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EXIT Chart Based Joint Code-Rate and Spreading-Factor Optimisation of Single-Carrier Interleave Division Multiple Access

Rong Zhang and Lajos Hanzo
School of ECS., Univ. of Southampton, SO17 1BJ, UK.
Tel: +44-23-80-593 125, Fax: +44-23-80-593 045
Email: lh@ecs.soton.ac.uk, http://www-mobile.ecs.soton.ac.uk

Abstract— **In this paper, we consider the joint code-rate and spreading-factor optimisation of turbo-style iterative joint detection and decoding assisted single-carrier interleave division multiple access (SC-IDMA) systems using different-rate convolutional codes and Extrinsic Information Transfer (EXIT) charts, when communicating over Additive White Gaussian Noise (AWGN) channels. More explicitly, we study the extrinsic information exchange between two serial concatenated components and maximise the number of users supported by the SC-IDMA system under the constraint of a fixed bandwidth expansion factor, while maintaining a predefined Bit Error Ratio (BER) versus E_b/N_0 performance. We found that an optimum code-rate and spreading-factor combination can be found for the SC-IDMA system at low E_b/N_0 values, where maintaining a low BER inevitably requires the employment of channel coding. By contrast, at high E_b/N_0 the system performs best, when no channel coding is used, i.e. DS-spreading is the only means of bandwidth expansion.**

I. Introduction

In wireless spread spectrum communications [1], the employment of channel coding is crucial. Viterbi's classic work [2] suggests that bandwidth expansion dedicated to low-rate channel coding has the potential of fully exploiting the achievable processing gain, while simultaneously offering a high coding gain as well as approaching the capacity of the multiple access channel contaminated by Additive White Gaussian Noise (AWGN). The terms of *Fourier-bandwidth* and *Shannon-bandwidth* [1] were defined by Massey [3] as early as 1994, recognising that channel coding and spreading may be regarded as two relatives, benefiting somewhat differently from a given bandwidth expansion. As a further development, it was suggested by Veeravalli and Mantravadi in [4] that a given total bandwidth expansion can be considered to be the result of concatenating of both channel coding as well as DS spreading. These considerations lead to the joint optimisation of the channel code-rate and Spreading-Factor (SF) in spread spectrum systems, [4]-[8]. In the early work of Hui [5], it was shown that the system's effective information throughput was maximised by low-rate channel coding, where a bandwidth expansion was imposed purely due to channel coding. However, their conclusion was based on single-user matched filtering aided detection, where a high residual multiuser interference had to be mitigated by the channel codec. Further insightful theoretical analysis based on multiuser detection was then provided by Verdu in [6], confirming the findings of [5] that indeed bandwidth expansion entirely dedicated to channel coding is favoured for single-user matched filter aided detection. By contrast, there is a beneficial tradeoff between the channel coding rate and SF, when linear Multiuser Detection (MUD), such as for example decorrelating or Minimum Mean Square Error (MMSE) MUD is used. These concepts were then further developed in a multicell context in [4]. In [7], Yue and Wang analysed the associated coding and spreading tradeoffs in the context of an iteratively detected CDMA system using Low Density Parity Check (LDPC) codes. These investigations underline the importance of joint code-rate and spreading-factor optimisation in the context of Single-Carrier Interleave Division Multiple Access (SC-IDMA), which is the novel objective of this study.

The concept of IDMA accrued from the philosophy of *code-spread* CDMA [8] as well as *chip-interleaved* CDMA [9] and hence inherited all the attractive properties of CDMA. The IDMA philosophy was then further developed by Ping and his team [10], as well as by Höher and Schöneich [11]. As alluded to above, IDMA entails reversing the position of Direct Sequence (DS) spreading and interleaving, leading to chip-interleaving instead of bit-interleaving. Then the different users can be distinguished by their unique user-specific chip-interleavers combined with turbo-style iterative joint detection as well as channel decoding [12]. Owing to its meritorious properties, IDMA has been proposed for numerous applications, such as next-generation cellular uplink systems [11] as well as for time-hopping Ultra Wide Band (UWB) systems [13]. In this treatise, we invoke EXtrinsic Information Transfer (EXIT) charts [14] to analyse the joint code-rate and spreading-factor optimisation of a SC-IDMA system communicating over an AWGN channel.

The rest of the paper is organised as follows. In Section II, we present the transceiver architecture of IDMA. In Section III, we elaborate on the joint code-rate and spreading-factor optimisation of SC-IDMA communicating over AWGN channels using EXIT chart analysis. In Section IV, this is followed by our simulation results and we conclude our discourse in Section V.

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[1]For example, a Direct Sequence (DS) spread spectrum system requires a significantly higher Fourier-bandwidth than Shannon-bandwidth, since the effective information throughput, which is related to the DS chip-rate by the Spreading Factor (SF), is at least SF times lower than the total Fourier-bandwidth required.

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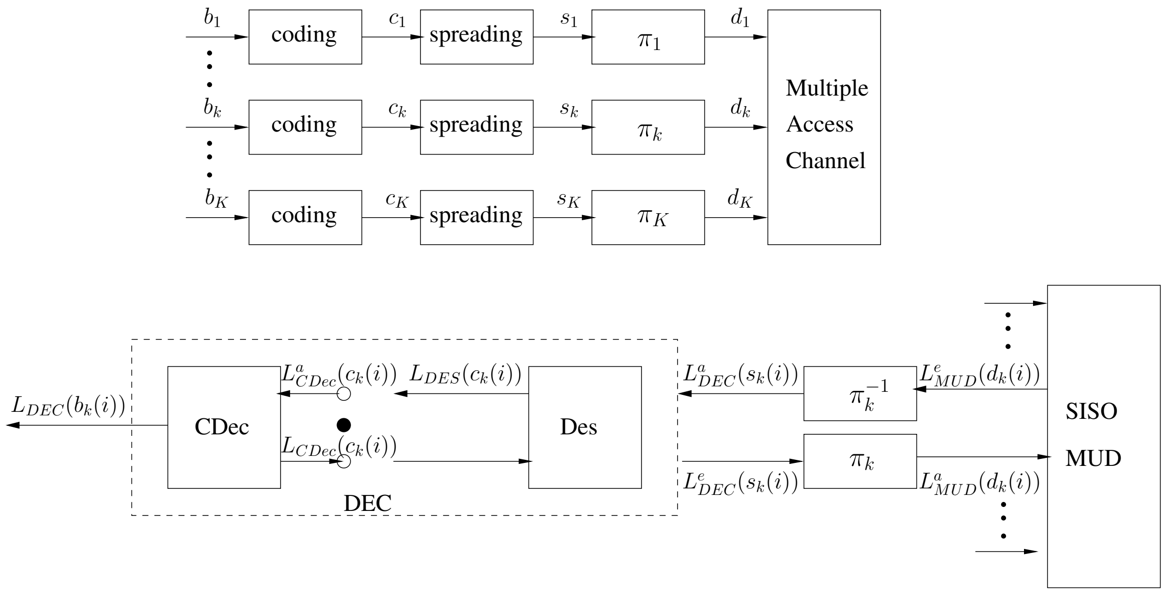

Fig. 1. Transmitter of single-carrier IDMA and its joint soft-in-soft-out multiuser detector, de-spreader (Des) and channel decoder (CDec)

II. Transceiver of Single-Carrier IDMA

Fig. 1 shows the transceiver of an IDMA system. Assuming BPSK modulation, the kth user 's transmitted bit stream $\{b_k(i); i = 1, 2, \cdots, M\}$ is firstly channel encoded at a rate of $R = M/N$, yielding the encoded stream $\{c_k(i); i = 1, 2, \cdots, N\}$. Then the resultant channel encoded stream is DS spread by a common spreading code having a spreading gain G, yielding the sequence $\{s_k(i); i = 1, 2, \cdots, P\}$, which is then chip-interleaved using a user-specific random chip-interleaver Π_k, in order to form the chip stream $\{d_k(i); i = 1, 2, \cdots, P\}$. The chip stream is then transmitted through the multiple access channel supporting K users. This is in contrast to the concept of a classic CDMA system, where the interleaver is between the Forward Error Correction (FEC) coding and DS-spreading stages. The latter classic philosophy corresponds to bit-interleaving.

For the sake of attaining the best possible performance, an iterative receiver is employed [10] [12]. The receiver of Fig. 1 consists of a Soft In Soft Out (SISO) MUD and a bank of K individual SISO decoders (DEC), which are constituted by the combined Channel Decoder (CDec) and De-spreader (Des). The soft information exchanged between the receiver components is constituted by the extrinsic Log-Likelihood Ratios (LLRs) [15]. At each iteration, the SISO MUD of Fig. 1 generates the extrinsic output information $L^e_{MUD}(d_k(i))$ and deinterleaves it in order to create the stream $L^a_{DEC}(s_k(i))$, which is forwarded as *a priori* information to the SISO DEC, as seen in Fig. 1. In the feedback loop, the SISO DEC computes the extrinsic information stream of $L^e_{DEC}(s_k(i))$, which is then interleaved for the sake of generating the stream $L^a_{MUD}(d_k(i))$ as *a priori* information for the SISO MUD of Fig. 1. The SISO MUD employs Soft Interference Cancellation (SoIC) on a chip-by-chip basis.

Again, the SISO DEC of Fig. 1 consists of two components, namely the SISO Des and the SISO CDec, where the SISO Des first computes its *a posteriori* information $L_{DES}(c_k(i))$ seen in Fig. 1 by Maximal Ratio Combining (MRC) each *a priori* soft chip contribution $L^a_{DEC}(s_k(i))$, which is multiplied by the corresponding spreading codes. Then the SISO CDec of Fig. 1 uses the output LLRs received from the SISO Des as its *a priori* information $L^a_{CDec}(c_k(i))$, in order to perform standard *A Posteriori* Probability (APP) based detection [15] and two *a posteriori* LLRs are computed, namely the codeword LLR stream $L_{CDec}(c_k(i))$ acting as the *a priori* information for the SISO Des and the data LLRs $L_{DEC}(b_k(i))$ representing the transmitted bits $b_k(i)$. The SISO Des of Fig. 1 outputs its extrinsic information $L^e_{DEC}(s_k(i))$ for each chip by subtracting the *a priori* information $L^a_{DEC}(s_k(i))$ from the codeword LLRs $L_{CDec}(c_k(i))$. The iterations are terminated, when a predefined termination criterion is satisfied. Finally, the LLRs $L_{DEC}(b_k(i))$ of Fig. 1, which represent the original information bits are subjected to a hard decision, yielding $d_k(i)$. In this treatise, we use the Log-MAP algorithm [15] for the channel decoder.

III. EXIT Chart Analysis of Joint Code-rate and Spreading-factor Optimisation

A. Normalised User-load

We commence our discussion of joint code-rate and spreading-factor optimisation by firstly defining the optimisation problem. As in [4], given a fixed total bandwidth expansion factor Ω, we separately treat the bandwidth expansion imposed by FEC coding having a code-rate of R and by DS-spreading having a spreading factor of G, yielding:

$$\Omega = \frac{G}{R}. \tag{1}$$

For the sake of characterising the achievable throughput of the system, we define the normalised *user-load* as:

$$\eta = \frac{K_{max}}{\Omega}, \tag{2}$$

where K_{max} is the maximum number of users supported given a fixed bandwidth expansion factor Ω and E_b/N_0 value, where E_b is the bit-energy and N_0 is the power spectral density of AWGN. Note that given a fixed bandwidth expansion factor Ω and a channel code-rate R, the affordable DS-spreading factor is determined. Hence, K_{max} is a function of Ω, R and E_b/N_0. Equivalently, the joint code-rate and spreading-factor optimisation is essentially equivalent to finding the best code-rate, which maximises the effective normalised *user-load*.

B. Extrinsic Information Transfer Charts

In this subsection, we use EXIT charts for analysing the associated FEC coding versus DS-spreading tradeoffs in the context of IDMA. EXIT charts were introduced by ten Brink [14]. This technique relies on computing the mutual information of two constituent components, namely that of the SISO MUD and the SISO DEC, which are denoted by I_i^{MUD}, I_o^{MUD}, I_i^{DEC}, I_o^{DEC}. The mutual information $I(L;X)$ between the random variable L and the BPSK modulated coded bits X is defined as [14]:

$$\frac{1}{2}\sum_{x\in(-1,+1)}\int_{-\infty}^{+\infty} p(l|x)log_2\frac{2p(l|x)}{p(l|+1)+p(l|-1)}dl, \qquad (3)$$

where $I(L;X) \in [0,1]$ and $p(l|x)$ is the distribution of the *a priori* or extrinsic information. It was shown in [14] that the PDF of the mutual information may be approximated by a Gaussian distribution, where the mutual information is a function of the Gaussian distribution's variance σ_i^2.

Next we evaluate the nonlinear function $I_o = \chi(I_i)$, which maps the input *a priori* mutual information $I_i \in [0,\cdots,1]$ to the output extrinsic mutual information $I_o \in [0,\cdots,1]$, where the amount of output extrinsic mutual information $I_o \in [0,\cdots,1]$ gleaned from the input *a priori* mutual information determines the convergence behaviour of this SISO component. Finally, since the extrinsic information generated by the first SISO component acts as the *a priori* information for the second SISO component and vice versa, in the EXIT charts we alternately swap the abscissa and ordinate axes, depending on which of the two components acts as the source of *a priori* information, corresponding to the abscissa, as discussed in [14]. To elaborate a little further, EXIT charts provide us with an insight into the mutual information exchange between the constituent SISO components, where the chip-interleaver allows us to decouple the constituent SISO components, which can be hence analysed separately.

C. EXIT Chart Analysis

Figs. 2 and 3 shows the EXIT charts of our coded IDMA system associated with a total fixed bandwidth expansion of $\Omega = 8$. A rate $R = 1/2$ convolutional code having a constraint length of 3 and octal generator polynomials of [5,7] was used. The piece-wise linear curve represents the EXIT curve of the outer SISO DEC of Fig. 1. Furthermore, the dashed line shows the EXIT curve of the outer SISO DEC of Fig. 1 constituted by the de-spreader associated with a spreading factor of $G = 8$, but using no FEC and thus representing uncoded IDMA. More

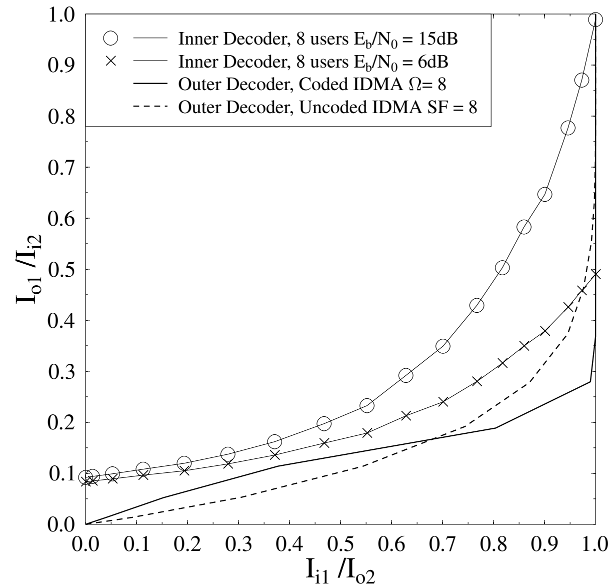

Fig. 2. EXIT chart analysis of coded IDMA over AWGN channels for a bandwidth expansion of $\Omega = 8$, $K = 8$ users and various E_b/N_0 values.

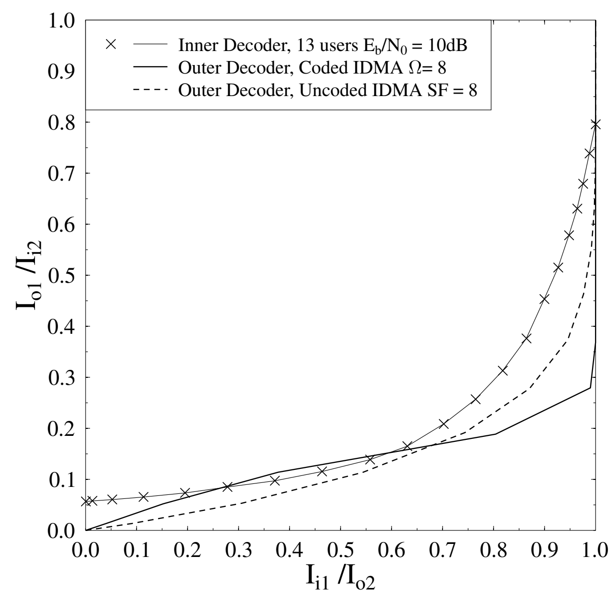

Fig. 3. EXIT chart analysis of coded IDMA over AWGN channels for a bandwidth expansion of $\Omega = 8$, 13 users and E_b/N_0 = 10dB.

explicitly, it represents an IDMA system using DS-spreading as the only means of bandwidth expansion.

As seen in these two figures, initially the FEC coded EXIT curve indicated by the piece-wise linear curve has a less rapidly increasing nature, but exhibits a more rapidly increasing nature in the vicinity of $I_{i1}/I_{o2} > 0.68$ than the EXIT curve of the uncoded system characterised by the dashed line having the same bandwidth expansion factor of $\Omega = 8$. Observe in Fig. 2 that at $E_b/N_0 = 6$dB, the EXIT curve of the SISO MUD marked by the cross legends has an intersection with the EXIT curve of the uncoded IDMA system using $SF = 8$, while an open tunnel is observed with respect to the FEC coded EXIT curve. By contrast, in

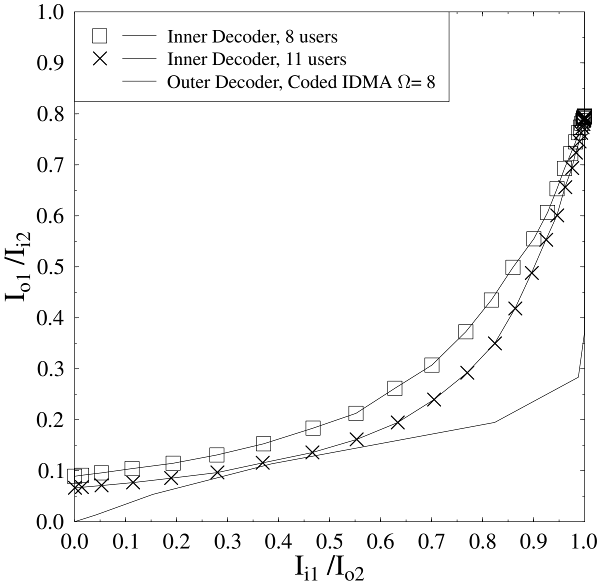

Fig. 4. EXIT charts for different number of users supported in coded IDMA over AWGN channels at a bandwidth expansion factor of $\Omega = 8$ and E_b/N_0 = 10dB, using $R = 1/2$ convolutional coding

Fig. 3, where the number of users is $K = 13$ and we have $E_b/N_0 = 10$dB, an intersection is observed between the EXIT curve of the SISO MUD and the FEC coded EXIT curve. However, the uncoded IDMA system represented by the EXIT curve marked by the dashed line is capable of supporting such a high number of user. These observations suggest that at this relatively high E_b/N_0 value of 10 dB, the FEC coded system becomes unable to support an equally high number of users as the uncoded system at the same bandwidth expansion factor of $\Omega = 8$. These observations reflect the plausible fact that at high E_b/N_0 values, dedicating bandwidth expansion to FEC coding ultimately reduces the normalised *user-load*. By contrast, at low E_b/N_0 values, the employment of FEC coding is inevitable.

Fig. 2 shows the EXIT curve of the SISO MUD (inner decoder) for different E_b/N_0 values. Upon increasing the E_b/N_0 value from 6 to 15dB, for low values of I_{i1}/I_{o2}, initially the EXIT curve of the SISO MUD remains similar. However, as expected, the inner decoder's curve corresponding to $E_b/N_0 = 15$dB provides a widely open EXIT tunnel, while that recorded at $E_b/N_0 = 6$dB has an intersection with the EXIT curve of the uncoded IDMA. This indicates the plausible fact that an infinitesimally low BER may be achieved at an $E_b/N_0 = 15$dB with a low number of decoding iterations. Observe furthermore in Fig. 2 that the half-rate coded IDMA outer decoder's EXIT curve exhibits a narrow EXIT tunnel opening at low I_{i1}/I_{o2} values, but a wider tunnel opening at I_{i1}/I_{o2} values in excess of about 0.68. Hence, the FEC-coded scheme is clearly less likely to experience intersections at relatively low E_b/N_0 values. However, the price of this benefit is the higher complexity of each decoding iteration. Figs. 2 and 3 demonstrate that indeed, an optimum code rate and SF exists, when jointly designing the FEC code and the spreading scheme.

Code Rate	Octal Generator	Constraint length	SF (G)	No. of Iterations / Trellis States (Complexity)
$R = 1/8$	[7 7 5 5]	3	-	10 / 4 (40)
$R = 2/8$	[5 7 7 7]	3	2	10 / 4 (40)
$R = 1/3$	[5 7 7]	3	3	10 / 4 (40)
$R = 4/8$	[5 7]	3	4	10 / 4 (40)
$R = 2/3$	[4 2 6] [1 4 7]	3	5	10 / 4 (40)
$R = 3/4$	[6 2 2 6] [1 6 0 7] [0 2 5 5]	5	6	10 / 4 (40)

TABLE I
TABLE OF CONVOLUTIONAL CODES USED IN JOINT CODE-RATE AND SPREADING-FACTOR OPTIMISATION

IV. JOINT CODE-RATE AND SPREADING-FACTOR OPTIMISATION

Let us now continue our investigations by fixing the bandwidth expansion factor Ω, the channel code-rate R as well as the E_b/N_0 values and investigate the number of users supported, while maintaining a given target BER of 10^{-5}. As before, we can solve the joint code-rate and SF optimisation by generating the EXIT curves of both the SISO MUD and the SISO DEC of Fig. 1 and then finding the specific number of users, for which an open EXIT-tunnel exists. More explicitly, we plot the EXIT curve of the SISO MUD for various number of users and the maximum number of user is found, where there is no intersection between the inner and outer codes' EXIT curves. In Fig. 4, the bandwidth expansion factor was fixed to $\Omega = 8$. A rate $R = 1/2$ convolutional code was used and E_b/N_0 = 10dB was assumed. We found that the maximum number of user supported by the IDMA system was $K_{max} = 11$, corresponding to a normalised *user-load* of $\eta = K/\Omega = 11/8$, where the inner and outer codes' EXIT curves 'just' touch each other.

Let us now proceed by investigating the joint code-rate and SF design by employing the range of different-rate convolutional codes summarised in Table I, which have similar constraint lengths and were selected from [16]. Importantly, please also observe in Table I that the detection complexity of all the different scenarios are adjusted to be the same in terms of the total number of trellis states, which was determined by the product of the number of trellis states and the number of iterations. We set the target BER to $P_e = 10^{-5}$.

Fig. 5 shows the normalised *user-load* supported versus various code-rate and SF combinations for two different E_b/N_0 scenarios. It can be seen in Fig. 5 that at $E_b/N_0 = 5$ dB, the normalised user-load was maximised by a combination of the rate-5/8 code and the SF of $G = 5$ of Table I, when supporting $K_{max} = 9$ users. By contrast, the IDMA system favours the employment of pure spreading without FEC coding at E_b/N_0 = 10dB, as can be seen from Fig. 5, reaching $\eta = 1.75$ at a rate of $R = 1$.

Fig. 6 portrays the E_b/N_0 value required for supporting a normalised *user-load* of both $\eta = 1.00$ and $\eta = 1.25$. It can be observed that having a high code-rate results in a low coding gain and hence a high transmit power is required for supporting the target *user-load*. The optimum code-rate is $R =$

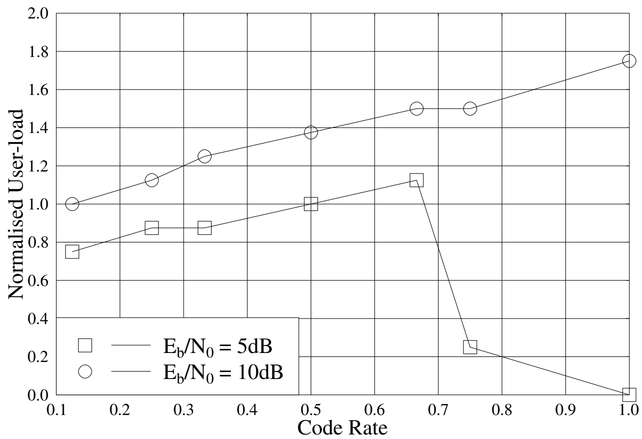

Fig. 5. Normalised user-load versus code-rate for a fixed bandwidth expansion factor of $\Omega = 8$ and for $P_e = 10^{-5}$ using the convolutional codes of Table I

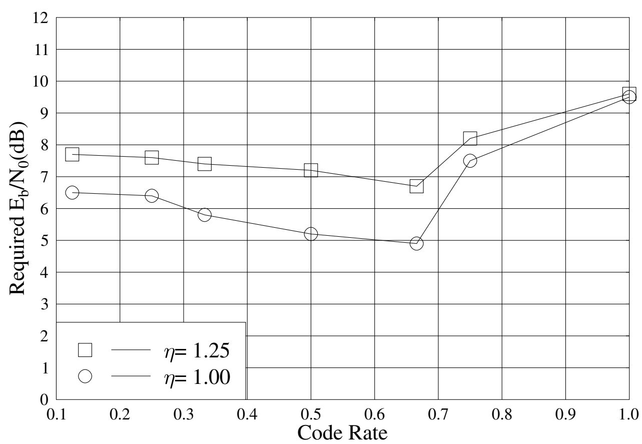

Fig. 6. The E_b/N_0 values required for supporting a normalised user-load of η versus the code-rate at a fixed bandwidth expansion factor of $\Omega = 8$ and for $P_e = 10^{-5}$ using the convolutional codes of Table I

$5/8$ in this scenario. However, having a code-rate R below the optimum also requires an increased E_b/N_0 for supporting the target *user-load*.

These observations were found to be consistent with our previous EXIT chart analysis provided in Figs 2 and 3. At high E_b/N_0 values, where the noise is insignificant, the SISO MUD and the SISO de-spreader become capable of almost completely eliminating the Multiple User Interference (MUI), while at low E_b/N_0 values the output extrinsic information $L_{DES}(c_k(i))$ of the SISO de-spreader seen in Fig. 1 is gravely noise contaminated, which can only be improved by the relatively low-rate and hence powerful SISO channel decoder. Increasing the code-rate beyond the optimum point weakens the channel decoder and the resultant feedback extrinsic information $L_{CDec}(c_k(i))$ remains noise contaminated, which reduces the achievable *user-load*. By contrast, when reducing the code-rate below the optimum point, an extra coding gain is provided, which benefits the system in terms of requiring a reduced number of iterations, but also reduces the number of users that may be supported.

V. Conclusion

In this paper, we considered the joint code-rate and spreading-factor optimisation of SC-IDMA system communicating over AWGN channels with the aid of EXIT chart analysis. Different-rate convolutional codes were employed. At high E_b/N_0 values, DS-spreading dispensing with FEC was capable of supporting the highest normalised *user-load*. By contrast, at low E_b/N_0 values the employment of coding became crucial, since it improved both the achievable BER and the attainable multiple access capability. Our further research is related to the EXIT chart analysis aided joint code-rate and spreading-factor optimisation of multi-carrier IDMA using LDPC codes.

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