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APPLICATION OF PATTERN SEARCH METHOD TO POWER SYSTEM ECONOMIC LOAD DISPATCH

J S Alsumait, J K Sykulski
University of Southampton
Electronics and Computer Sience School
Electrical Power Engineering Group
Highfield Southampton SO17 1BJ
jas2@soton.ac.uk jks@soton.ac.uk

A K Alothman
The Public Authority for Applied Education and Training
College of Technical Studies
Electrical Engineering Department
Shuwaik,Jamal Abdullnaser St Kuwait
alothman@paaet.edu.kw

ABSTRACT
Direct Search (DS) methods are evolutionary algorithms used to solve constrained optimization problems. DS methods do not require information about the gradient of the objective function while searching for an optimum solution. One of such methods is *Pattern Search* (PS) algorithm. This study examines the usefulness of a constrained pattern search algorithm to solve well-known power system Economic Load Dispatch problem (ELD) with a valve-point effect. For illustrative purposes, the proposed PS technique has been applied to various test systems to validate its effectiveness. Furthermore, convergence characteristics and robustness of the proposed method have been assessed and investigated through comparison with results reported in literature. The outcome is very encouraging and suggests that pattern search (PS) may be very useful in solving power system economic load dispatch problems.

KEY WORDS
Economic Load Dispatch, Valve-Point effect, Direct Search method, Pattern Search method, Evolutionary Algorithms (EA) and optimization.

1. Introduction

Scarcity of energy resources, increasing power generation costs and ever-growing demand for energy necessitate optimal economic dispatch in modern power systems. The main objective of economic dispatch is to reduce the total power generation cost while satisfying various equality and inequality constraints. Traditionally, in economic dispatch problems, the cost function for generating units has been approximated as a quadratic function.

A wide variety of optimization techniques have been applied to solving Economic Load Dispatch problems (ELD). Some of these techniques are based on classical optimization methods while others use artificial intelligence methods or heuristic algorithms. Many references present the application of classical optimization methods, such as linear programming or quadratic programming, to solve ELD problems [1, 2]. Such classical optimization methods are highly sensitive to staring points and often converge to local optimum or diverge altogether. Linear programming methods are fast and reliable but have a disadvantage associated with the piecewise linear cost approximation. Non-linear programming methods have known problems of convergence and algorithmic complexity. Newton based algorithms have difficulty with handling a large number of inequality constraints [3]. Methods based on artificial intelligence techniques, such as artificial neural networks, have also been applied successfully and are reported for example in [4, 5]. Lately, many heuristic search techniques, such as particle swarm optimization [3] and genetic algorithms [6], have been considered in the context of the ELD problems. Finally, hybrid methods have been developed [7], where the conventional Lagrangian relaxation approach, first order gradient method and multi-pass dynamic programming are combined together.

Recently, a particular family of global optimization methods, introduced and developed by researchers in 1960 [8], has received a great attention, namely the Direct Search methods. Direct Search methods are simply structured to explore a set of points, around the current position, looking for a point that has smaller objective value than the current one. This family includes Pattern Search (PS) algorithms, Simplex Methods (SM) (different from the simplex used in linear programming), Powell Optimization (PO) and others [9].

Direct Search methods, as opposed to more standard optimization methods, are often called derivative-free as they do not require any information about the gradient or higher derivatives of the objective function to search for an optimal solution. Therefore Direct Search methods may very well be used to solve non-continuous, non-differentiable and multimodal (i.e. multiple local optima) optimization problems. Since the economic dispatch is one of such problems, then the proposed method appears to be a good candidate to tackle the ELD tasks.
The main objective of this study is to introduce the use of Pattern Search (PS) optimization technique to the subject

of power system economic load dispatch. In this paper, the PS method has been employed to solve economic dispatch problem with a valve-point effect. A valve-point effect is the rippling effect added to the generating unit curve when each steam admission valve in a turbine starts to open. Moreover, to assure accurate results for this model, an additional term representing the valve-point effect should be added to the cost function [22]. The addition of the valve-point effect poses a more challenging task to the proposed method since it increases the non-linearity of the search space as well as the number of local minima.

The paper is organized as follows: Section 2 introduces the problem formulation; Section 3 presents a description of the proposed PS algorithm; analysis and test results are presented in Section 4, followed by concluding remarks.

2. Problem Formulation

The traditional formulation of the economic load dispatch problem is a minimization of summation of the fuel costs of the individual dispatchable generators subject to the real power balanced with the total load demand as well as the limits on generators outputs. In mathematical form the problem can be stated as:

$$F = \sum_{i=1}^{N} F_i(P_i) \tag{1}$$

with the incremental fuel cost functions of the generation units with valve-point loading represented as [10]

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i + \left| e_i \times \sin(f_i \times (P_{i\min} - P_i)) \right| \tag{2}$$

subject to

$$\sum_{i=1}^{N} P_{gi} = P_D + P_L \tag{3}$$

$$P_{gi(\min)} < P_{gi} < P_{gi(\max)}, i \in N_s \tag{4}$$

where

F is the system overall cost function

N the number of generators in the system

d_i, b_i, c_i the constants of fuel function of generator number *i*

e_i, f_i the constants of the valve-point effect of generator number *i*

P_{gi} the active power generation of generator number *i*

P_D the total power system demand

P_L the total system transmission losses

$P_{gi(\min)}$ the minimum limit on active power generation of generator *i*

$P_{gi(\max)}$ the maximum limit on active power generation of generator *i*

N_s the set of generators in the system

The sinusoidal term added to the fuel cost function which models the valve-point effect introduces ripples to heat-rate curve, thus introducing more local minima to the search space.

Finally, we should mention that the system losses will be ignored for all test cases considered in this study for simplification purposes.

3. Pattern Search Method

The Pattern Search (PS) optimization routine is an evolutionary technique that is suitable to solve a variety of optimization problems that lie outside the scope of the standard optimization methods. Generally, PS has the advantage of being very simple in concept, easy to implement and computationally efficient. Unlike other heuristic algorithms, such as genetic algorithms [11, 12], PS possesses a flexible and well-balanced operator to enhance and adapt the global and fine tune local search. A useful review of direct search methods for unconstrained optimization is presented in [9], where the authors give a modern perspective on the classical family of derivative-free algorithms, focusing on the development of direct search methods.

The Pattern Search (PS) algorithm proceeds by computing a sequence of points that may or may not approach the optimal value. The algorithm starts by establishing a set of points called a mesh, around the given point. This current point could be the initial starting point supplied by the user or it could be computed from the previous step of the algorithm. The mesh is formed by adding the current point to a scalar multiple of a set of vectors called a pattern. If a point in the mesh is found to improve the objective function at the current point, the new point becomes the current point at the next iteration.

The details of the above process are as follows. First, the Pattern search begins at the initial point X_0 that is given as a starting point by the user. At the first iteration, with a scalar equal to 1 called the mesh size, the pattern vectors are constructed as [1 0], [0 1], [−1 0] and [0 −1]; they may be called the direction vectors. Then the Pattern Search algorithm adds the direction vectors to the initial point X_0 to compute the following mesh points:

$$X_0 + \begin{bmatrix}1 & 0\end{bmatrix}$$

$$X_0 + \begin{bmatrix}0 & 1\end{bmatrix}$$

$$X_0 + [-1 \quad 0]$$
$$X_0 + [0 \quad -1]$$

Figure 1 illustrates the formation of the mesh and pattern vectors. The algorithm computes the objective function at the mesh points in the order shown.

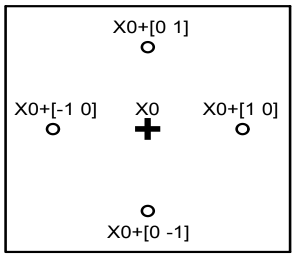

Figure 1: PS Mesh points and the Pattern

The algorithm polls the mesh points by computing their objective function values until it finds the one with a value smaller than the objective function value of X_0. If there is such a point, then the poll is successful and the algorithm sets this point as equal to X_1.

After a successful poll, the algorithm steps to the second iteration and multiplies the current mesh size by 2 (this is called the expansion factor and normally has a default value of 2). The mesh at iteration two contains the following points: 2*[1 0] + X_1, 2*[0 1] + X_1, 2*[−1 0] + X_1 and 2*[0 −1] + X_1. The algorithm polls the mesh points until it finds the one whose value is smaller than the objective function value of X_1. The first such point it finds is called X_2, and the poll is successful. Because the poll is successful, the algorithm multiplies the current mesh size by 2 to get a mesh size of 4 at the third iteration because the expansion factor equals 2.

Secondly, if iteration 3 (mesh size = 4) ends up being an unsuccessful poll, i.e. none of the mesh points have a smaller objective function value than the value at X_2, the algorithm does not change the current point at the next iteration. That is, $X_3 = X_2$. At the next iteration, the algorithm multiplies the current mesh size by 0.5, a contraction factor, so that the mesh size at the next iteration is smaller. The algorithm then polls with a smaller mesh size.

The Pattern search optimization algorithm will repeat the illustrated steps until it finds the optimal solution for the minimization of the objective function. The algorithm stops when any of the following conditions occurs:

- The mesh size is less than mesh tolerance.
- The number of iterations performed by the algorithm reaches a predefined value.
- The total number of objective function evaluations performed by the algorithm reaches a pre-set maximum number of function evaluations.
- The distance between the point found at one successful poll and the point found at the next successful poll is less than a set tolerance.
- The change in the objective function from one successful poll to the next successful poll is less than a function tolerance.

All the parameters involved in the Pattern search optimization algorithm can be pre-defined subject to the nature of the problem being solved.

3.1 Constraints handling

Many ideas have been put forward to ensure that the solution satisfies the imposed constraints [13]. For example, the constraint may be augmented with the objective function using Lagrange multipliers. In this way the size of the problem will increase by introducing new parameters. In this study, the Pattern Search (PS) method handles constraints by using augmented Lagrangian to solve the nonlinear constrained economic dispatch problem [14-17]. The variables' bounds and linear constraints are handled separately from nonlinear constraints. Thus a sub-problem is formulated and solved, (having the objective function and nonlinear constraint function), using the Lagrangian and the penalty factors. Such a sub-problem is minimized using a pattern search method, where the linear constraints and bounds are satisfied. For more explanation on how PS handles constraints refer to [16, 18, 19].

4. Numerical results

In order to assess the effectiveness and robustness of the proposed Pattern Search method, three test cases of economic load dispatch with a valve-point effect have been considered. For simplicity, transmission losses are ignored in all cases (P_L in Equation 3 is set to zero). The non-linear minimization problem formulation of all test cases has been solved using the predefined function pattern search incorporated in the GA & DS toolbox of Matlab [19]. This function implements the Pattern Search algorithm described in section 3. Thus, cost coefficients of the fuel cost and the combined objective function for the considered test cases were coded in Matlab environment. The three test cases differ in the number of generating units, which were assumed as 3, 13 and 40 respectively. Lack of space allows only for the first two cases to be described in detail, but for the 40-generator case the tendencies and the properties of the algorithm are similar to those observed when studying Case II.

Initially, several runs have been carried out with different values of the key parameters of PS, such as the initial mesh size and the mesh expansion and contraction factors. In this study, the mesh size and the mesh expansion and contraction factors are selected as 1, 2 and 0.5 respectively. In addition, a vector of initial points, i.e. X_0, was randomly generated (each initial point is bounded within the generators limits) to provide an initial guess for the PS to proceed. As for the stopping criteria, all tolerances were set to 10^{-6} and the maximum number of iterations and function evaluations were set to 1000. All runs have been conducted on a modest 1GHz Pentium 3 processor with 256 MB of RAM laptop computer, so the comparisons of computing times with those given in literature should be fair.

4.1 Case I: Three Generating Units

This test case consists of three generating units with quadratic cost function combined with the effects of valve-point loading. The units data (upper and lower bounds) along with the cost coefficients for the fuel cost (a, b, c, e, and f) for the three generators with valve-point loadings are given in [10, 20]. The Pattern Search algorithm has been executed 100 times with different starting points to study its performance and effectiveness. The solutions obtained using the PS method and the execution times for the 100 runs were compared with the outcome of other evolutionary methods, for example Genetic Algorithm (GA) and Evolutionary Programming (EP), applied to the same test system in [20]. The comparison of performance of PS with the other methods is in terms of dispatching costs and convergence speed. Table 1 shows the optimal solutions determined by PS for the three units while the execution time and cost comparisons are shown in Table 2. The definition of the various methods (GAB, GAF, etc) may be found in [20].

Table 1: Generator loading and fuel cost determined by PS with total load demand of 850 MW

Generator	Generator Production (MW)
Pg_1	300.2663
Pg_2	149.7331
Pg_3	399.9996
Σ Pgi = 850 MW	Total cost: $8234.05

Table 2: Comparison of PS and EP

Evolution Method	Mean time (sec)	Best time (sec.)	Mean cost ($)	Maximum cost ($)	Minimum cost ($)
GAB	35.80	32.46	------	------	8234.08
GAF	24.65	23.03	------	------	8234.07
CEP	20.46	18.35	8235.97	8241.83	8234.07
FEP	4.45	3.79	8234.24	8241.78	8234.07
MFEB	8.00	6.31	8234.71	8241.80	8234.08
IFEP	6.78	6.11	8234.16	8234.54	8234.07
PS	0.81	0.62	8352.41	8453.00	8234.05

All methods give a similar 'best' solution, whereas 'mean' and 'maximum' costs differ. The PS algorithm is significantly faster than methods described in [20].

The convergence of optimal solution using PS is shown in Figure 2, where only about 22 iterations were needed to find the optimal solution. However, PS may be allowed to continue the search in the neighborhood of the optimal point to increase the confidence in the result. PS stops after 44 more iteration and returns the optimal value.

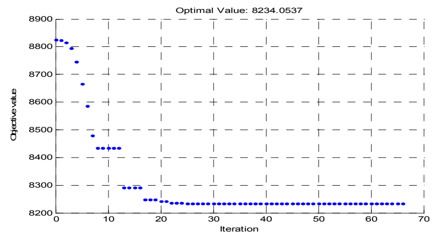

Figure 2: Convergence of PS for Case I

Figure 3 depicts the mesh size throughout the convergence process. It is apparent that the mesh size decreases until the algorithm terminates, in this case at a mesh size of 1.5259e-005 which is more than the stopping criteria, thus indicating that this particular run did not terminate using the mesh size tolerance. Figure 3 shows that for the first 8 iterations the poll was successful since the mesh size keeps increasing as the algorithm had to expand the scope of the search. This is accomplished by multiplying the current mesh size by the expansion factor, in this study taken as 2. This scenario continued until iteration number 8 when the mesh size reached 256. At iteration number 9 the mesh size decreased by half due to multiplying the current mesh size by the contracting factor, indicating an unsuccessful poll in the previous iteration. This process continues until reaching one of the termination criteria.

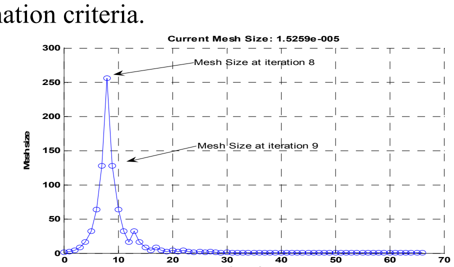

Figure 3: Convergence of PS Mesh Size (Case I)

It is worth mentioning that the mean and the maximum costs are higher than those of the other methods, and this is a certain drawback of the performance of PS in this test. Moreover, it has been observed that the algorithm is quite sensitive to the initial (starting) point and how far it is from the global optimal solution. Figure 4 illustrates the sensitivity of PS where a hundred solutions were obtained by PS with different initial values. The optimal solution has been reached a number of times for initial points around run number 80. The total execution time for the 100 runs was 80.75 sec. Other quality answers occurred for runs between 32 to 40 and 84 to 100. However, there were also several less successful results as illustrated in Figure 4.

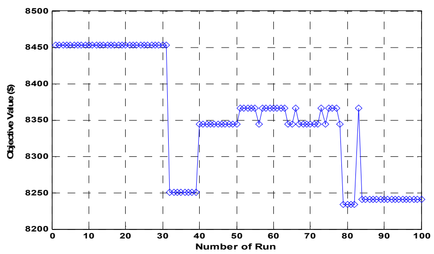

Figure 4: Objective Function Value for 100 different Starting Points (Case I)

4.2 Case II: 13 Generating Units

This test assumes 13 generating units with quadratic cost function combined with the effects of valve-point loading. The units data (upper and lower bounds) and cost coefficients for the fuel cost (a, b, c, e, and f) for the 13 generators with valve-point loading are given in [20, 21].

Table 3: Generator loading and fuel cost determined by PS with total load demand of 1800 MW

Generator	Generator Production (MW)
Pg_1	538.5587
Pg_2	224.6416
Pg_3	149.8468
Pg_4	109.8666
Pg_5	109.8666
Pg_6	109.8666
Pg_7	109.8666
Pg_8	109.8666
Pg_9	109.8666
Pg_{10}	77.4666
Pg_{11}	40.2166
Pg_{12}	55.0347
Pg_{13}	55.0347
Σ Pgi = 1800 MW	**Total cost: $17969.17**

The Pattern Search algorithm has been executed 50 times with different starting points and similar comparisons as for Case I are summarized by Tables 3 and 4. The results for all the 'EP' methods are taken from [20] and [21].

Table 4: Comparison of PS and EP

Evolution Method	Mean time (sec)	Best time (sec.)	Mean cost ($)	Maximum cost ($)	Minimum cost ($)
CEP	294.96	293.41	18190.32	18404.04	18048.21
FEP	168.11	166.43	18200.79	18453.82	18018.00
MFEP	317.12	315.98	18192.00	18416.89	18028.09
IFEP	157.43	156.81	18127.06	18267.42	17994.07
PS	5.88	1. 65	18088.84	18233.52	17969.17

In this case the PS method outperforms all other algorithms in terms of all costs: minimum, mean and maximum, while at the same time offering significant saving in computing times.

The convergence of the PS algorithm is shown in Figure 5. As before, the search continues beyond the 70 iterations (when the optimal solution has been reached) to improve the confidence in the result. A total of 122 iterations have been performed.

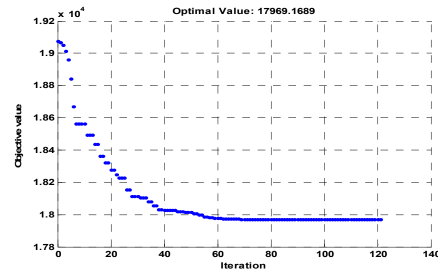

Figure 5: Convergence of PS for the 13 Generating Units

The dynamics of the mesh size is depicted by Figure 6. As before, the initial polling is successful leading to mesh size increases, whereas subsequently the mesh size is being reduced (with the exception of iterations 11 and 23) indicating unsuccessful polls. As for Case I, the termination criteria for the mesh size have not been reached.

Although the PS has achieved the 'best' optimum only on three occasions out of 50 runs (see Figure 7), the overall minimum and mean costs are still better than those obtained by other methods. The total execution time for 50 runs is 294.06 s, which is comparable to just one run using the other techniques.

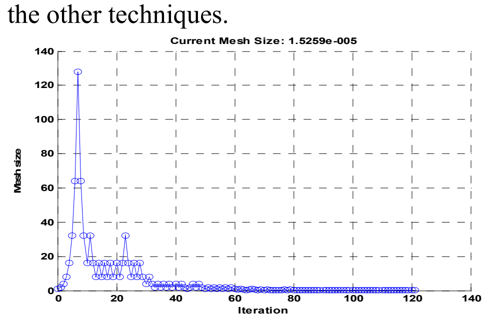

Figure 6: Convergence of PS Mesh Size for the 13 Generating Units

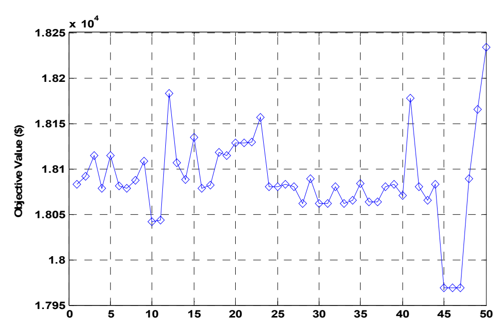

Figure 7: Objective Function Value for 50 different Starting Point (Case II)

5. Conclusion

This paper introduces a new approach based on Pattern Search (PS) optimization to study the power system economic dispatch with valve-point effect, which is formulated as a constrained optimization problem. The proposed method has been applied to two test cases. When compared with Evolutionary Programming (EP), and in one case also with a Genetic Algorithm (GA), the analysis results have demonstrated that PS outperforms the other methods in terms of a better optimal solution and significant reduction of computing times. On the other hand, the PS is more sensitive to the initial guess and appears to rely on how close the given initial point is to the global solution. This makes the PS method possibly more susceptible to getting trapped in local minima. However, the much improved speed of computation allows for additional searches to be made to increase the confidence in the solution. It should also be noted that GA and EP methods normally start with a population of starting points, rather than a single initial point like the PS, thus require even more computational effort. Overall, the PS algorithm has been shown to be very helpful in studying optimization problems in power systems.

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