometry
- Labels
  - Robin Milner’s work on bigraphs
Conclusions

• Barbs
  • study interesting reduction rules

• Labels
  • understand relationship between the contribution of contexts and parameters
  • derive asynchronous labels (Honda-Tokoro)
