

# MIMO Detection in Analog VLSI

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**Abstract**—In this paper we propose an analog VLSI approach to maximum a posteriori (MAP) detection in Multiple-Input Multiple-Output (MIMO) systems. This detector can be seen as an extension of the well known analog decoding concept for error correcting codes, as it is constructed using similar building blocks. Therefore, it can naturally interact with analog decoders in order to perform turbo detection in MIMO systems. First transistor-level simulations for a small analog MIMO detector in a 0.25 $\mu$ m BiCMOS process agree well with floating-point digital simulations.

## I. INTRODUCTION

In recent years, analog VLSI has emerged as an advantageous way of implementing efficient signal processing modules. There seems to be a perfect match between translinear circuits and probability computations [1], and it has been exploited in order to construct analog belief propagation networks. These networks have been typically designed to decode error correcting codes, such as turbo or LDPC codes [2], [3].

Analog decoders have the potential to outperform their digital counterparts in terms of power and silicon area [1]. This fact makes them good candidates for wireless devices where power consumption must be kept to a minimum. Another challenge in wireless systems, with ever increasing transmission rates, is to provide high capacity with limited bandwidth. A promising technique to tackle this problem is the use of Multiple-Input Multiple-Output (MIMO) links [4].

The nature of these systems, in which signals are transmitted over a matrix channel, doesn't seem to fit into the analog decoding framework. However, MIMO detection can be expressed as a probabilistic problem and therefore can benefit from the aforementioned qualities of analog decoding circuits. It is the aim of this contribution to bring both concepts together, proposing an analog MIMO detector.

The rest of the paper is organized as follows: section II introduces spatial multiplexing and MAP detection algorithm. In section III analogue implementation is outlined, and section IV presents the first simulation results. Conclusions are drawn in section V.

## II. SYSTEM DESCRIPTION AND DETECTION ALGORITHM

Consider the MIMO system diagram of Fig. 1. At the transmitter side,  $K$  information bits  $b_{1:K}$  are encoded to  $N > K$  coded bits  $c_{1:N}$ , randomly interleaved, and finally modulated and mapped onto  $N_T$  transmit antennas. This basic form of space-time signalling is known as V-BLAST or Spatial-Multiplexing [5].

The resulting symbol vector  $\mathbf{x}$  is transmitted from all  $N_T$  antennas over a narrowband channel  $\mathbf{H}$  of size  $N_R \times N_T$ , where  $N_R$  is the number of receive antennas, and each entry  $h_{i,j}$  represents the channel between transmit antenna  $j$  and receive antenna  $i$ . The system is typically modeled as

$$\mathbf{y} = a\mathbf{H}\mathbf{x} + \mathbf{n}, \quad (1)$$

where  $\mathbf{y}$  is the received vector of length  $N_R$ ,  $a$  is a normalization constant that makes the total energy per symbol equal to unity, and the vector  $\mathbf{n}$  represents the Gaussian noise of zero mean and variance  $\sigma_n^2$ .

The receiver's task is to detect the information bits  $b_{1:K}$  given  $\mathbf{y}$  and  $\mathbf{H}$ . Specifically, we are interested in the full set of marginal posterior probabilities (MPPs)

$$\{f(b_1|\mathbf{y}, \mathbf{H}), f(b_2|\mathbf{y}, \mathbf{H}), \dots, f(b_K|\mathbf{y}, \mathbf{H})\}.$$

In order to ease the computational burden at the receiver, it is common to carry out this task by means of a sub-optimal procedure called turbo-detection. In such case, detection becomes an iterative process in which the MIMO detector computes

$$\{f(x_1|\mathbf{y}, \mathbf{H}), f(x_2|\mathbf{y}, \mathbf{H}), \dots, f(x_{N_T}|\mathbf{y}, \mathbf{H})\},$$

and the channel decoder obtains

$$\{f(b_1|c_{1:N}), f(b_2|c_{1:N}), \dots, f(b_K|c_{1:N})\}.$$

Finally, since both are estimating the same set of information bits, they can use each other's result (just the extrinsic information) as prior in subsequent iterations, in order to obtain an improved estimation.

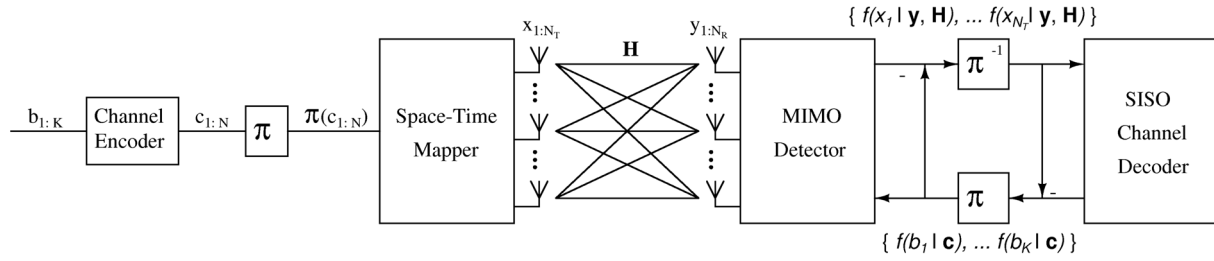


Figure 1. Diagram of a coded MIMO system.

Analogue implementation of the channel decoder part of the receiver has already been widely studied [1]–[3], so in this contribution we will focus on the MIMO detection block. The optimal MAP detector calculates the MPPs of interest by first obtaining the joint posterior distribution (JPD) for the full received vector

$$f(\mathbf{x} | \mathbf{y}) \propto f(\mathbf{y} | \mathbf{x}) \cdot f(\mathbf{x}) = \prod_{i=1}^{N_R} f(y_i | \mathbf{x}) \cdot \prod_{j=1}^{N_T} f_{\text{prior}}(x_j), \quad (2)$$

and then marginalizing out each variable as follows:

$$f(x_i | \mathbf{y}) = \sum_{(-i)} f(\mathbf{x} | \mathbf{y}), \quad (3)$$

where for brevity the dependence on  $\mathbf{H}$  is implicit, and the notation  $(-i)$  stands for “all except  $i$ ”.

In a general case, due to the fully-connected nature of the factor graph representation of the problem, message passing algorithms offer poor performance. In order to calculate any MPP, the best way is to obtain the terms of the joint posterior distribution (at least some of them) and then marginalize as prescribed by (3).

Given our assumption about the noise, and considering for simplicity BPSK modulation (i.e.  $x_i \in [-1, 1]$ ), the JPD can be written as

$$f(\mathbf{x} | \mathbf{y}) \propto \prod_{i=1}^{N_R} \exp\left(-\frac{1}{\sigma_n^2} \left|y_i - a \sum_{j=1}^{N_T} h_{i,j} x_j\right|^2\right) \cdot \prod_{j=1}^{N_T} f_{\text{prior}}(x_j) \quad (4)$$

and further simplified to

$$f(\mathbf{x} | \mathbf{y}) \propto \prod_{i=1}^{N_T} \exp(x_i(z_i + L_i)) \cdot \prod_{i=1}^{N_T-1} \prod_{j=i+1}^{N_T} \exp(x_i x_j r_{i,j}), \quad (5)$$

where  $L_i$  is the prior information about bit  $i$  in log-likelihood ratio representation, and  $r_{ij}$  and  $z_i$  are respectively the terms of the channel’s cross-correlation matrix  $\mathbf{R}$  and the output vector  $\mathbf{z}$  of the matched filter ( $\mathbf{H}^T$ ):

$$L_i = \log\left(\frac{P_{\text{prior}}(x_i = 1)}{P_{\text{prior}}(x_i = -1)}\right) \quad (6)$$

$$\mathbf{R} = -\frac{2a^2}{\sigma_n^2} \mathbf{H}^T \mathbf{H} \quad (7)$$

$$\mathbf{z} = \frac{2a}{\sigma_n^2} \mathbf{H}^T \mathbf{y}. \quad (8)$$

From the previous equations it follows that we can express the JPD in terms of products of simple binary distributions which are given, in log-domain, by  $\mathbf{L}$ ,  $\mathbf{z}$  and  $\mathbf{R}$ . Since the cross-correlation matrix is symmetric, and moreover, the terms in the main diagonal are not relevant, the total number of input binary distributions to our BPSK MIMO detector will be  $2N_T + N_T(N_T - 1)/2$ .

Once the full distribution is calculated, the MPPs can be obtained as in (3). It is worth noting that the sizes of  $\mathbf{z}$  and  $\mathbf{R}$  are independent of  $N_R$ , and the effects of receiver diversity ( $N_R > N_T$ ) will only make the cross-correlation matrix more orthogonal (i.e. off-diagonal terms will be comparatively smaller). This means that the design of our analog MIMO detector will be independent of the number of receive antennas.

### III. ANALOG IMPLEMENTATION

#### A. Joint Posterior Distribution

As aforementioned, the analog MIMO detector is constructed with building blocks similar to those found on analog channel decoders. Indeed, a generalized Gilbert multiplier topology, widely adopted for analog decoder implementation, can be conveniently used to obtain the joint posterior distribution terms in our MIMO case.

Fig. 2 shows a generic circuit [1] able to obtain all the pair-wise products of two probability mass functions  $\mathbf{u}$  and  $\mathbf{v}$ . These functions can be represented either by currents (probabilities) or voltages (log-likelihoods). The conversion between both domains is straightforward given the exponential current-voltage characteristic of BJT and subthreshold MOS transistors.

For convenience, we will assume that the input function  $\mathbf{v}$  is given in voltage domain, whereas  $\mathbf{u}$  and the product function  $\mathbf{w}$  are represented by currents. Translinear analysis of this circuit leads to the following expression for the output currents:

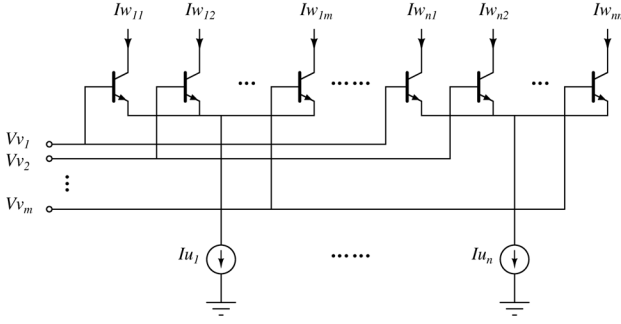


Figure 2. Gilbert-based probability multiplier.

$$Iw_{i,j} = Iu_i \frac{\exp\left(\frac{Vv_j}{V_T}\right)}{\sum_{k=1}^M \exp\left(\frac{Vv_k}{V_T}\right)}, \quad (9)$$

where  $V_T$  is the thermal voltage, and the denominator is a mere normalization constant. Comparing (5) and (9) it is obvious how to combine several of these blocks in order to obtain the full distribution of interest, just by defining the input voltages as:

$$\Delta V_{Zi} = 2V_T z_i \quad (10)$$

$$\Delta V_{Rij} = 2V_T r_{i,j}. \quad (11)$$

As an example, a symbolic representation of a detector for 3 transmit antennas and BPSK modulation is depicted in Fig. 3. An equivalent current-mode solution can be obtained by using probabilities, instead of log-ratios, as inputs. The terms of the distribution, considering no prior information is available, are given by all eight possible values of

$$f(\mathbf{x} | \mathbf{y}, \mathbf{H}) \propto \exp(x_1 z_1) \cdot \exp(x_2 z_2) \cdot \exp(x_3 z_3) \cdot \exp(x_1 x_2 r_{12}) \cdot \exp(x_1 x_3 r_{13}) \cdot \exp(x_2 x_3 r_{23}) \quad (12)$$

For practical reasons, in a real implementation the resulting tree would be split into several smaller blocks, and the currents after the last stages (where some of them are discarded) would be normalized back to a reference current.

### B. Marginalization

The remaining task is to obtain the MPPs, given in log-likelihood representation by

$$L(x_i) = \log \left( \frac{\sum_{x_j=-1}^1 f(\mathbf{x} | \mathbf{y}, \mathbf{H})}{\sum_{x_j=-1}^1 f(\mathbf{x} | \mathbf{y}, \mathbf{H})} \right). \quad (13)$$

It can be done by simply obtaining  $N_T$  copies of the JPD with current mirrors, and summing appropriately. However, we opted for a different approach due to a precision problem at high signal-to-noise ratios (SNR).

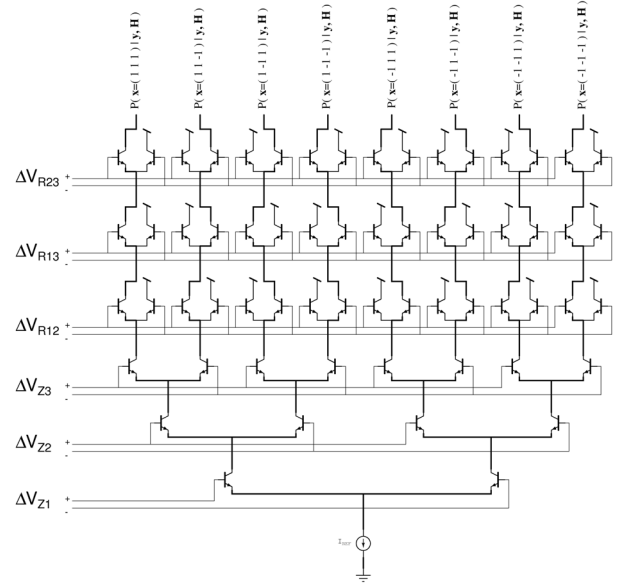


Figure 3. MIMO detection tree for  $N_1=3$ .

The values for  $\mathbf{R}$  and  $\mathbf{z}$  given by (7) and (8) are inversely proportional to the noise variance  $\sigma_n^2$ . This means that, at high SNR, our detector works most of the time with probabilities close either to zero or one. In Fig. 3 it can be observed that, in the last stages of the detector, half of the output currents generated (unwanted terms) are discarded. In the event of all the high terms ( $P \approx 1$ ) being discarded, our detector is left with a probability density function that in practice has lost all precision, even if we normalize back to a high total current after each stage.

One possible way around this is to limit the minimum value of  $\sigma_n^2$  that is used for the inputs. However, if the detector works with a noise variance ( $\sigma_d^2$ ) bigger than the real one ( $\sigma_n^2$ ), the posterior distribution obtained is given by

$$\hat{f}(\mathbf{x} | \mathbf{y}, \mathbf{H}) = [f(\mathbf{x} | \mathbf{y}, \mathbf{H})]^Q \quad (14)$$

$$Q = \frac{\sigma_n^2}{\sigma_d^2} < 1. \quad (15)$$

In other words, we obtain a probability distribution that is a flattened version of the desired one, and when marginalizing according to (13), errors are introduced. One way to avoid them is to pick out just the highest probabilities:

$$\hat{L}(x_i) = \log \left( \frac{\text{MAX}_{x_j=-1}^1 (\hat{f}(\mathbf{x} | \mathbf{y}, \mathbf{H}))}{\text{MAX}_{x_j=-1}^1 (\hat{f}(\mathbf{x} | \mathbf{y}, \mathbf{H}))} \right). \quad (16)$$

This solution introduces no errors in the hard decisions, and provides a good approximation to the real soft values:

$$\hat{L}(x_i) \approx \frac{\sigma_n^2}{\sigma_d^2} L(x_i). \quad (17)$$

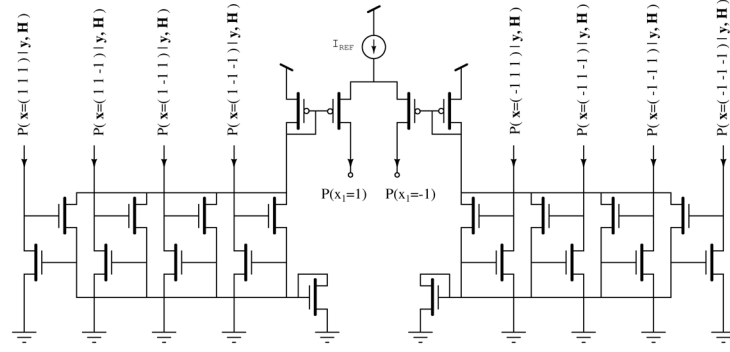


Figure 4. Marginalization circuit for  $N_T=3$ .

Fig. 4 shows the schematic of a marginalizer circuit for our example. It comprises two maximum selection circuits [6], two current mirrors and a current source for normalization.

#### IV. SIMULATION RESULTS

The proposed detector was designed and simulated in a  $0.25\mu\text{m}$  BiCMOS process. The relatively small circuit size allowed transistor-level simulation to be performed in an acceptable amount of time.

A supply voltage of 3.3V and a bias current per block of  $200\mu\text{A}$  were used, and bit-error-rate curves for three different configurations ( $N_T \times N_R = 3 \times 3, 3 \times 4, 3 \times 5$ ) were obtained at 75 Mbps.

Fig. 5 presents the results obtained, along with those of the optimal maximum-likelihood (ML) detector implemented in MATLAB. As expected, the difference between them is minimal since our detector obtains the full posterior distribution and therefore is performing an equivalent ML search.

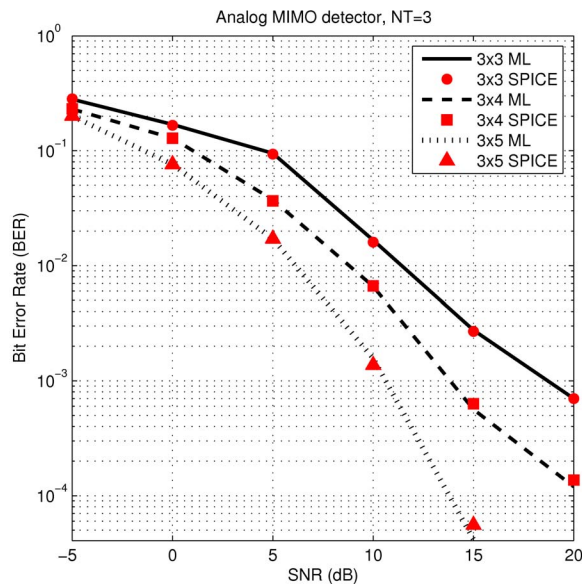


Figure 5. Simulation results for  $N_T \times N_R = 3 \times 3, 3 \times 4$  and  $3 \times 5$ .

The effect of process variations and transistor mismatch on the results remains to be analyzed. One of the nice properties of analog decoders is that they achieve great system-level accuracy despite the inherent inaccuracy of the smaller building blocks that work in parallel. However, in our case, where the structure is tree-like and no iterations are performed, mismatch effects can be worse. Fortunately, MIMO detection will always be accompanied by channel decoding, and the typical length of an error correcting code is much larger than the number of transmit antennas ( $N \gg N_T$ ), so at system-level, the structure of the resulting turbo receiver is iterative and highly parallelized.

#### V. CONCLUSIONS

In this contribution we have proposed an all analog solution to perform MAP detection in MIMO systems. On the downside, due to the optimal nature of the algorithm, the number of transistors required increases exponentially with the number of transmit antennas and modulation size, so its applicability is reduced to small MIMO systems with simple modulation formats.

Future research will include fabrication and testing of a proof-of-concept MIMO detector, as well as the design of reduced-complexity high-performance algorithms that could be implemented in analog VLSI networks.

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