Fading Performance Evaluation of an Adaptive MSER Beamforming Receiver for QAM Systems

Andrew Livingstone

Sheng Chen, Lajos Hanzo

Detica

University of Southampton
Motivation
System Model

L – number of antenna array elements
S – number of users
\( b(k) \) – symbols transmitted by each user at time instant \( k \)
\( P(k) \) – describes the propagation channel and physical arrangement of antenna elements
\( n(k) \) – additive white Gaussian noise
\( x(k) \) – vector of received signal samples
\( w(k) \) – beamformer weights
\( y(k) \) – beamformer soft output
MMSE – Minimum Mean-Squared-Error

Minimise the error between estimated and transmitted signal waveforms

If \( \varepsilon(k) \) is the instantaneous error, then

\[
\text{MSE} \, \xi = E \left[ |\varepsilon(k)|^2 \right]
\]

The MMSE problem can be defined

\[
\mathbf{w}_{MMSE} = \arg \min_{\mathbf{w}} \xi(\mathbf{w})
\]

Its solution is the minimum of the MSE cost function (right), the Wiener solution

\[
\mathbf{w}_{MMSE}(k) = \left( \mathbf{P}(k)\mathbf{P}^H(k) + \frac{2\sigma_n^2}{\sigma_b^2} \mathbf{I}_L \right)^{-1} \mathbf{p}_1(k)
\]
LMS Algorithm

LMS – Least Mean Squares

LMS is a stochastic gradient adaptive algorithm that converges towards the solution on each update

\[
\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \{ b_1(k) - y(k) \}^* \mathbf{x}(k)
\]

\( \mu \) – the adaption constant
**MSER Beamforming**

**MSER** – Minimum Symbol-Error-Rate

Minimise the probability of incorrectly decoding a signal

The probability of an error occurring at a particular time instant \( k \) is

\[
P_{EB}(w) = P\{\hat{b}_1(k) \neq b_1(k)\}
\]

The MSER problem is therefore defined as

\[
w_{MSER} = \arg \min_w P_{EB}(w)
\]

Its solution is the minimum of the SER cost function (right).

There is no closed form solution

An iterative conjugate-gradient algorithm can be used to find the solution
Adaptive Beamforming: LSER

**LSER – Least Symbol Error Rate**

LSER is a practically usable stochastic gradient adaptive algorithm that converges towards the solution on each update

\[ \mathbf{w}(k + 1) = \mathbf{w}(k) + \mu \left( -\nabla \tilde{P}_{EB}(\mathbf{w}(k), k) \right) \]

\( \mu \) – adaption constant

\( \rho_n \) – kernel width

\[ \nabla \tilde{P}_{EB}(\mathbf{w}(k), k) = \nabla \tilde{P}_{ER}(\mathbf{w}(k), k) + \nabla \tilde{P}_{EI}(\mathbf{w}(k), k) \]

\[ \nabla \tilde{P}_{ER}(\mathbf{w}(k), k) = \frac{\gamma}{2\sqrt{2\pi}\rho_n} \exp \left( -\frac{(y_R(k) - \hat{c}_{R_1}(k)(b_{R_1}(k) - 1))^2}{2\rho_n^2} \right) (\mathbf{x}(k) + (b_{R_1}(k) - 1)\hat{p}_1) \]

\[ \nabla \tilde{P}_{EI}(\mathbf{w}(k), k) = \frac{\gamma}{2\sqrt{2\pi}\rho_n} \exp \left( -\frac{(y_I(k) - \hat{c}_{I_1}(k)(b_{I_1}(k) - 1))^2}{2\rho_n^2} \right) (j\mathbf{x}(k) + (b_{I_1}(k) - 1)\hat{p}_1) \]
Simulation Study

- 3-element antenna array
- 4 users with equal power
- M-QAM modulation

- Time-varying channel (correlated Rayleigh fading over 250 fades)
- Frame structure

![Diagram showing antenna array and channel conditions with interferers and source angles.]
Results: SER Performance

- Min. angular separation: $\theta = 27^\circ$
- Normalised Doppler frequency $f_D = 10^{-4}$ and $10^{-3}$
- Modulation: 64-QAM
- LMS: $\mu = 0.0002$
- LSER: $\mu = 0.00005$, $\rho_n = 4\sigma_n$
Results: Parameter Tuning

- Min. angular separation: $\theta = 44^\circ$
- Normalised Doppler frequency $f_D = 10^{-4}$
- Modulation: 64-QAM
- LMS: $\mu = 0.0002$; LSER: $\mu = 0.00005$, $\rho_n = 4\sigma_n$
Results: Averaged SER Performance

- Min. angular separation: $\theta$ averaged over $[20^\circ, 50^\circ]$
- Normalised Doppler frequency $f_D = 10^{-3}$
- Modulation: 64-QAM
- LMS: $\mu = 0.0002$
- LSER: $\mu = 0.00005$, $\rho_n = 4\sigma_n$
Conclusions

• LSER is an adaptive implementation of the MSER beamforming solution

• LSER algorithm can operate successfully in
  – Fast fading conditions
  – With bandwidth-efficient M-QAM modulation
  – An SDMA environment with more users than antenna elements

• LSER algorithm consistently outperforms the adaptive LMS algorithm bench marker

• Benefits
  – Higher network capacity
  – Higher data rates
  – Longer range
  – Lower transmit power