Exact BER Analysis of OFDM Systems Communicating over Frequency-Selective Fading Channels Subjected to Carrier Frequency Offset

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***Abstract*— Orthogonal Frequency Division Multiplexing (OFDM) has been employed in numerous wireless standards. However, the performance of OFDM systems is degraded by both the Carrier Frequency Offset (CFO) and the Phase Estimation Error (PER). Hence new exact closed-form expressions are derived for calculating the average BER of OFDM systems in the presence of both CFO and PER in the context of frequency-selective Nakagami-m fading channels. Our simulation results verify the accuracy of our exact BER analysis. By contrast, the Gaussian approximation slightly over-estimates the average BER, especially when the normalized CFO is small, the number of OFDM subcarriers is low and when the fading is less severe.**

I. Introduction

In recent years Orthogonal Frequency Division Multiplexing (OFDM) has attracted intensive research efforts and, as a result, has found its way into numerous wireless standards [1]. However, OFDM is sensitive to the effects of Carrier Frequency Offset (CFO), which destroys the orthogonality of the OFDM subcarriers and inflicts Inter-Carrier Interference (ICI). The performance degradation imposed by the CFO has been extensively studied in the open literature [2]–[14].

As a benefit of the associated mathematical simplicity, the Signal-to-Interference-plus-Noise Ratio (SINR) or Signal-to-Noise Ratio (SNR) have been the predominantly quantified performance metrics over the past decade [2]–[7]. Nevertheless, the most important performance evaluation metrics are the Bit Error Ratio (BER) [8]–[10] and the Symbol Error Ratio (SER) [7], [11], which characterize the associated performance degradation more accurately.

One of the difficulties in the analysis of the performance degradation caused by CFO in OFDM systems is the statistical characterization of the ICI. Typically the ICI is assumed to be approximately Gaussian distributed [7]–[10], [12], which is based on the Central Limit Theorem (CLT) [15]. However, the accuracy of the Gaussian Approximation (GA) is limited [8], especially when the SNR encountered is high. This is because at high SNRs the effects of ICI become more dominant, particularly at low Doppler frequencies, when the ICI is typically constituted by a low number of immediately adjacent subcarriers. In this scenario the CLT is not satisfied. The exact analysis of the ICI distribution was provided based on either multiple integrals [10] or on the Characteristic Function (CF) [11]–[13].

At the time of writing the exact analytical BER/SER degradation caused by CFO in OFDM systems has been quantified in the context of Additive White Gaussian Noise (AWGN) channels [11]–[13], Rayleigh fading channels [10], [12], [14] and Ricean fading channels [10]. Moreover, to the best of the authors' knowledge, there are no studies in the open literature on the exact analytical BER performance of OFDM systems in the context of communicating over Nakagami-m fading channels in the presence of CFO. *Against this background, the novel contribution of this paper is that we provide a closed-form expression, rather than a sum of the infinite series in [11], [12], for the average BER calculation of OFDM systems in the presence of both CFO and Phase Estimation Error (PER) in the context of frequency-selective Nakagami-m fading channels.*

This paper is organized as follows. In Section II an OFDM system using the BPSK modulation communicating over frequency-selective Nakagami-m channels in the presence of CFO is presented. Then, in Section III its exact BER performance is investigated based on the CF approach. Our numerical results are presented in Section IV and finally, our conclusions are provided in Section V.

II. System Model

We consider an OFDM system having N subcarriers using the BPSK modulation for communicating over frequency-selective Nakagami-m slow-fading channels.

Let us assume that the data symbols $\{\widetilde{A}_k\}_{k=0}^{N-1}$ transmitted over the N subcarriers are mutually independent and selected from the constellation set according to a uniform probability distribution. The transmitted equivalent baseband OFDM signal $\widetilde{s}(t)$ can be expressed in the time-domain as [14]:

$$\widetilde{s}(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \widetilde{A}_k e^{j\frac{2\pi kt}{T_u}}, -T_g \le t < T_u, \qquad (1)$$

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where T_g and T_u are the duration of the cyclic prefix and the useful data signal segment, respectively, $\widetilde{A}_k$ is the information symbol modulating the kth subcarrier in the Frequency-Domain (FD) and N is the number of OFDM subcarriers.

We consider a time-invariant frequency-selective fading channel. Typically the length T_g of the cyclic prefix is designed so that it becomes longer than the maximum propagation delay. Hence the Inter-Symbol Interference (ISI) between consecutive OFDM symbols is considered to be negligible in our analysis. Then the received signal $\widetilde{r}(t)$ encountered in the presence of CFO can be expressed as:

$$\widetilde{r}(t) = \frac{1}{\sqrt{N}} e^{j2\pi f_\Delta t} \sum_{k=0}^{N-1} \widetilde{A}_k \widetilde{h}_k + \widetilde{\eta}(t), -T_g \leq t < T_u, \quad (2)$$

where f_Δ denotes the carrier frequency offset between the transmitter and the receiver, and $\widetilde{\eta}(t)$ denotes the zero-mean complex-valued AWGN. More explicitly, $\widetilde{h}_k = h_k e^{j\theta_k}$ represents the complex-valued Frequency-Domain (FD) channel transfer function (FDCHTF) of the kth subcarrier, which are considered to be mutually independent. In practice this assumption becomes valid, when the channel is quite dispersive and hence the CIR becomes significantly longer than the bit duration at the input of the OFDM modem. The FDCHTF h_k is characterized with the aid of the parameters $\{m_k, \Omega_k\}$, where m_k and Ω_k are the Nakagami-m fading parameter and the average power of the kth subcarrier, respectively[1] [16]. The associated Probability Density Function (PDF) and Characteristic Function (CF) of the FDCHTF h_k were given in [17], while the fading phase θ_k is typically assumed to be uniformly distributed over the interval of $[0, 2\pi)$ [18].

The received signal is sampled within an OFDM symbol at the time instants:

$$t_n = \frac{nT_u}{N}, n = 0, 1, ..., N-1. \quad (3)$$

Then the N Time-Domain (TD) samples $\{\widetilde{b}_n\}_{n=0}^{N-1}$ within an OFDM symbol are given by:

$$\widetilde{b}_n = \widetilde{r}(t_n) = \frac{1}{\sqrt{N}} e^{j\frac{2\pi n\epsilon}{N}} \sum_{k=0}^{N-1} \widetilde{A}_k e^{j\frac{2\pi kn}{N}} \widetilde{h}_k + \widetilde{\eta}_n, \quad (4)$$

where $\epsilon = f_\Delta T_u$ is the normalized CFO and $\widetilde{\eta}_n = \widetilde{\eta}(t_n)$ is the noise sample at the nth sampling instant within an OFDM symbol.

Upon performing Fast Fourier Transform (FFT) based demodulation [1], the Frequency-Domain (FD) symbols $\{\widetilde{B}_{k'}\}_{k'=0}^{N-1}$ can be expressed as:

$$\begin{aligned}\widetilde{B}_{k'} &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \widetilde{b}_n e^{-j\frac{2\pi k' n}{N}} \\ &= \sum_{k=0}^{N-1} \widetilde{A}_k \widetilde{d}_{k-k'} \widetilde{h}_k + \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \widetilde{\eta}_n e^{-j\frac{2\pi k' n}{N}}, \quad (5)\end{aligned}$$

[1]When the time-domain fading is Nakagami-m distributed, it is not strictly true that the frequency-domain fading is also Nakagami-m distributed. Nevertheless, this is a reasonable approximation, as it was argued in [16] and hence it will be exploited in this treatise.

where the frequency-offset-dependent complex-valued FDCHTF-contribution $\widetilde{d}_k$ [9]–[11] induced by the CFO of the kth subcarrier is given by:

$$\widetilde{d}_k = d_k e^{j\psi_k} = \frac{\sin[\pi(k+\epsilon)]}{N \sin[\frac{\pi}{N}(k+\epsilon)]} e^{j\pi(1-\frac{1}{N})(k+\epsilon)}. \quad (6)$$

If the receiver is capable of compensating for the aggregate phase-shift $(\psi_0 + \theta_{k'})$ of the k'th subcarrier experiencing a phase estimation error of $\varsigma_{k'}$, the decision statistics $\widetilde{Z}_{k'}$ may be written as:

$$\widetilde{Z}_{k'} = \widetilde{D}_{k'} + \sum_{k=0, k\neq k'}^{N-1} \widetilde{I}_{k'k} + \widetilde{\Lambda}_{k'}, \quad (7)$$

where the signal component $\widetilde{D}_{k'}$, the ICI component $\widetilde{I}_{k'k}$ imposed on the kth subcarrier and the noise component $\widetilde{\Lambda}_{k'}$ are given by:

$$\widetilde{D}_{k'} = \widetilde{A}_{k'} d_0 h_{k'} e^{j\varsigma_{k'}} \quad (8)$$

$$\widetilde{I}_{k'k} = \widetilde{A}_k \widetilde{d}_{k-k'} \widetilde{h}_k e^{-j(\psi_0+\theta_{k'})+j\varsigma_{k'}} \quad (9)$$

$$\widetilde{\Lambda}_{k'} = \frac{1}{\sqrt{N}} e^{-j(\psi_0+\theta_{k'})+j\varsigma_{k'}} \sum_{n=0}^{N-1} \widetilde{\eta}_n e^{-j\frac{2\pi k' n}{N}}. \quad (10)$$

If we assume furthermore that the noise samples $\widetilde{\eta}_n$, $n = 0, 1, ..., N-1$ are independently and identically distributed zero-mean complex Gaussian variables having a variance of $2\sigma_\eta^2$, $\widetilde{\Lambda}_{k'}$ may be shown to be a zero-mean complex-valued Gaussian random variable having a variance of $2\sigma_\eta^2$.

III. BER Analysis

A. Exact BER Analysis

If BPSK modulation is used, we may judiciously assume that the data symbol $\widetilde{A}_k$ transmitted over the kth subcarrier obeys the symmetric Bernoulli distribution [15], i.e. we have $P\left\{\widetilde{A}_k = \pm 1\right\} = \frac{1}{2}$. Since the CF of $\Re\{\widetilde{h}_k\}$ is known when $\widetilde{h}_k$ is a complex-valued Nakagami-m distributed variable [19], the conditional CF $\Phi_{\Re\{\widetilde{I}_{k'k}\}|\widetilde{A}_k}(\omega)$ of the real ICI component imposed by the kth subcarrier may be expressed as [19]:

$$\Phi_{\Re\{\widetilde{I}_{k'k}\}|\widetilde{A}_k}(\omega) = {}_1\mathbb{F}_1\left(m_k; 1; -\frac{\Omega_k}{4m_k} d_{k-k'}^2 \omega^2\right), \quad (11)$$

where ${}_1\mathbb{F}_1(\alpha;\beta;x)$ is the confluent hypergeometric function [20]. It becomes explicit from Equation 11 that $\Phi_{\Re\{\widetilde{I}_{k'k}\}|\widetilde{A}_k}(\omega)$ is independent of the transmitted symbol $\widetilde{A}_k$. Hence the CF $\Phi_{\Re\{\widetilde{I}_{k'k}\}}(\omega)$ of the real ICI component imposed by the kth subcarrier may be formulated as:

$$\Phi_{\Re\{\widetilde{I}_{k'k}\}}(\omega) = \Phi_{\Re\{\widetilde{I}_{k'k}\}|\widetilde{A}_{k'}}(\omega). \quad (12)$$

It may be readily shown that the ICI contributions $\widetilde{I}_{k'k}$, $k = 0, 1, ..., N-1$ and $k \neq k'$ imposed by the different subcarriers are mutually independent. Upon defining the total interference plus noise $\widetilde{\xi}_{k'}$ experienced by the k'th subcarrier as:

$$\widetilde{\xi}_{k'} = \sum_{k=0, k\neq k'}^{N-1} \widetilde{I}_{k'k} + \widetilde{\Lambda}_{k'}, \quad (13)$$

it transpires that both the PDF $f_{\Re\{\widetilde{\xi}_{k'}\}}(x)$ and the CF $\Phi_{\Re\{\widetilde{\xi}_{k'}\}}(\omega)$ of $\Re\{\widetilde{\xi}_{k'}\}$ are even functions. The CF $\Phi_{\Re\{\widetilde{\xi}_{k'}\}}(\omega)$ may be expressed as:

$$\Phi_{\Re\{\widetilde{\xi}_{k'}\}}(\omega) = \Phi_{\Re\{\widetilde{\Lambda}_{k'}\}}(\omega) \prod_{k=0,k\neq k'}^{N-1} \Phi_{\Re\{\widetilde{I}_{k'k}\}}(\omega). \tag{14}$$

Upon performing the inverse Fourier transform on $\Phi_{\Re\{\widetilde{\xi}_{k'}\}}(\omega)$, we arrive at the PDF $f_{\Re\{\widetilde{\xi}_{k'}\}}(x)$ of $\Re\{\widetilde{\xi}_{k'}\}$. Then, upon integrating $f_{\Re\{\widetilde{\xi}_{k'}\}}(x)$, we generate the Cumulative Distribution Function (CDF) $F_{\Re\{\widetilde{\xi}_{k'}\}}(x)$ of $\Re\{\widetilde{\xi}_{k'}\}$. The average BEP $P_{e|\widetilde{A}_{k'},h_{k'}}(k')$ conditioned on the transmitted symbol $\widetilde{A}_{k'}$ and the fading amplitude $h_{k'}$ is given by:

$$P_{e|\widetilde{A}_{k'},h_{k'}}(k') = 1 - F_{\Re\{\widetilde{\xi}_{k'}\}}\left(\left|\Re\{\widetilde{D}_{k'}\}\right|\right). \tag{15}$$

Upon integrating $P_{e|\widetilde{A}_{k'},h_{k'}}(k')$ over $\widetilde{A}_{k'}$ and $h_{k'}$, we arrive at the average BEP $P_e(k')$ of the k'th subcarrier as:

$$P_e(k') = \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{1}{\omega}\Phi_{\Re\{\widetilde{\xi}_{k'}\}}(\omega)\Im\left\{\Phi_{h_{k'}}(d_0\omega\cos\varsigma_{k'})\right\} \mathrm{d}\omega, \tag{16}$$

where $\Phi_{h_{k'}}(\omega)$ is the imaginary part of the CF of the fading amplitude $h_{k'}$ of the k'th subcarrier, which is given by Table II of [17]. Upon exploiting the integral identity of Equation 5.2.4.35 in [21], we arrive at the closed-form version of Equation 16 in the form of:

$$P_e(k') = \frac{1}{2} - \frac{\Gamma(m_{k'}+\frac{1}{2})}{\Gamma(m_{k'})}\sqrt{\frac{\gamma_{k'}}{\pi m_{k'}}} \times \mathbb{F}_A^{(N)}\left(\frac{1}{2}, \alpha_0, ..., \alpha_{N-1}; \beta_0, ..., \beta_{N-1}; -\frac{\gamma_0}{m_0}, ..., -\frac{\gamma_{N-1}}{m_{N-1}}\right), \tag{17}$$

where $\mathbb{F}_A^{(n)}(\lambda, \alpha_1, ..., \alpha_n; \beta_1, ..., \beta_n; x_1, ..., x_n)$ is the Lauricella function of n variables [21]. The coefficients of α_k, β_k and γ_k, $k = 0, 1, ..., N-1$, are given by:

$$\alpha_k = \begin{cases} m_{k'} + \frac{1}{2}, k = k', \\ m_{k'}, \quad k \neq k', \end{cases} \tag{18}$$

$$\beta_k = \begin{cases} \frac{3}{2}, k = k', \\ \frac{1}{2}, k \neq k', \end{cases} \tag{19}$$

$$\gamma_k = \begin{cases} \dfrac{\Omega_{k'}}{2\sigma_\eta^2} d_0^2 \cos^2\varsigma_{k'}, k = k', \\ \dfrac{\Omega_k}{2\sigma_\eta^2} d_{k-k'}^2, \quad k \neq k'. \end{cases} \tag{20}$$

If there is no CFO, i.e. we have $\epsilon = 0$, there will be no ICI and the system is equivalent to a single-carrier and single-user Nakagami-m fading model [22]. Accordingly, Equation 17 reduces to

$$P_e(k') = \frac{1}{2} - \frac{\Gamma(m_{k'}+\frac{1}{2})}{\Gamma(m_{k'})}\sqrt{\frac{\gamma_{k'}}{\pi m_{k'}}}\,{}_2\mathbb{F}_1\left(m_{k'}+\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; -\frac{\gamma_{k'}}{m_{k'}}\right), \tag{21}$$

where ${}_2\mathbb{F}_1(\alpha, \beta; \gamma; x)$ is the hypergeometric function [20]. Equation 21 is equivalent to Equation 8.106 in [22] for BPSK, except that there is a multiplicative factor of $\cos^2\varsigma_{k'}$ in Equation 20, which is induced by the phase estimation error.

B. Special Case: Rayleigh Fading

When all subcarriers are subjected to Rayleigh fading, i.e. we have $m_k = 1$ for $k = 0, 1, ..., N-1$, Equation 11 reduces to:

$$\Phi_{\Re\{\widetilde{I}_{k'k}\}|\widetilde{A}_k}(\omega) = \exp\left(-\frac{1}{4}d_{k-k'}^2\Omega_k\omega^2\right). \tag{22}$$

Furthermore, Equation 17 reduces to:

$$P_e(k') = \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + \frac{1}{\gamma_{k'}} + \sum_{k=0,k\neq k'}^{N-1}\frac{\gamma_k}{\gamma_{k'}}}}\right). \tag{23}$$

C. Gaussian Approximation

The ICI is typically assumed to be Gaussian distributed [7]–[10], [12] and hence it may be treated as additional AWGN. Hence the CF $\Phi_{\Re\{\widetilde{I}_{k'k}\}}(\omega)$ of the real ICI component imposed by the kth subcarrier may be approximated using Equation 22 for BPSK modulation. Therefore, this scenario is also equivalent to the single-carrier and single-user Nakagami-m fading model, except that the noise is replaced by the combined interference-plus-noise component. The average BER expressed in Equation 17 may be shown to be approximated as:

$$P_e(k') \approx \frac{1}{2} - \frac{\Gamma(m_{k'}+\frac{1}{2})}{\Gamma(m_{k'})}\sqrt{\frac{\gamma_{k'}}{\pi m_{k'}\left(1+\sum_{k=0,k\neq k'}^{N-1}\gamma_k\right)}} \times {}_2\mathbb{F}_1\left[m_{k'}+\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; -\frac{\gamma_{k'}}{m_{k'}\left(1+\sum_{k=0,k\neq k'}^{N-1}\gamma_k\right)}\right]. \tag{24}$$

In fact, the GA corresponds to the case, when all interfering subcarriers are subjected to Rayleigh fading, although there is no reason for the desired subcarrier to suffer from fading statistics different from those of other subcarriers. If all subcarriers are subjected to Rayleigh fading, Equation 24 reduces to Equation 23. In this case the results obtained by the Gaussian approximation coincide with those obtained by our exact analysis and this has been reported in [9], [10].

IV. Numerical Results

In this section we will verify the accuracy of our exact BER analysis provided in Sections III-A for the BPSK modulation, as well as demonstrate the relatively high accuracy of the Gaussian ICI approximation provided in Section III-C. Since the effects of PER only introduce a multiplicative factor of $\cos^2\varsigma_{k'}$ in Equation 20, we will focus our attention on considering the effects of the CFO, i.e. we assume having no PER, yielding $\varsigma_{k'} = 0$.

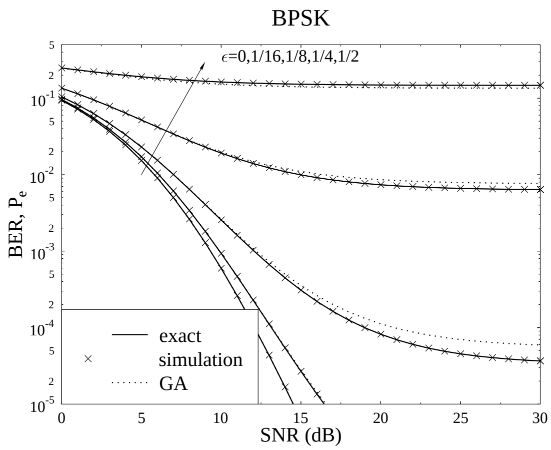

Fig. 1. BER versus the per-bit SNR in a BPSK-modulated OFDM system subjected to frequency-selective Nakagami-m fading. All OFDM subcarriers experience the same fading distribution, i.e. we have $m_k = m = 5$. The number of OFDM subcarriers is $N = 64$. The normalized CFO is $\epsilon = 0$, $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{2}$, respectively. Perfect channel estimation is assumed, i.e. we have $\varsigma = 0$.

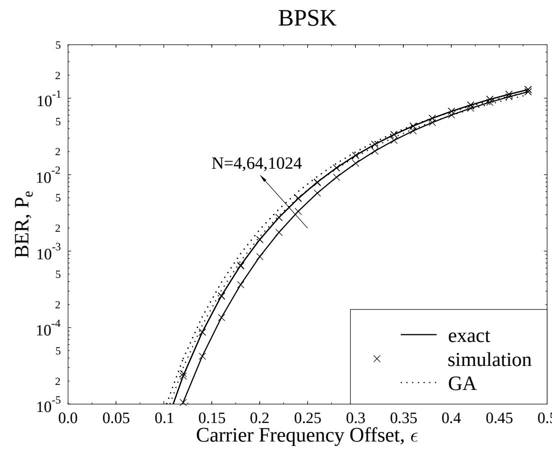

Fig. 2. BER versus the normalized CFO in a BPSK-modulated OFDM system subjected to frequency-selective Nakagami-m fading. All OFDM subcarriers experience the same fading distribution, i.e. we have $m_k = m = 5$. The number of OFDM subcarriers is $N = 4$, 64 and 1024, respectively, but the associated BER curves of $N = 64$ and 1024 are indistinguishable for both the exact results and for the GA. Perfect channel estimation is assumed, i.e. we have $\varsigma = 0$. The background noise is ignored, i.e. we have $\sigma_\eta = 0$.

Figure 1 illustrates the average BER performance versus the per-bit SNR in conjunction with various normalized CFO values in a BSPK-modulated OFDM system in the presence of CFO and frequency-selective Nakagami-m fading. We included no GA results in Figure 1, when we have $\epsilon = 0$, since there is no ICI when there is no CFO. We can see from Figure 1 that generally speaking the BER decreases, as the SNR value increases, but eventually remains limited by the ICI, leading to an error floor. There are two exceptions, namely when we have $\epsilon = 0$ and $\frac{1}{2}$. When there is no CFO, i.e. we have $\epsilon = 0$, the BPSK-modulated OFDM system is noise-limited. By contrast, when the CFO is relatively high, i.e. we have $\epsilon = \frac{1}{2}$, the system behaves interference-limited. Furthermore, Figure 1 shows that the results obtained from our exact BEP analysis and those accruing from simulations match well for various normalized CFO values. The GA is also fairly accurate, but it slightly over-estimates the BER in Figure 1, especially, when the SNR is high and when the normalized CFO is low.

Figure 2 illustrates the achievable average BER versus the normalized CFO performance in conjunction with various numbers of OFDM subcarriers in a BSPK-modulated OFDM system in the context of frequency-selective Nakagami-m fading. Note that there should be no bit errors in the context of a single-carrier BPSK-modulated OFDM system, when the effects of noise are ignored. However, when multi-carrier modulation is adopted, the ICI induced by the CFO limits the achievable BER performance. Nevertheless, as the number of OFDM subcarriers is increased, the BER performance degradation is not aggravated further, provided that the number of OFDM subcarriers is sufficiently high. Upon their further increase, the associated BER curve of 64-subcarrier OFDM and of 1024-subcarrier OFDM becomes indistinguishable in the BPSK-modulated OFDM system considered. We can see from Figure 2 that the CFO significantly degrades the achievable BER performance of OFDM systems. When there is no CFO, there are no bit errors, since we stated before that we have ignored the effects of background noise. When the normalized CFO is low, the BER increases exponentially with the normalized CFO. Again, Figure 2 shows that the results obtained from our exact analysis and those accruing from our simulations match well for various numbers of OFDM subcarriers. Similarly, the GA estimates the BER performance in Figure 2 fairly accurately, although the BER is slightly over-estimated, especially when the normalized CFO is low and when the number of OFDM subcarriers is low.

Figure 3 illustrates the achievable average BER performance of a BSPK-modulated OFDM system versus the normalized CFO in conjunction with various Nakagami-m fading parameters in the context of frequency-selective Nakagami-m fading. Although we do not plot the curves for small values of the normalized CFO, i.e. for $\epsilon < 0.02$, the BERs recorded for all Nakagami-m fading parameters should be zero, when there is no CFO, i.e. for $\epsilon = 0$, since we have ignored the effects of noise. Therefore, we can see from Figure 3 that the CFO has a more detrimental impact on the system suffering from more severe fading, i.e. when the Nakagami-m parameter is lower. When the normalized CFO is low, the BER increases exponentially with the normalized CFO. Again, Figure 3 shows that the results obtained from our exact analysis and those accruing from our simulations match well for the various numbers of OFDM subcarriers considered. As stated in Section III-C, the results obtained by the GA in Figure 3 match with those

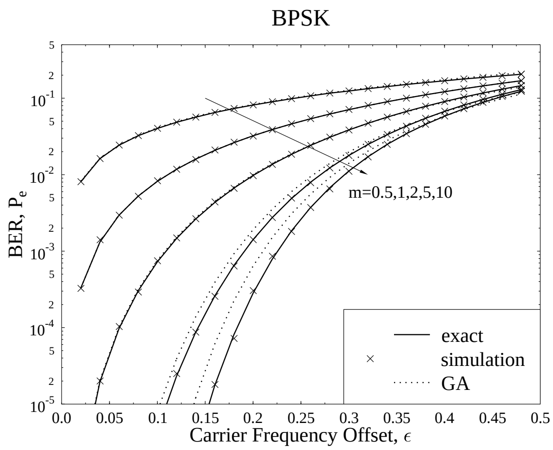

Fig. 3. BER versus the normalized carrier frequency offset in a BPSK-modulated OFDM system subjected to frequency-selective Nakagami-m fading. All OFDM subcarriers experience the same fading distribution, i.e. we have $m_k = m$. The Nakagami-m parameter is $m = 0.5$, 1, 2, 5 and 10, respectively. Note that the BER curves obtained by our exact analysis and by the Gaussian approximation are not distinguishable when $m = 0.5$, 1 and 2. The number of OFDM subcarriers is $N = 64$. Perfect channel estimation is assumed, i.e. we have $\varsigma = 0$. The background noise is ignored, i.e. we have $\sigma_\eta = 0$.

generated by our exact analysis, when the fading is Rayleigh, i.e. when we have $m = 1$. However, the GA slightly over-estimates the BER, when the fading is less severe, i.e. when the Nakagami-m fading parameter is high, especially when the normalized CFO is low.

V. Conclusion

We have analyzed the BER degradation of a BPSK-modulated OFDM system induced by both the CFO and the PER in the context of frequency-selective Nakagami-m fading channels. In contrast to the sum of the infinite series provided in [11], [12], a closed-form expression was provided for calculating the average BER of such OFDM systems, which were derived based on the CF approach. Our simulation results verified the accuracy of our exact BER analysis for various combinations of the normalized CFO value, the number of OFDM subcarriers and the Nakagami-m fading parameter. The Gaussian approximation of the ICI also estimates the average BER fairly accurately, although when the per-bit SNR is high, the normalized CFO is small, the number of OFDM subcarriers is low and when the fading is less severe, the GA slightly over-estimates the BER.

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