A VLSI Array Architecture for Realization of DFT, DHT, DCT and DST

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Abstract:

An unified array architecture is described for computation of DFT, DHT, DCT and DST using a modified CORDIC (CoOrdinate Rotation DIgital Computer) arithmetic unit as the basic Processing Element (PE). All these four transforms can be computed by simple rearrangement of input samples. Compared to five other existing architectures, this one has the advantage in speed in terms of latency and throughput. Moreover, the simple local

neighborhood interprocessor connections make it convenient for VLSI implementation. The architecture can be extended to compute transformation of longer length by judicially cascading the modules of shorter transformation length which will be suitable for Wafer Scale Integration (WSI). CORDIC is designed using Transmission Gate Logic (TGL) on sea of gates semicustom environment. Simulation results show that this architecture may be a suitable candidate for low power/low voltage applications.

Keyword: VLSI, CORDIC, Wavefront array, DFT, DHT, DCT, DST.

1. Introduction:

In Digital Signal Processing (DSP) orthogonal transforms e. g., Discrete Fourier Transform (DFT), Discrete Hartley Transform (DHT), Discrete Cosine Transform (DCT) and Discrete Sine Transform (DST) are useful for spectrum analysis, data compression, speech processing and in many other applications [1, 2]. Since all these transforms are computationally intensive and involve complicated data manipulations, the help of parallel processing array architectures is much sought for their implementation. Design of array architecture for computing a number of such transforms in a unified manner, through simple rearrangement of input samples may be particularly useful for a specific application. Different array architectures for these transforms have been proposed over the past decade [3-8]. Most of the architectures are of systolic type and a major part of them use Multiplier-Accumulator (MAC) based PE. However, this method essentially requires a large storage of the trigonometric co-efficients for each of the transform to be computed. One way to overcome this difficulty is to use CORDIC arithmetic processor as the basic PE that facilitates trigonometric multiplication with elegance by using only shift-and-add method and rules out the requirement of co-efficient storage. Accordingly,

several CORDIC based array architectures have been proposed for computation of the above mentioned transforms.

Despite its elegance in computing trigonometric multiplication, the conventional CORDIC element suffers from the drawbacks like requirement of the scale factor compensation and lower operation speed. Intense attentions have been paid to develop special purpose CORDIC processors in order to overcome these drawbacks of the conventional CORDIC method.

In this paper, the authors first develop a new CORDIC processor, which is scaling free in nature and the error arising from the approximation of the trigonometric function is analyzed. Subsequently, this processor is used as the basic PE to develop a unified, asynchronous, wavefront array type architecture for fast computation of DXT - a collective term representing either of the DFT, DHT, DCT or DST. Being multiplierless, the modified CORDIC arithmetic unit is expected to reduce the hardware cost considerably compared to that of the MAC based designs. Keeping an eye on the low power/low voltage applications, the datapath of the basic PE has been designed and simulated using TGL in sea of gates semicustom environment. The paper is structured as follows: the theoretical background of the modified CORDIC and the DXT have been developed in Section 2. A brief introduction to the modified CORDIC [9] is also provided in this section along with its design, layout of the datapath and simulated performance. The proposed array architecture is described in Section 3. In Section 4, the authors describe the design and furnish the result of simulated power consumption of the modified CORDIC unit. The salient features of the DXT architecture along with a

comparison with five other existing architectures are also described in this section.

Conclusions are drawn in Section 5.

2. Theoretical Information:

2.1. The CORDIC unit:

The transforms of DXT family involve trigonometric operations on the input sample sequences that can be expressed in terms of plane rotations. The CORDIC unit [10] can iteratively rotate an input vector $\mathbf{A} = [A_x \ A_y]$ through a target angle ψ (say in clockwise direction) through small steps of elementary angles ψ_i (so that $\Sigma_i \ \psi_i = \psi$) to generate an output vector $\mathbf{B} = [B_x \ B_y]$. This operation can be represented mathematically as

i. e.,
$$\mathbf{B} = \mathbf{A} \operatorname{Rot}(\mathbf{\psi})$$
 (2)

For fast convergence of the algorithm to the final small angle value, a simple modification [9] of the conventional CORDIC algorithm is applied here which represents the small incremental angle by $\sin\psi_i\approx\psi_i=2^{-i}$ and expresses the basic Rot (ψ) operation

as Rot
$$(\psi_i) = \begin{bmatrix} 1 - 2^{-(2i+1)} & -2^{-i} \\ 2^{-i} & 1 - 2^{-(2i+1)} \end{bmatrix}$$
; for a 16 bit machine, $i \in \{4, 5,, 15\}$.

(3)

Thus, the rotation through the target angle ψ may be expressed as,

Rot
$$(\psi) = \prod_{i=0}^{b-1} \begin{bmatrix} 1 - 2^{-(2i+1)} & -2^{-i} \\ 2^{-i} & 1 - 2^{-(2i+1)} \end{bmatrix}$$
 (4)

Where b is the wordlength of the machine in which the operation is supposed to be implemented. Fig. 1 shows the structure of the datapath of the elementary CORDIC unit.

It is evident from the above formulation that the modified CORDIC unit is free from the scale factor compensation step which saves a significant amount of hardware compared to the conventional CORDIC unit and subsequently makes the CORDIC operation faster.

Error analysis:

The potential sources of error for the hardware realization of an algorithm are two folds namely, the error due to the quantization of the input word and due to the finite wordlength. In case of CORDIC another potential error source in the error due to approximation of the target angles. In our scheme, the errors due to first two sources are same as the conventional implementation of the circular CORDIC. Thus, the detailed analysis of the third error source is presented here with the consideration that the implementation has been done with 16-bit wordlength.

Let θ_c be the computed angle whereas θ is the ideal target angle. Then from equation (1),

$$B_{x} = A_{x} \cos \theta_{c} + A_{y} \sin \theta_{c} \tag{3a}$$

$$B_{y} = -A_{x} \sin \theta_{c} + A_{y} \cos \theta_{c} \tag{3b}$$

Where $\theta_c = \theta - \epsilon$ and ϵ is the error due to angle approximation. From equation (4) it may be readily seen that the value of ϵ is $O(2^{-16})$. On simplification, equation (3a) and (3b) yields,

$$B_{xerror} = (A_x \cos\theta_c + A_y \sin\theta_c)(1 - \cos\epsilon) + (-A_x \sin\theta_c + A_y \cos\theta_c) \sin\epsilon$$
 (4a)

$$B_{\text{yerror}} = (-A_x \sin\theta_c + A_y \cos\theta_c)(1 - \cos\epsilon) - (A_x \cos\theta_c + A_y \sin\theta_c) \sin\epsilon$$
 (4b)

Since $\varepsilon = O(2^{-16})$, $(1 - \cos \varepsilon)$ and $\sin \varepsilon$ will be $O(2^{-33})$ and $O(2^{-16})$ respectively. Thus the maximum error that can occur due to the angle approximation according to the proposed

scheme is $O(2^{-16})$ which seems to be quite within limit. The simulation result for the errors due to approximation of different target angles is shown in Figure 2.

To design the low power CORDIC unit, TGL style [11] is used in 1.6 µm double layer sea of gates semicustom environment. The area of the designed CORDIC chip is (4.4642 x 3.9538) mm². The performance of the circuit according to the Switch Level timing Simulator (SLS) at 5 V supply is, datapath delay = 54 nsec., power dissipation = 37 mW, Power-Delay Product (PDP) = 1.998 nJ and Energy-Delay Product (EDP) = 1.078 x 10⁻¹⁶ J-sec. To get the effect of worst case power consumption, the activity factor has been taken as 1. At 5 V supply, the calculated values for delay, dynamic power consumption, PDP and EDP of the simulated circuit modules used to construct the processor chip are given in Table 1. Layout of one 4-bit conditional sum adder/subtractor is shown in Fig. 2. These results indicate that this CORDIC processor may be a suitable choice for low power/low voltage operation domain. It is expected that with proper transistor sizing and threshold voltage scaling, the processor will show a better performance and consume less power [12].

2.2. Theoretical background of the transforms:

Considering a real sample sequence f(n), $n \in \{0, 1, ..., N-1\}$ the DXT can be defined as

DFT:
$$F(k) = \sum_{n=0}^{N-1} f(n) [\cos(2\pi / N) kn - j \sin(2\pi / N) kn] = F_x(k) + j F_y(k)$$
 (5)

DHT:
$$H(k) = \sum_{n=0}^{N-1} f(n) [\cos(2\pi / N) kn + \sin(2\pi / N) kn]$$
 (6)

DCT:
$$C(k) = \sum_{n=0}^{N-1} f(n) \cos [\pi k(2n + 1) / 2N]$$
 (7)

DST:
$$Z(k) = \sum_{n=0}^{N-1} f(n+1) \sin [\pi k(2n+1) / 2N]$$
 (8)

In terms of the operator Rot (*), the equations can be represented as

DFT:
$$[F_x F_y] = \sum_{n=0}^{N-1} [f(n) 0] Rot (m\theta)$$
 (9)

DHT:
$$[H(k) H(N-k)] = \sum_{n=0}^{N-1} [f(n) f(n)] \text{ Rot } (m\theta)$$
 (10)

DCT:
$$[C(k) - C(N-k)] = [\sum_{n=0}^{N-1} [r(n) \ 0] \text{ Rot } (m\theta)] \text{ Rot } (k\phi)$$
 (11)

where $k=0,\,1,\,.....,\,N-1;\,m=kn$ modulo N (=<kn>_N), $\theta=2\pi/N$ and $\varphi=\theta/4$ and

$$r(n) = f(2n)$$
, for $n = 0, 1,, (N-1)/2$
= $f(2N - 2n - 1)$, for $n = (N + 1)/2, ..., (N-1)$ (12)

The comparison between equations (7) and (8) shows that DCT and DST are obtainable from either of the two by proper data ordering.

Thus, all of the above four transforms can be clubbed into a single equation as

$$Y(k) = \left[\sum_{n=0}^{N-1} h(n) \operatorname{Rot} (m\theta)\right] \operatorname{Rot} (k\phi)$$
(13)

where m = <kn>_N , θ = 2\pi/N, φ = 0 for DFT/DHT and φ = $\theta/4$ for DCT and DST.

3. Array architecture:

Fig. 3 shows the proposed unified architecture for N=5. To achieve the operation $Rot(m\theta)$ on the given sample set, the input samples for θ array has been scheduled according to the permutation sequence of $\langle nk \rangle_N$ (ref. to Table 2). Computation of the final result has been carried out by rotating the partial results emerging from the θ array by an angle $k\phi$. The data from the FIFO can be loaded to the registers at the input of the adders with the help of one controlling counter and a mode control switch. The 'ON' and

'OFF' states of the mode control switch decide whether the data from the FIFO will go to both the adders or just to one adder. In steady state, the whole operation is summarized in the following pseudocode,

For i = 1 to N, do in parallel /* i = column (for DFT and DHT i varies from 1 to (N-1)) */

For j = 1 to (N-1), do /* j = row */

Load data (j, i); /* Load j th data from the i th FIFO to the register of (j, i) th adder */

For all j, do in parallel

Wait_data (j, i-1); /* Wait until data from the output of (j, i-1) th processor is available */

Add_data (j, i-1) and (j, i); /* For all $j \in \{1, 2, ..., (N-1)\}$ simultaneous addition of (j, i-1) th and (j, i) th data */

Signal_start (Processors (j, i)) /* Issue start signal to all j th processor at i th column when all data from the (j, i) adders are available at the input of i th processor column after addition */

It reveals from Fig. 3 that in order to exploit the inherent pipelinability of the structure, the clock period T_c of the controlling counter must maintain the relationship, $A_d \leq T_c \leq P_d / 4$, where A_d is the FIFO access time and P_d is the delay of a single processor. The processed data move through the array like a plane wavefront. However, a suitable handshaking protocol is to be adopted for data transaction between consequent processor columns.

For clarity, using the unified architecture, the computation of 5 point DCT is explained. The data scheduling according to the permutation sequence of $\langle nk \rangle_N$ (ref. to Table 2) is depicted in Table 3 where U'(k) = U(k) - f(0), the terms U(k)s are the intermediate results after the rotation in the θ plane. The U(0) term, which is a mere accumulation (dc term) of the input samples, is omitted in Table 3. The U'(k) terms after addition with the f(0) terms undergo rotation $k \phi$ to generate the complete set of Y(k). Here the x-component of Y(k) gives rise to the desired DCT of the given sample sequence. The mode control switch is kept in the 'OFF' state here to route the data from the FIFO to the adder at the x inputs of the processors.

Following the same line, the DST can also be obtained by replacing f(n) th sample of DCT with f(n+1) th sample and considering the y-component of the output vector.

Letting $\phi = 0$, the same architecture is capable to compute DHT and DFT. Data distribution for the 5 point DHT is shown in Table 4. For computation of DHT, the mode control switch is kept in the 'ON' state. DFT of the sample set can be obtained by inserting f(n) at the x-input of the input vector and putting 0 at the y input. The real and imaginary parts are obtained from the x and y components of the output vector respectively.

The proposed architecture can be extended for computation of long transform sequence by simply cascading the modules with lower N. As an example, to compute DCT for N=11, the architecture for N=5 is considered as the basic cascading block and coined as generalized module. Table 5 (a) shows the data distribution for 11 point DCT which are partitioned into different blocks (D s) as shown by the solid lines in Table 5 (b). Each partitioned data block can be computed separately using one generalized

module. The resulting structure is shown in Fig. 4 where D s are the inputs corresponding to different blocks of data shown in Table 5.

To compute DFT of complex input data sequence, equation (3) may be rewritten as

$$[F_x(k) F_y(k)] = \sum_{n=0}^{N-1} [f_x(n) f_y(n)] \operatorname{Rot}(m\theta)$$
 (14)

In this case, the operation procedure of the architecture is same as explained earlier, with the difference that $f_x(n)$ and $f_y(n)$ components (the real and imaginary parts of f(n)) are stored in a pair of FIFO dedicated for each processor column and they are simultaneously retrieved during operation. The x and y components of the output vector contain the real and imaginary parts of the result respectively.

4. Salient features and comparison with other architectures:

The proposed architecture is compared with five existing architectures and is summarized in Table 6. The proposed one exhibits local neighborhood connections and is selftimed in nature. Being asynchronous, it is more advantageous than the other systolic designs from fault tolerance perspective. Moreover, the data skewing and subsequent glitch problems are suppressed considerably. The efficient floor utilization is possible due to its nature of simple interprocessor connections. This architecture exhibits same order of latency as that of Pan et al. [3] and Mandal et al. [8]. This architecture is inherently a pipelined one and in the steady state its throughput rate is O(N). Thus, the steady state throughput rate has been considerably enhanced than that of other architectures. The area - time complexity (AT^2) of the proposed architecture is $O(N^4)$.

Another feature of the proposed architecture is its expandability. The modules calculating DXT for shorter transform lengths can be cascaded straightway to compute longer transform sequences. However, in this type of cascading applications, 100% processor

utilization may not be possible for all N. Being cascadable in nature, the proposed architecture is compatible with the WSI (Wafer Scale Integration) technology.

This architecture may be a suitable candidate for low power/low voltage applications. The architecture is based on modified CORDIC method which implies reduction of power consumption at the algorithmic level since the number of operations have been reduced. The presence of block level pipelining and parallelism reduce power consumption at the architectural level. The asynchronous operation mode suppresses the glitches and thus a considerable amount of power can be saved. The CORDIC is designed using TGL and thus reduction of circuit level power consumption is achieved. Therefore, the proposed architecture can save power in the algorithm, architecture and circuit level.

6. Conclusions:

The proposed architecture is an asynchronous, wavefront array type capable of computing DXT. Being data driven and self timed in nature, it eliminates the data skewing and subsequent glitch problems. However, a suitable handshaking protocol is to be adopted to maintain the data driven property. The advantage of modified CORDIC PE stems from an expected lower area consumption than the MAC based PE. Use of CORDIC eliminates the requirement of precomputation of cosθ and sinθ. The simple local interprocessor connection leads to the efficient utilization of the floor space and subsequent reduction in routing area. The presence of inherent pipelining and concurrency in this architecture are advantageous so far the low power/low voltage operation is concerned. Construction of the CORDIC module using TGL methodology enhances the power saving at circuit level. Thus, the proposed structure exhibits power saving at algorithm, architecture and at the circuit level.

A comparative study of the proposed architecture with five other existing architectures reveals its superiority in speed (in terms of latency and throughput) and in processor interconnection over the others. It does not require any global timing synchronization or complicated data manipulations and can straightway be cascaded to realize longer transformation sequences and thus is WSI compatible. The same architectural methodology can be adopted for multidimensional DXT computation and for non prime transform length also.

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Cell name	Delay	Power (mW)	PDP (pJ)	EDP (J-sec)
	(nsec)	@ 18.5 MHz		
Multiplexer bank	0.07	1.2536	0.0877	6.1426 X 10 ⁻²⁴
16 bit register	0.7	3.0889		1.5135 X 10 ⁻²¹
Shifter	19	0.8289		2.9923 X 10 ⁻¹⁹
Adder/Subtractor	7.1	4.0538	28.7823	2.0435 X 10 ⁻¹⁹

Table 1

k n	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

Table 2

	1θ	2θ	3θ	40
U'(1)	f(2)	f(4)	f(3)	f(1)
U'(2)	f(3)	f(2)	f(1)	f(4)
U'(3)	f(4)	f(1)	f(2)	f(3)
U'(4)	f(1)	f(3)	f(4)	f(2)

Table 3

	1θ	2θ	3θ	40
Y'(1)	f(1)	f(2)	f(3)	f(4)
Y'(2)	f(3)	f(1)	f(4)	f(2)
Y'(3)	f(2)	f(4)	f(1)	f(3)
Y'(4)	f(4)	f(3)	f(2)	f(1)

Table 4

	1θ	2θ	3θ	4θ	5θ	6θ	7θ	80	90	10θ
Y'(1)	f(2)	f(4)	f(6)	f(8)	f(10)	f(9)	f(7)	f(5)	f(3)	f(1)
Y'(2)	f(9)	f(2)	f(7)	f(4)	f(5)	f(6)	f(3)	f(8)	f(1)	f(10)
Y'(3)	f(8)	f(5)	f(2)	f(10)	f(3)	f(4)	f(9)	f(1)	f(6)	f(7)
Y'(4)	f(6)	f(9)	f(3)	f(2)	f(8)	f(7)	f(1)	f(4)	f(10)	f(5)
Y'(5)	f(3)	f(7)	f(10)	f(6)	f(2)	f(1)	f(5)	f(9)	f(8)	f(4)
Y'(6)	f(4)	f(8)	f(9)	f(5)	f(1)	f(2)	f(6)	f(10)	f(7)	f(3)
Y'(7)	f(5)	f(10)	f(4)	f(1)	f(7)	f(8)	f(2)	f(3)	f(9)	f(6)
Y'(8)	f(7)	f(6)	f(1)	f(9)	f(4)	f(3)	f(10)	f(2)	f(5)	f(8)
Y'(9)	f(10)	f(1)	f(8)	f(3)	f(6)	f(5)	f(4)	f(7)	f(2)	f(9)
Y'(10)	f(1)	f(3)	f(5)	f(7)	f(9)	f(10)	f(8)	f(6)	f(4)	f(2)

Table 5 (a)

Y'(1) Y'(2)	D11	D12	D13
Y'(3)			
Y'(4)			
Y'(5)			
Y'(6)	D21	D22	D23
Y'(7)			
Y'(8)			
Y'(9)	D31	D32	D33
Y'(10)			

Table 5 (b)

	Cho & Lee	Chang & Wu	Lee [6]	Pan [3]	Mandal [8]	Proposed
	[4]	[5]				
Nature of	Systolic	Systolic	Systolic	Systolic	Systolic	Wavefront
architecture						

Nature of operation	Sequential	sequential	sequential	sequential	concurrent	concurrent
Number of operational blocks	4 multipliers	2 multipliers	8 multipliers	2 multipliers	multiplierless CORDIC (4 adders and 4 shifters)	multiplierless CORDIC (4 adders and 4 shifters)
Number of PE	N + 1	N + 1	N –1	N/2	N x (N - 1)	N x (N - 1)
Total number of multipliers	4 (N +1)	2 (N – 1)	8 N	N	no requirement of real multiplication operation	no requirement of real multiplication operation
Latency	O(2N)	O (2N - 2)	$O[2(\sqrt{N+1}) + \sqrt{N+2}]$	O (N)	O (N)	O (N)
Throughput	O(1)	O(1)	O(1)	O(1)	O(1)	O(N)

Table 6

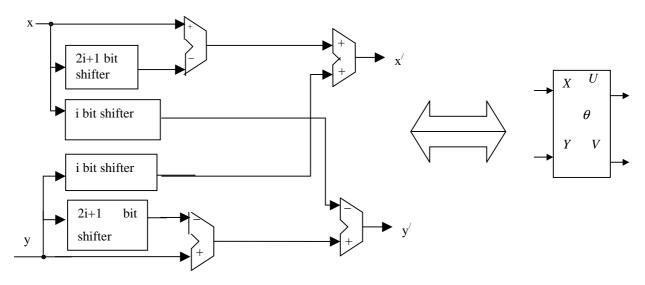
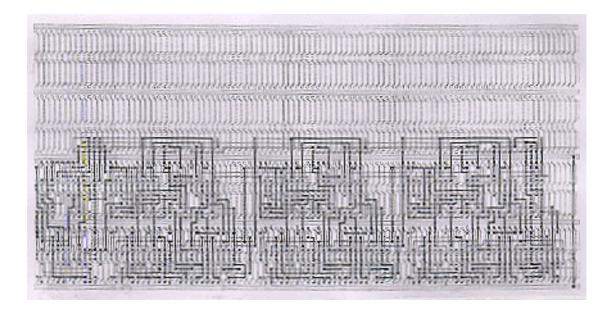


Fig. 1



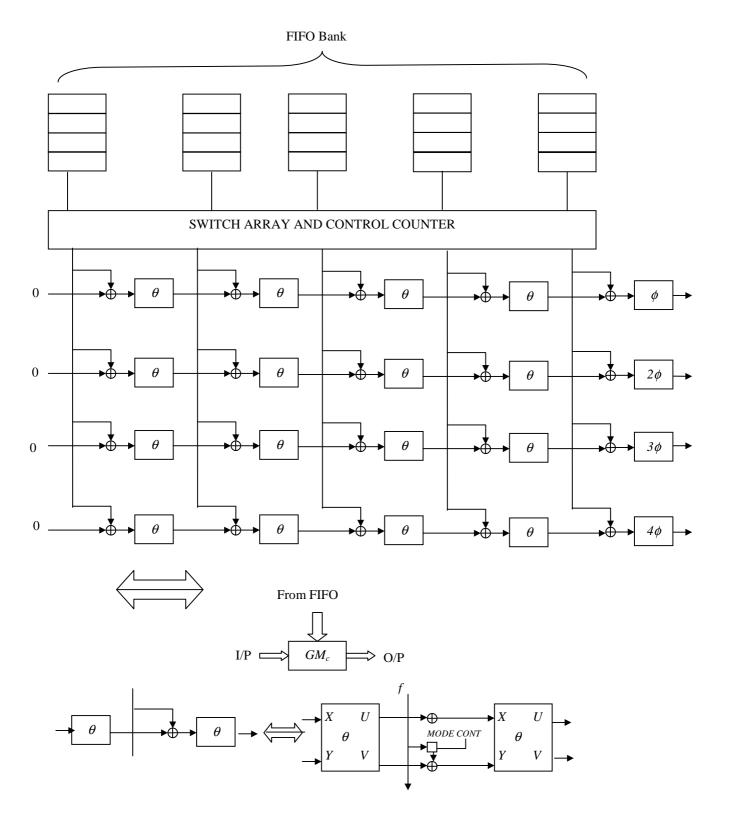


Fig. 3

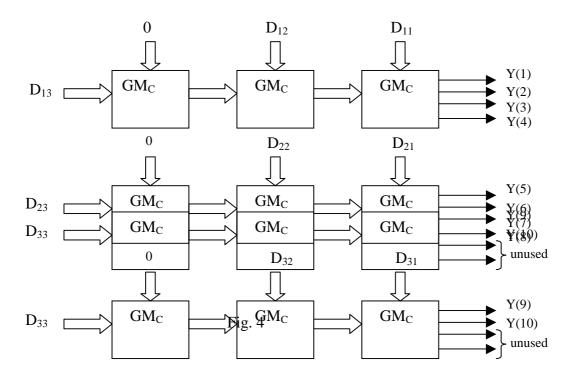


Figure 4

Fig. caption

- Fig. 1. The block diagram of the modified CORDIC unit.
- Fig. 2. Layout of 4 bit adder/subtractor (conditional sum type).
- Fig. 3. The architecture for N = 5 and the symbol (GM_C).
- Fig. 4. The cascaded architecture for 11 point DCT using 5 point DCT modules as the basic block.

Table caption

- Table 1. Power performance of different blocks used to construct the CORDIC unit.
- Table 2. The permutation table of nk modulo N for N = 5.
- Table 3. The data scheduling for 5 point DCT derived from Table 1.
- Table 4. The data scheduling for 5 point DHT derived from Table 1.
- Table 5. The data scheduling for 11 point DCT and its representation in the form of block data (D*).
- Table 6. Comparison with other five architectures.