Decision Directed Channel Estimation Employing Projection Approximation Subspace Tracking

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Abstract— The attainable capacity and integrity of a state-of-the-art broadband multi-carrier communication system is highly sensitive to the accuracy of the information available concerning the channel conditions encountered. The majority of existing advanced channel estimation schemes assume knowledge of the channel's Power Delay Profile (PDP) in order to estimate the corresponding instantaneous Channel Impulse Response (CIR). It is evident however that in realistic mobile channels, where at least one of the communicating terminals is in motion, the power delay profile will also be time-variant and thus may not be known *a priori* at the receiver. In this paper we propose a decision directed channel estimation scheme employing the so-called Projection Approximation Subspace Tracking (PAST) algorithm. The PAST algorithm is utilised for the sake of achieving low-complexity recursive tracking of the channel's PDP. The achievable perormance of the proposed method is documented in the context of an OFDM system communicating in realistic channel conditions characterized by a time-variant fractionally-spaced PDP.

I. INTRODUCTION

The ever-increasing demand for high data-rates in wireless networks requires the efficient utilisation of the limited bandwidth available, while supporting a high grade of mobility in diverse propagation environments. On the other hand, Orthogonal Frequency Devision Multiplexing (OFDM) [1] is capable of satisfying these requirements. This is a benefit of its ability to cope with highly time-variant wireless channel characteristics. However, as pointed out in [2], the capacity and the achievable integrity of communication systems is highly dependent on the system's knowledge concerning the channel conditions encountered. Thus, the provision of an accurate and robust channel estimation strategy is a crucial factor in achieving a high performance.

The family of well-documented *decision directed* channel estimation (DDCE) methods [1], [3]–[5] provides a suitable solution for the problem of channel estimation in OFDM-based systems. The major benefit of the DDCE scheme is that in contrast to purely *pilot assisted* channel estimation methods [6], [7] both the pilot symbols as well as all the information symbols are utilised for channel estimation [1]. The simple philosophy of this method is that in the absence of transmission errors we can benefit from the availability of 100% pilot information by using the detected subcarrier symbols as an *a posteriori* reference signal. The employment of this method allows us to reduce the number of pilot symbols required.

The majority of the aforementioned channel estimation methods rely on the a priori knowlege of the channel statistics commonly characterized by the channel's Power Delay Profile (PDP) for the sake of estimating the instantenious Channel Impulse Response and the corresponding frequency-domain Channel Transfer Function (CTF). It is evident however that in realistic wireless mobile channels,

where at least one of the communicating terminals is in motion, the channel's PDP will also become time-variant and thus may not be a priori known at the receiver. In this paper we propose a decision-directed channel estimation method, which employs the so-called Projection Approximation Subspace Tracking (PAST) algorithm [8] for the sake of recursive tracking of the channel's PDP and subsequent estimation of the instantaneous CTF.

II. SYSTEM MODEL

The discrete frequency-domain model of the OFDM system can be described as in [9]

$$y[n,k] = H[n,k]x[n,k] + w[n,k], \qquad (1)$$

for $k = 0, \ldots, K-1$ and all n, where $y[n,k], x[n,k]$ and $w[n,k]$ are the received symbol, the transmitted symbol and the Gaussian noise sample respectively, corresponding to the kth subcarrier of the nth OFDM block. Furthermore, $H[n,k]$ represents the complex-valued Channel Transfer Function (CTF) coefficient associated with the kth subcarrier and time instance n. In the case of an M-QAM modulated OFDM system, $x[n,k]$ corresponds to the M-QAM symbol accommodated by the kth subcarrier.

As it was pointed out in [5], in OFDM systems using a sufficiently long cyclic prefix and adequate synchronisation, the subcarrier-related CTF can be expressed as

$$H[n,k] \triangleq H(nT, k\Delta f) = C(k\Delta f) \sum_{l=1}^{L} \alpha_l[n] W_K^{k\tau_l/T_s}, \qquad (2)$$

where $\boldsymbol{\alpha}[n]$ is the fractionally-spaced CIR and $C(f)$ is an aggregate transfer function of the transmitter-receiver pulse-shaping filter pair. Note that in realistic channel conditions associated with non-sample-spaced time-variant path-delays $\tau_l(n)$ the fractionally-spaced CIR (FS-CIR) $\alpha_l[n] \triangleq \alpha_l(nT)$ will be constituted by a low number of L non-zero statistically independent taps associated with distinctive propagation paths. The corresponding PDP $\lambda_l[n]$ constituted by the Mean Square (MS) values $\mathrm{E}\{|\alpha_l[n]|^2\}$ may be identified as the vector of the eigenvalues of the channel's covariance matrix

$$C_H = \mathrm{E}\{H[n]H^{\mathrm{H}}[n]\} = W[n]\mathrm{diag}\left(\lambda_l[n]\right) W^{\mathrm{H}}[n], \qquad (3)$$

where W is a $(K \times L)$-dimensional matrix having the eigenvectors of the channel covariance matrix $C_H[n]$ as its columns. The columns of the transformation matrix W span the so-called *signal subspace* of the random vector process associated with the subcarrier-related CTF $H[n]$. Furthermore, Equation (2) can be expressed in a matrix form as

$$H[n] = W[n]\boldsymbol{\alpha}[n]. \qquad (4)$$

III. PARAMETRIC FS-CIR ESTIMATION

A. Projection Approximation Subspace Tracking

Let $H[n] \in \mathbb{C}^K$ be the vector of the subcarrier-related CTF coefficients associated with the channel model of Equation (1). As

described in Section II, the CIR associated with the CTF coefficient vector $H[n]$ is constituted by a relatively low number of $L \ll K$ statistically-independent Rayleigh fading paths. The corresponding CIR components are related to the CTF coefficients $H[n, k]$ by means of Equation (2). The motivation for employing the so-called subspace technique [10] here is that usually we have $L \ll K$ and thus it is more efficient to estimate a low number of CIR-related taps in the low-dimensional signal subspace than estimating all the K FD-CTF coefficients.

Let λ_l and u_l be the eigenvalues and the corresponding eigenvectors of the CTF's covariance matrix C_H of Equation (3). Then, we have $C_H = U\Sigma U^H$, where $\Sigma = \text{diag}(\lambda_l)$ and $U = [u_1 \cdots u_K]$.

The eigenvalues aligned in a descending order may be expressed as

$$\lambda_1 \geq \cdots \geq \lambda_L > \lambda_{L+1} = \cdots = \lambda_K = \sigma_w^2, \tag{5}$$

where the first L dominant eigenvalues $\lambda_1, \cdots, \lambda_L$ in conjunction with the L corresponding eigenvectors $u_1, \cdots, u_L$ may be termed as the *signal* eigenvalues and eigenvectors, respectively [8]. The remaining eigenvalues $\lambda_{L+1}, \cdots, \lambda_K$ and eigenvectors $u_{L+1}, \cdots, u_K$ are termed the *noise* eigenvalues and eigenvectors. The resultant sets of *signal* and *noise* eigenvectors, which are column vectors, span the mutually orthogonal *signal* and *noise* subspaces U_S and U_N, such that we have

$$U_S = [u_1, \cdots, u_L] \quad \text{and} \quad U_N = [u_{L+1}, \cdots, u_K]. \tag{6}$$

The corresponding time-domain-related L-tap estimate of the FS-CIR vector $\alpha[n]$ may be obtained as follows

$$\hat{\alpha} = U_S^H[n]\tilde{H}[n]. \tag{7}$$

Furthermore, the reduced-noise estimate of the CTF vector $H[n]$ may reconstructed using

$$\hat{H}[n] = U_S[n]\hat{\alpha}[n]. \tag{8}$$

For the sake of evaluating and tracking the potentially time-variant signal subspace $U_S[n]$ we employ subspace tracking method developed by Yang [8]. More specifically, we consider the following real-valued scalar objective function having the matrix argument of $W \in \mathbb{C}^{K \times L}$

$$J(W) = \text{E}\left\{\|H - WW^H H\|^2\right\} \\ = \text{tr}(C_H) - 2\,\text{tr}(W^H C_H W) + \text{tr}(W^H C_H W \cdot W^H W) \tag{9}$$

As demonstrated by Yang in [8], the objective function $J(W)$ of Equation (9) exhibits the following important properties

1) W is a stationary point of $J(W)$ if and only if we have $W = U_L Q$, where $U_L \in \mathbb{C}^{K \times L}$ contains any L distinct eigenvectors of C_H and $Q \in \mathbb{C}^{L \times L}$ is an arbitrary unitary matrix. Furthermore, at each stationary point, $J(W)$ equals the sum of these particular eigenvalues, whose eigenvectors are not involved in U_L [8, Theorem 1].
2) All stationary points of $J(W)$ are local saddle points except, when U_L contains the L dominant eigenvectors of C_H. In this case, $J(W)$ attains the global minimum [8, Theorem 2].
3) The global convergence of W is guaranteed by using iterative minimization of $J(W)$ and the columns of the resultant value of W will span the signal subspace of C_H.
4) The use of an iterative algorithm to minimize $J(W)$ will always converge to an orthonormal basis of the signal subspace of C_H without invoking any orthonormalization operations during the iterations.

5) The global minimum of $J(W)$, W does not necessarily contain the signal eigenvectors, but an arbitrary orthogonal basis of the signal subspace of C_H as indicated by the unitary matrix Q introduced in Property 1. In other words, we have $W = \text{argmin}\, J(W)$ if and only if $W = U_S Q$, where Q is an arbitrary unitary matrix.
6) For the simple scalar case of $L = 1$, the solution minimizing $J(W)$ is given by the most dominant normalized eigenvector of C_H.

Subsequently, Yang [8] proposes an iterative RLS algorithm for tracking of the signal subspace of the channel's covariance matrix C_H. Specifically, upon replacing the expectation value in Equation (9) by the exponentially weighted sum of the RLS algorithm, we arrive at the following new objective function

$$J(W[n]) = \sum_{i=1}^{n} \eta^{n-i} \|H[i] - W[n]W^H[n]H[i]\|^2, \tag{10}$$

where $\eta \in (0, 1)$ is the so-called *forgetting factor*, which accounts for possible deviations of the actual channel statistics encountered from the WSS assumption. Observe that the sole difference between the objective functions of Equations (9) and (10) is the introduction of the time-variant exponentially weighted sample covariance matrix [8], which may be expressed as

$$C_H[n] = \sum_{m=1}^{n} \eta^{n-m} H[m]H^H[m] = \eta C_H[n-1] + H[n]H^H[n] \tag{11}$$

instead of the time-invariant matrix $C_H = \text{E}\left\{HH^H\right\}$ of Equation (3).

The PAST algorithm may be derived by approximating the expression $W^H[n]H[m]$ in Equation (10), which may be interpreted as a projection of the vector $H[m]$ onto the column space of the matrix $W[n]$, by the readily available *a posteriori* vector $\alpha[m] = W^H[m]H[m]$. The resultant modified cost function may be formulated as

$$J'(W[n]) = \sum_{m=1}^{n} \eta^{n-m} \|H[m] - W[n]\alpha[m]\|^2. \tag{12}$$

As is argued in [8], for stationary or slowly varying signals, the aforementioned projection approximation, hence the name PAST, does not substantially change the error surface associated with the corresponding cost function of Equation (12) and therefore does not significantly affect the convergence properties of the derived algorithm.

Similarly to other RLS estimation schemes [11], [12], the cost function $J'(W[n])$ is minimized if

$$W = C_{H\alpha}[n]C_{\alpha\alpha}^{-1}[n], \tag{13}$$

where we have

$$C_{H\alpha}[n] = \sum_{i=1}^{n} \eta^{n-i} H[i]\alpha^H[i] = \eta C_{H\alpha}[n-1] + H[n]\alpha^H[n] \tag{14}$$

and

$$C_{\alpha\alpha}[n] = \sum_{i=1}^{n} \eta^{n-i} \alpha[i]\alpha^H[i] = \eta C_{\alpha\alpha}[n-1] + \alpha[n]\alpha^H[n]. \tag{15}$$

Following the RLS approach [8], a low-complexity solution of the computational problem associated with minimizing the cost function $J'(W[n])$ of Equation (12) may be obtained using recursive updates of the matrix $W[n]$. More specifically, we have

$$W[n] = W[n-1] + e[n]k^H[n], \tag{16}$$

Algorithm 1 Deflation PAST

$$H_1[n] = H[n] \tag{21a}$$

$\text{for} \quad l = 1, 2, \ldots, L \quad \text{do}$

$$\alpha_l[n] = w_l^{\mathrm{H}}[n-1]H_l[n] \tag{21b}$$

$$\lambda_l[n] = \beta\lambda_l[n-1] + |\alpha_l[n]|^2 \tag{21c}$$

$$e_l[n] = H_l[n] - w_l[n-1]\alpha_l[n] \tag{21d}$$

$$w_l[n] = w_l[n-1] + e_l[n](\alpha_l^*[n]/\lambda_l[n]) \tag{21e}$$

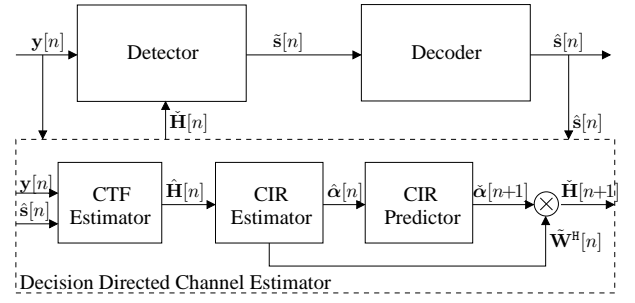$$H_{l+1}[n] = H_l[n] - w_l[n]\alpha_l[n] \tag{21f}$$

end for

Fig. 1. Schematics of a generic receiver employing Decision Directed Channel Estimator constituted by an *a posteriori* decision-directed CTF Estimator, followed by a CIR Estimator and an *a priori* CIR predictor.

where $e[n]$ is the estimation error vector, which may be recursively obtained as

$$e[n] = H[n] - W[n-1]\alpha[n-1], \tag{17}$$

while

$$k[n] = \frac{P[n-1]\alpha[n]}{\eta + \alpha^{\mathrm{H}}[n]P[n-1]\alpha[n]} \tag{18}$$

denotes the RLS gain vector. Furthermore, the matrix $P[n]$ is the inverse of the CIR-related taps' $(L \times L)$-dimensional covariance matrix $C_{\alpha\alpha}$, which can be recursively calculated as follows

$$P[n] = \frac{1}{\eta}\mathrm{Tri}\{(I - k[n]\alpha^{\mathrm{H}}[n])P[n-1]\}, \tag{19}$$

where the operator $\mathrm{Tri}\{\cdot\}$ indicates that only the upper triangular part of $P[n]$ is calculated and its Hermitian conjugate version is copied to the lower triangular part [8].

B. Deflation PAST

In this work, however, we aim for maintaining the lowest possible complexity hence we are particularly interested in the *deflation*-based version of the PAST algorithm derived in [8], which is referred to as the PASTD algorithm. The simple philosophy of the *deflation* method is the sequential estimation of the principal components of the CTF covariance matrix C_H [13]. Consequently, we first update the most dominant eigenvector $w_1[n]$ by applying the PAST method of Section III-A in conjunction with $L = 1$. Subsequently, the projection of the current sample vector $H[n]$ onto the updated eigenvector $w_1[n]$ is subtracted from itself, resulting in a modified (deflated) version of the CTF vector in the following form $H_2[n] = H[n] - w_1[n]w_1^{\mathrm{H}}[n]H[n]$. The second most dominant eigenvector $w_2[n]$ has now become the most dominant one and therefore may be updated similarly to $w_1[n]$. By repeatedly applying this procedure, all the desired eigencomponents may be estimated. The resultant PASTD method is summarized in Algorithm 1.

A particularly important property of the PASTD method of Algorithm 1 is that as opposed to the PAST method of Section III-A, it enables the explicit tracking of the time-variant eigencomponents of the channel covariance matrix $C_H[n]$, namely the eigenvectors $w_l[n]$ as well as of the corresponding eigenvalues $\lambda_l[n]$ according to

$$w_l[n] = w_l[n-1] + \frac{\alpha_l^*[n]}{\lambda_l[n]}(H_l[n] - w_l[n-1]\alpha_l[n]), \tag{20}$$

where we have $\alpha_l[n] = w_l^{\mathrm{H}}[n-1]H[n]$ and $\lambda_l[n] = \beta\lambda_l[n-1] + |\alpha_l[n]|^2$.

C. PASTD -Aided FS-CIR Estimation

In this section we would like to utilize the PASTD method detailed in Section III-B in the context of the decision-directed channel estimation scheme depicted in Figure 1.

More specifically, our channel estimator is constituted by what we refer to as an *a posteriori* decision-directed CTF estimator followed by an *a posteriori* FS-CIR estimator and an *a priori* CIR predictor [1]. We consider a PASTD -aided *a posteriori* FS-CIR estimator, which corresponds to the CIR Estimator module of Figure 1. As seen in Figure 1, the task of the CIR estimator is to estimate the FS-CIR taps of Equation (2).
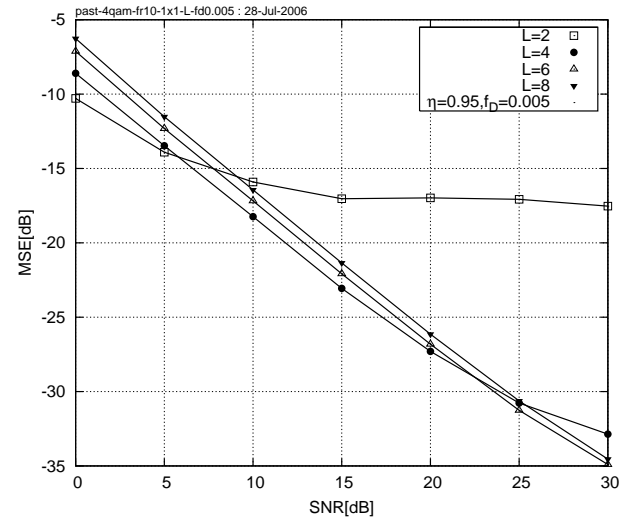

Fig. 2. MSE versus SNR performance exhibited by the 4QAM-OFDM system employing the **PASTD** CIR estimator of Algorithm 1 and tracking $L = 2, 4, 6$ and 8 CIR taps. The value of the PASTD forgetting factor was $\eta = 0.95$. The Doppler frequency was $f_D = 0.005$. Additional system parameters are summarized in Table I.

The achievable performance of the subspace tracking method of Section III-B is characterized in Figures 2, 3 and 4, where we define the Mean Square Error (MSE) performance criterion as follows

$$MSE = \mathbb{E}\left\{\sum_l |e_l[n]|^2\right\}, \tag{22}$$

where e_l is the FD-CTF tracking error defined by Equation 17. In our simulations we consider an OFDM system having $K = 128$ orthogonal QPSK-modulated subcarriers. The system characteristics are outlined in Table I. We employ an OFDM-frame-variant channel model having a time-variant 8-tap PDP characterized by the COST-207 BU channel model [14]. Additionally, each individual propa-
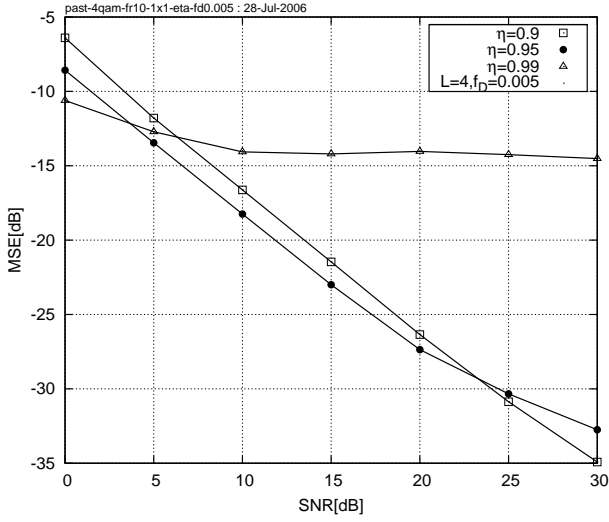
Fig. 3. MSE versus SNR performance exhibited by the 4QAM-OFDM system employing the **PASTD** method of Algorithm 1. The values of the PASTD forgetting factor wwere $\eta = 0.9, 0.95$ and 0.9. The Doppler frequency was $f_D = 0.005$. Additional system parameters are summarized in Table I.
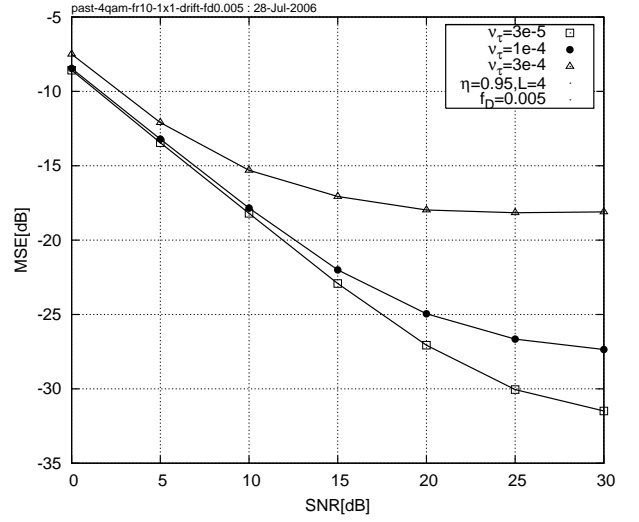

Fig. 4. MSE versus SNR performance exhibited by the 4QAM-OFDM system employing the **PASTD** method of Algorithm 1, while encountering different values of the PDP tap drift rate $\nu_\tau = 3 \cdot 10^{-5}, 10^{-4}$ and $3 \cdot 10^{-4}$ as well as the Doppler frequency of $f_D = 0.005$. Additional system parameters are summarized in Table I.

gation path undergoes fast Rayleigh fading with a corresponding OFDM-symbol-normalized Doppler frequency of $f_D = 0.005$. The resultant channel can be characterised as a mutli-path Rayleigh-fading channel with slowly-varying PDP, where the relative delays τ_l associated with different PDP taps vary with time at a rate determined by the drift rate parameter ν_τ

Firstly, Figure 2 characterizes the achievable FD-CTF MSE performance of the PASTD method of Algorithm 1 for different ranks L of the estimated subspace, while assuming a constant value of $\eta = 0.95$ for the forgetting factor. From Figure 2, we may conclude that a high CIR estimator performance may be achieved when assuming that the estimated CTF signal subspace has a rank of $L = 4$, regardless of the actual number of paths constituting the multi-path channel encountered.

Secondly, Figure 3 characterizes the achievable MSE performance of the PASTD method of Algorithm 1 for different values of the forgetting factor η, while assuming a constant rank of $L = 4$ for the estimated subspace. As may be concluded from Figure 3, the optimum value of the forgetting factor η is largely dependent on the SNR as well as on the Doppler frequency encountered. Nevertheless, the compromise value of $\eta = 0.95$ appears to constitute a relatively good choice in the practical ranges of both SNR values and Doppler frequencies.

Finally, Figure 4 characterizes the achievable MSE performance of the PASTD method of Section III-B for different values of the OFDM-symbol-normalized PDP tap drift rate $\nu_\tau = T\frac{v}{c}$, where T is the OFDM symbol's duration, while v and c denote the speeds of the communicating terminal and light, respectively. Observe that the specific values of the parameter ν_τ assumed in Figure 4 substantially exceed the maximum value considered in the base-line scenario outlined in Table I. Consequently, we may conclude that the CIR tracking method of Algorithm 1 exhibits an adequate performance over the entire range of practical channel conditions.

IV. PASTD AIDED DDCE

The detailed schematic of the channel estimation scheme proposed is depicted in Figure 5. Our channel estimator is constituted by a bank of the per-subcarrier *a posteriori* MMSE CTF estimators [1], followed by the PASTD -aided CIR estimator of Algorithm 1 as well as by
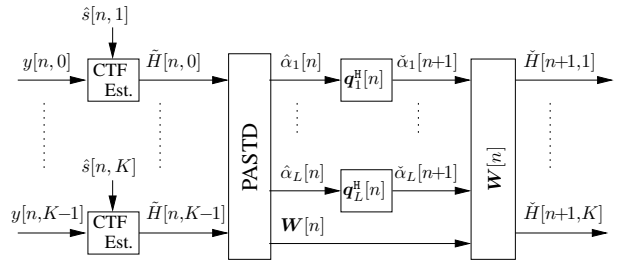
Fig. 5. Detailed structure of the 2D channel estimator corresponding to the DDCE module of Figure 1. The channel estimator comprises a PAST module, which performs recursive tracking of the CIR. The resultant CIR related taps $\hat{\alpha}_l[n]$ are filtered by the adaptive RLS-based prediction filter resulting in the *a priori* estimates of the CIR-related taps $\check{\alpha}_l[n+1]$. Finally, the *a priori* estimates of the subcarrier-related coefficients $\check{H}[n+1, k]$ are obtained by applying the transform matrix $W[n]$ provided by the PASTD module.

the *a priori* RLS CIR predictor [15]. The task of the CTF estimator seen in Figure 5 is to estimate the subcarrier-related CTF coefficients $H[n, k]$ of Equation (2). The resultant estimated subcarrier-related samples $\tilde{H}[n, k]$, which serve as an observation vector of the FD-CTF coefficients $H[n, k]$ are fed to the PASTD subspace-based tracking module, which performs recursive tracking of the channel's covariance matrix C_H signal subspace and the associated CIR-related taps. The output of the PASTD module is constituted by the instantaneous

TABLE I
SYSTEM PARAMETERS.

Parameter	Value
Carrier frequency f_c	2.5 GHz
Channel bandwidth B	8 MHz
Number of carriers K	128
FFT frame duration T_s	16 μs
OFDM symbol duration T	20 μs (4 μs of cyclic prefix)
Max. delay spread τ_{max}	4 μs
Max. terminal speed v	130 km/h
Norm. Max. Doppler spread f_D	$0.006 = T \cdot 300$ Hz
Norm. Max. PDP tap drift ν_τ	$2.4 \cdot 10^{-6}$ μs $= T \cdot 0.12$ μs/s

CIR-related tap estimates $\hat{\alpha}_l[n]$ and the corresponding estimate of the transformation matrix $W[n]$ of Equation (16). The CIR-related estimate vector $\hat{\alpha}_l[n]$ is then fed into the low-rank time-domain CIR-related tap predictor of Figure 5 for the sake of producing an *a priori* estimate $\breve{\alpha}_l[n+1]$, $l = 1, \cdots, L$ of the next CIR-related tap-vector on a tap-by-tap basis [1]. Finally, the predicted CIR is converted to the subcarrier-related CTF with the aid of the transformation matrix $W[n]$ provided by the PASTD module of Figure 5. The resultant FD-CTF is employed by the receiver for the sake of detecting and decoding of the next OFDM symbol. Note that this principle requires the transmission of a frequency-domain pilot-based channel sounding sequence, such as for example a pilot-assisted OFDM symbol, during the initialisation stage.

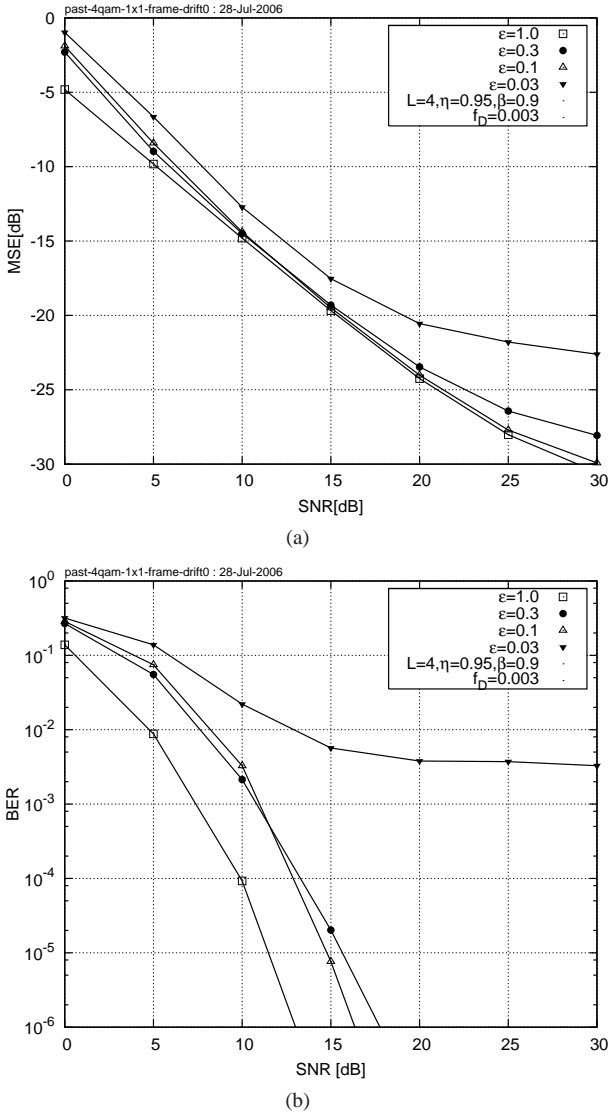*A. Performance Results and Conclusions*

(a)

(b)

Fig. 6. (a) MSE and (b) BER versus SNR exhibited by the 4QAM-OFDM system employing the **PASTD** -aided DDCE. The value of the parameters $L = 4, \eta = 0.95$ and $\beta = 0.9$ has been assumed. The Doppler frequency was $f_D = 0.003$. Additional system parameters are summarized in Table I.

The achievable performance of the PASTD aided DDCE scheme proposed is characterized in Figure 6. In our simulations we considered an OFDM system having $K = 128$ QPSK-modulated orthogonal subcarriers. The system characteristics are outlined in Table I. We employ an OFDM-frame-variant channel model associated with a time-variant 7-tap PDP characterized by the COST-207 BU channel model [14]. Additionally, each individual propagation path undergoes fast Rayleigh fading having an OFDM-symbol-normalized Doppler frequency of $f_D = 0.003$. We assumed the values $L = 4$ and $\eta = 0.95$ for the PASTD module-related subspace rank and forgetting factor parameters respectively, as well as the value of $\beta = 0.9$ for the RLS CIR-tap predictor-related forgetting factor. Additional system parameters are summarized in Table I.

Figure 6(a) portrays the achievable MSE performance of the PASTD aided DDCE for the pilot overhead ratios $\varepsilon = 0.03, 0.1, 0.3$ and 1.0, where $\varepsilon = 0.03$ and $\varepsilon = 1.0$ correspond to having 3% and 100% pilots, respectively. Furthermore, Figure 6(b) portrays the corresponding BER performance of the rate $\frac{1}{2}$ turbo-coded QPSK-modulated OFDM system.

In conclusion, in this paper we have discribed a PAST-aided DDCE scheme. We have documented the achievable performance of the PAST-DDCE method proposed in the context of an OFDM system communicating in realistic channel conditions characterized by a time-variant fractionally-spaced PDP. We have demonstrated, that the channel estimation method proposed exhibits good performance over the entire range of relevant channel conditions, while using as little as 10% of pilot overhead.

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