Fading Performance Evaluation of an Adaptive MSER Beamforming Receiver for QAM Systems

Andrew Livingstone

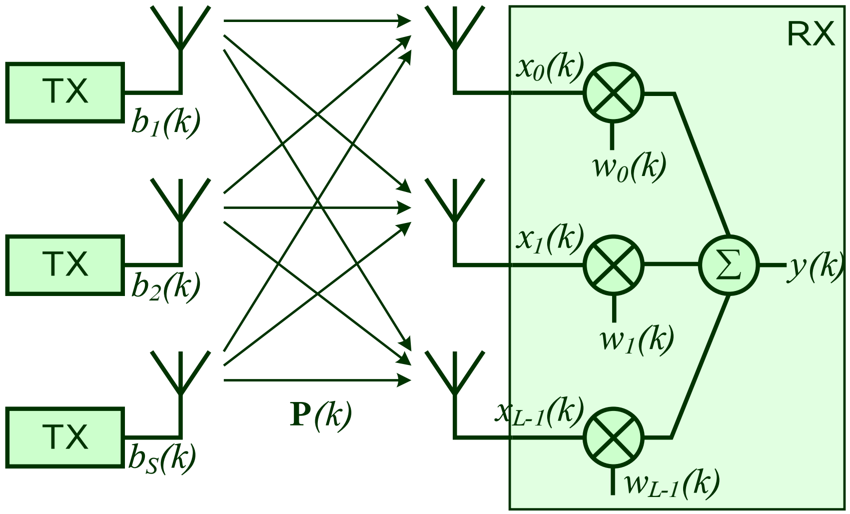
Detica

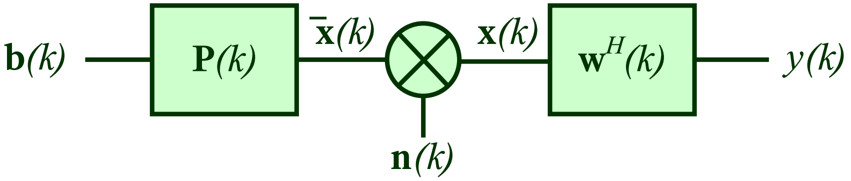
Sheng Chen, Lajos Hanzo

University of Southampton

Motivation

System Model





L – number of antenna array elements

S – number of users

$\mathbf{b}(k)$ – symbols transmitted by each user at time instant k

$\mathbf{P}(k)$ – describes the propagation channel and physical arrangement of antenna elements

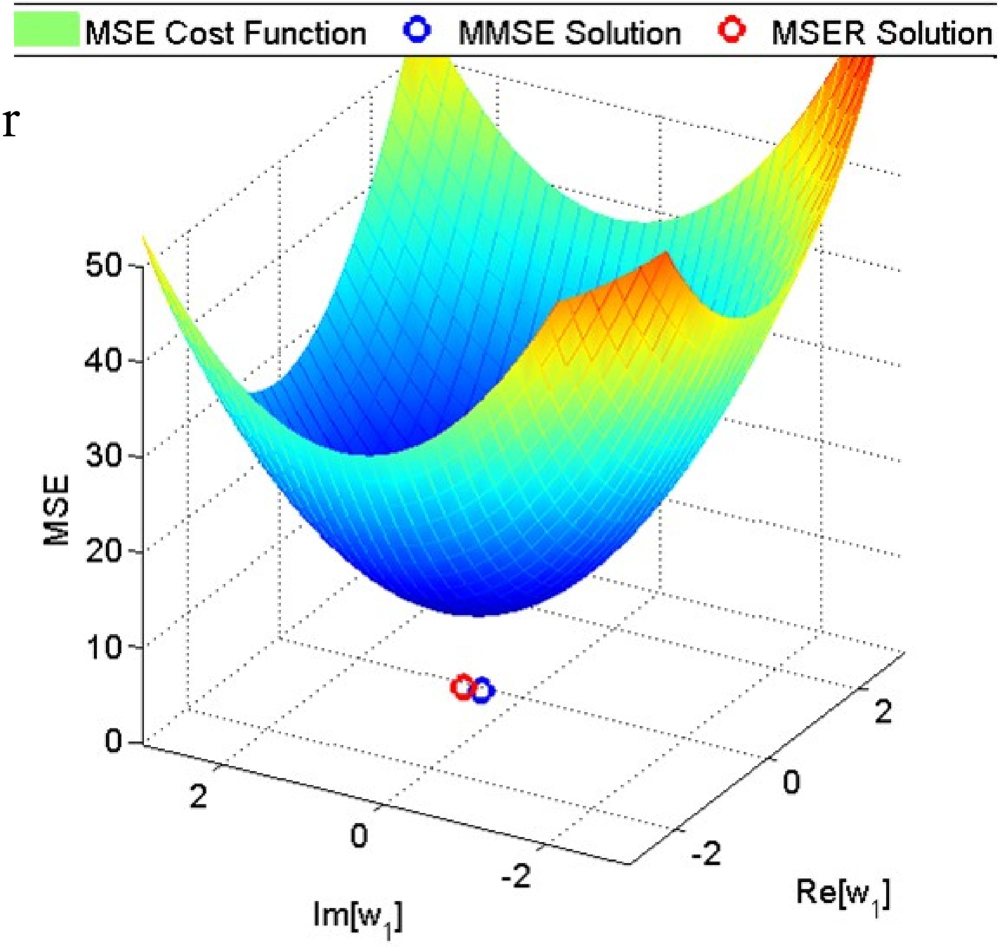
$\mathbf{n}(k)$ – additive white Gaussian noise

$\mathbf{x}(k)$ – vector of received signal samples

$\mathbf{w}(k)$ – beamformer weights

$y(k)$ – beamformer soft output

MMSE Beamforming



MMSE – Minimum Mean-Squared-Error

Minimise the error between estimated and transmitted signal waveforms

If *ε(k)* is the instantaneous error, then

$$\text{MSE } \xi = \text{E}\left[|\epsilon(k)|^2\right]$$

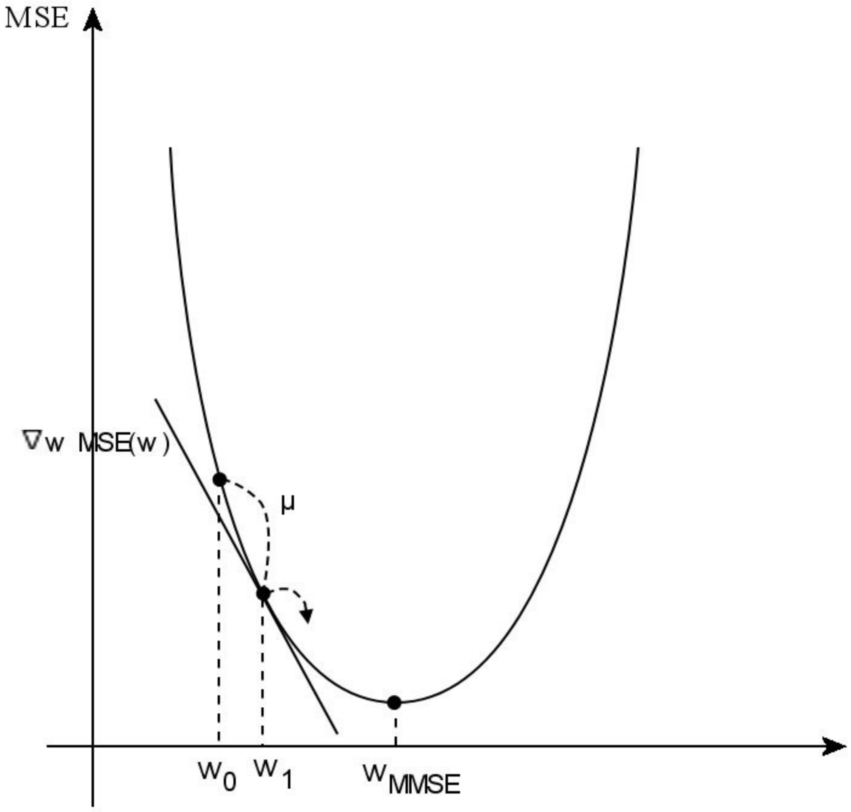
The MMSE problem can be defined

$$\mathbf{w}_{MMSE} = \arg\min_{\mathbf{w}} \xi(\mathbf{w})$$

Its solution is the minimum of the MSE cost function (right), the Wiener solution

$$\mathbf{w}_{MMSE}(k) = \left(\mathbf{P}(k)\mathbf{P}^H(k) + \frac{2\sigma_n^2}{\sigma_b^2}\mathbf{I}_L\right)^{-1}\mathbf{p}_1(k)$$

LMS Algorithm



LMS – Least Mean Squares

LMS is a stochastic gradient adaptive algorithm that converges towards the solution on each update

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\{b_1(k) - y(k)\}^*\mathbf{x}(k)$$

μ – the adaption constant

MSER Beamforming

MSER – Minimum Symbol-Error-Rate

Minimise the probability of incorrectly decoding a signal

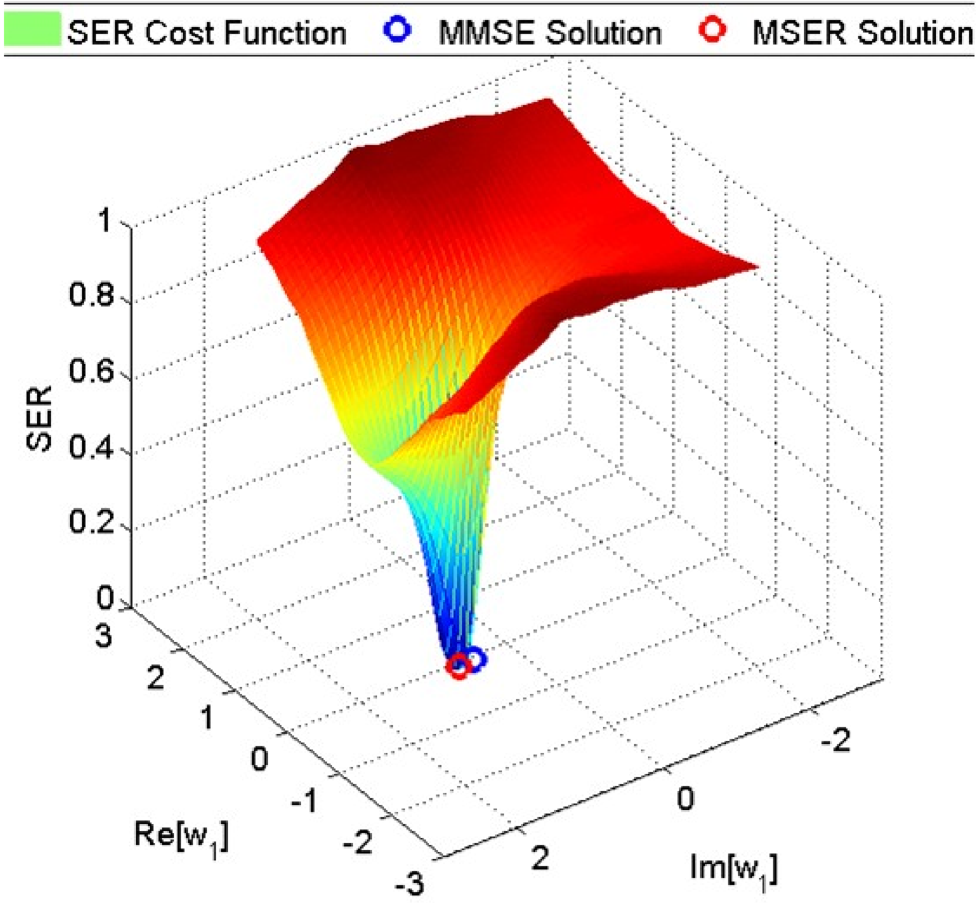
The probability of an error occurring at a particular time instant *k is*

$$P_{E_B}(\mathbf{w}) \quad = \quad P\{\hat{b}_1(k) \neq b_1(k)\}$$

The MSER problem is therefore defined as

$$\mathbf{w}_{MSER} = \arg\min_{\mathbf{w}} P_{E_B}(\mathbf{w})$$

Its solution is the minimum of the SER cost function (right).



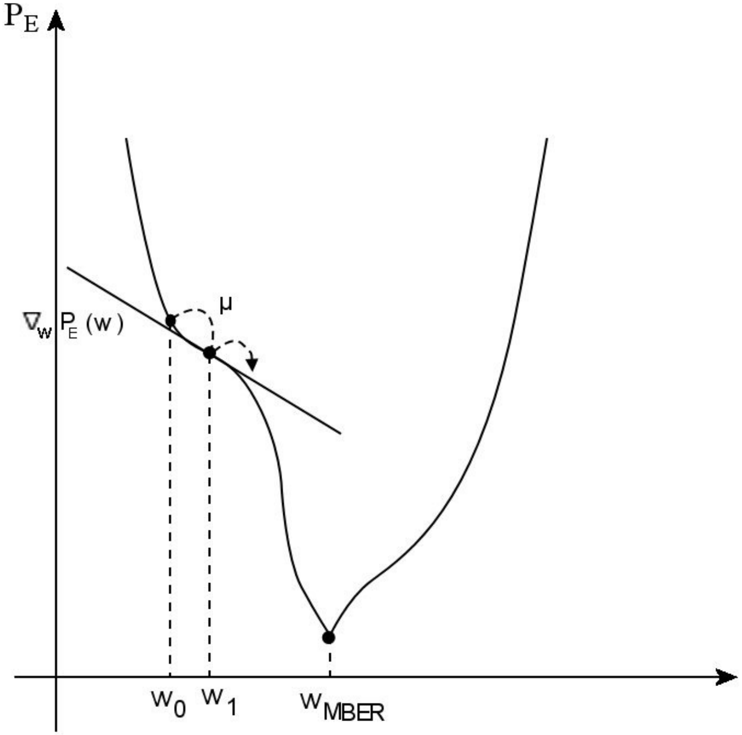
There is no closed form solution

An iterative conjugate-gradient algorithm can be use to find the solution

Adaptive Beamforming: LSER

LSER – Least Symbol Error Rate

LSER is a practically usable stochastic gradient adaptive algorithm that converges towards the solution on each update



$$\mathbf{w}(k+1) \quad = \quad \mathbf{w}(k) + \mu\left(-\nabla \tilde{P}_{E_B}(\mathbf{w}(k),k)\right)$$

μ – adaption constant

ρ_n – kernel width

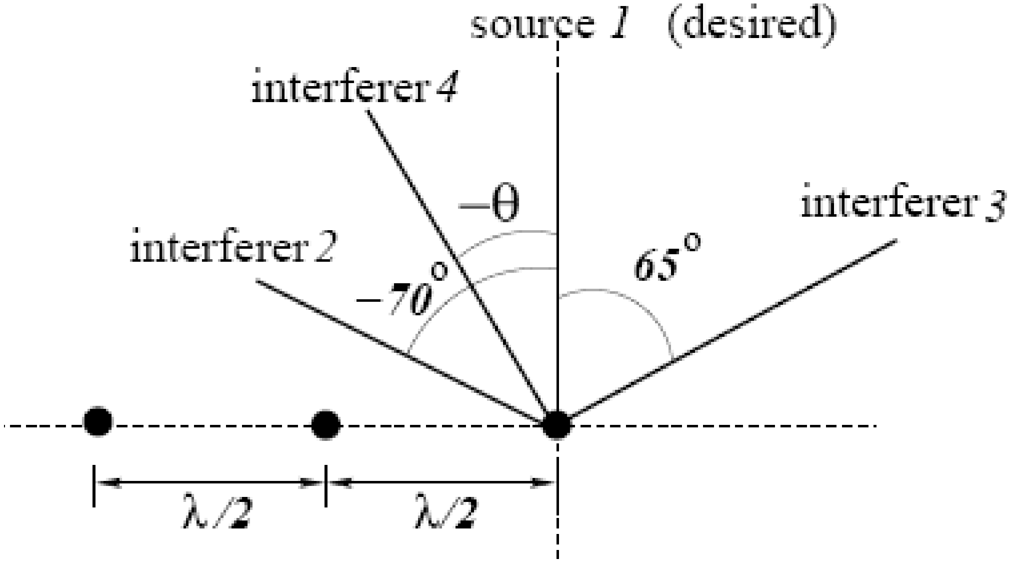
$$\nabla \tilde{P}_{E_B}(\mathbf{w}(k),k) = \nabla \tilde{P}_{E_R}(\mathbf{w}(k),k) + \nabla \tilde{P}_{E_I}(\mathbf{w}(k),k)$$

$$\nabla \tilde{P}_{E_R}(\mathbf{w}(k),k) \quad = \quad \frac{\gamma}{2\sqrt{2\pi}\rho_n}\exp\left(-\frac{(y_R(k)-\hat{c}_{R_1}(k)(b_{R_1}(k)-1))^2}{2\rho_n^2}\right)(-\mathbf{x}(k)+(b_{R_1}(k)-1)\hat{\mathbf{p}}_1$$

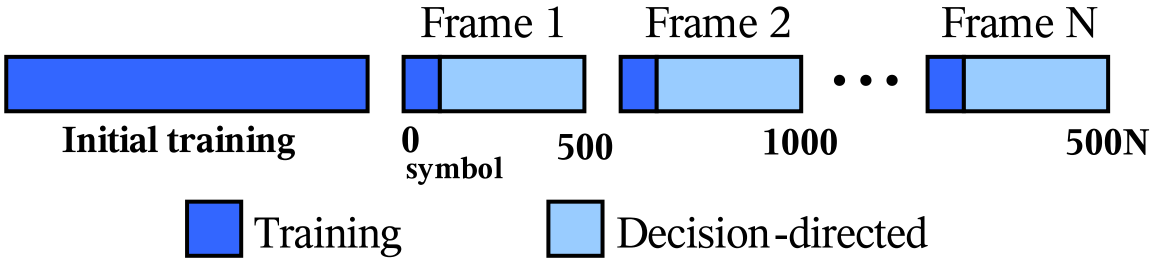
$$\nabla \tilde{P}_{E_I}(\mathbf{w}(k),k) \quad = \quad \frac{\gamma}{2\sqrt{2\pi}\rho_n}\exp\left(-\frac{(y_I(k)-\hat{c}_{I_1}(k)(b_{I_1}(k)-1))^2}{2\rho_n^2}\right)(j\mathbf{x}(k)+(b_{I_1}(k)-1)\hat{\mathbf{p}}_1)$$

Simulation Study

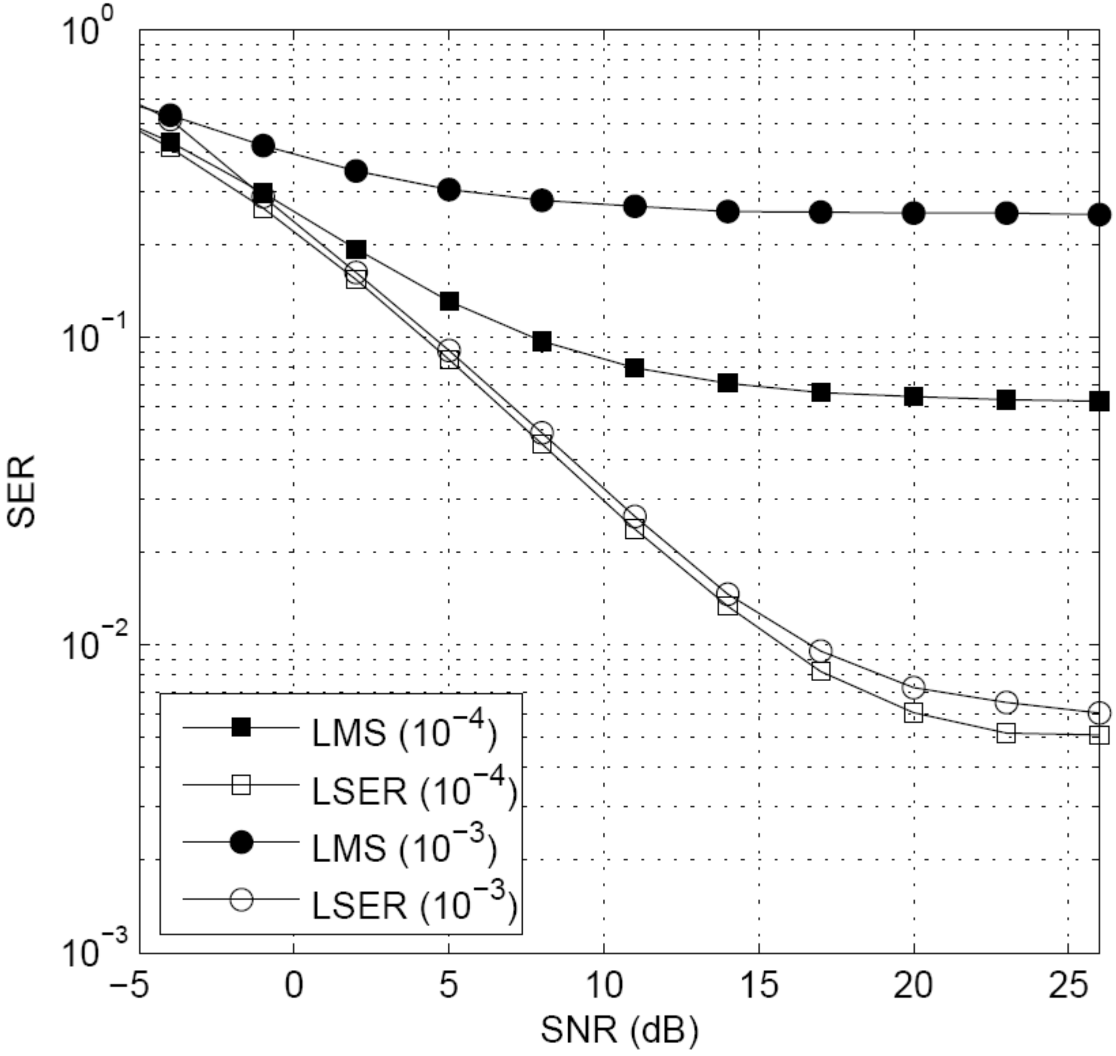
- 3-element antenna array
- 4 users with equal power
- M-QAM modulation



- Time-varying channel (correlated Rayleigh fading over 250 fades)
- Frame structure



Results: SER Performance

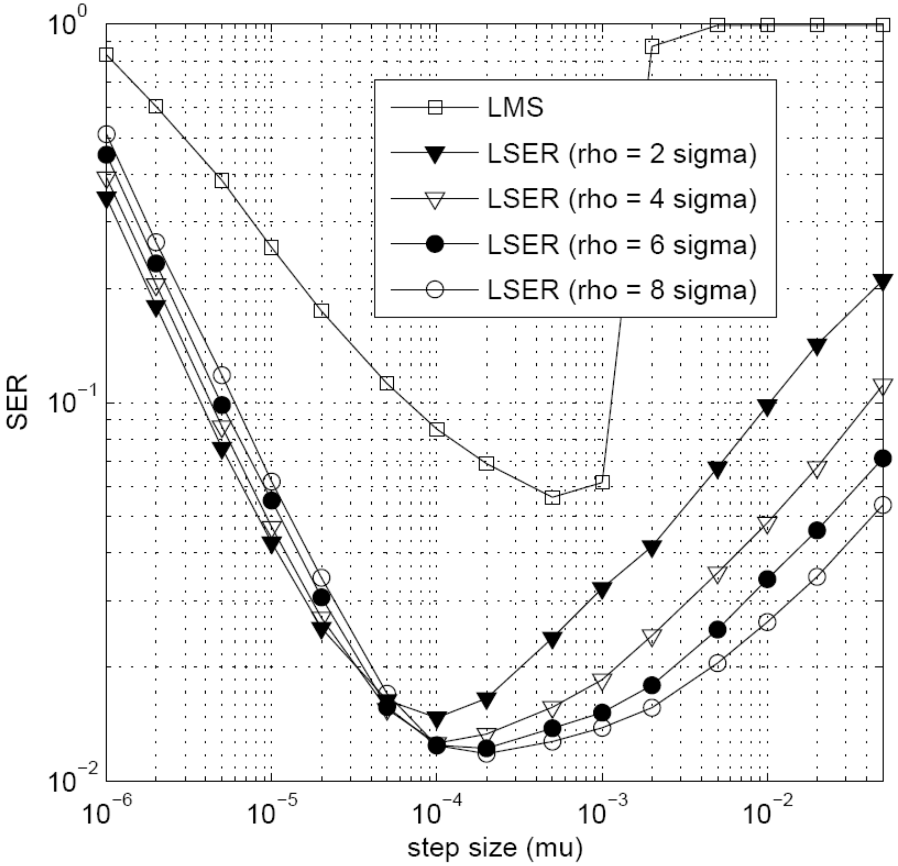


- Min. angular separation: $\theta = 27^{\circ}$
- Normalised Doppler frequency

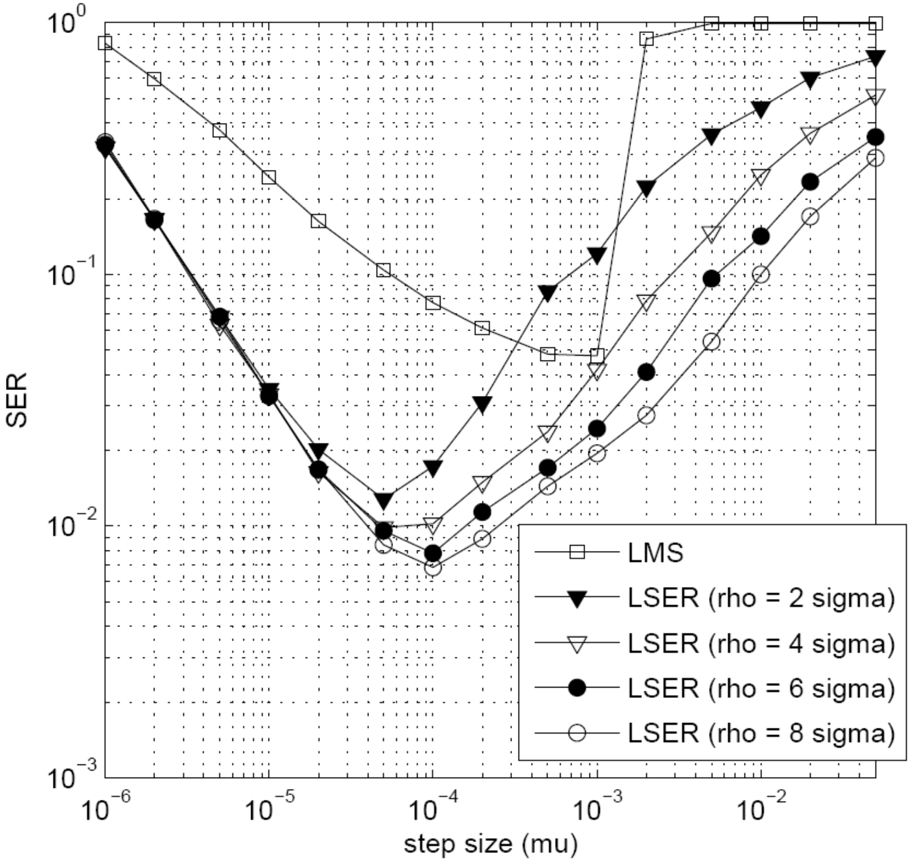
  $f_D = 10^{-4}$ and 10^{-3}
- Modulation: 64-QAM
- LMS: $\mu = 0.0002$
- LSER: $\mu = 0.00005$, $\rho_n = 4\sigma_n$

Results: Parameter Tuning

SNR 15dB

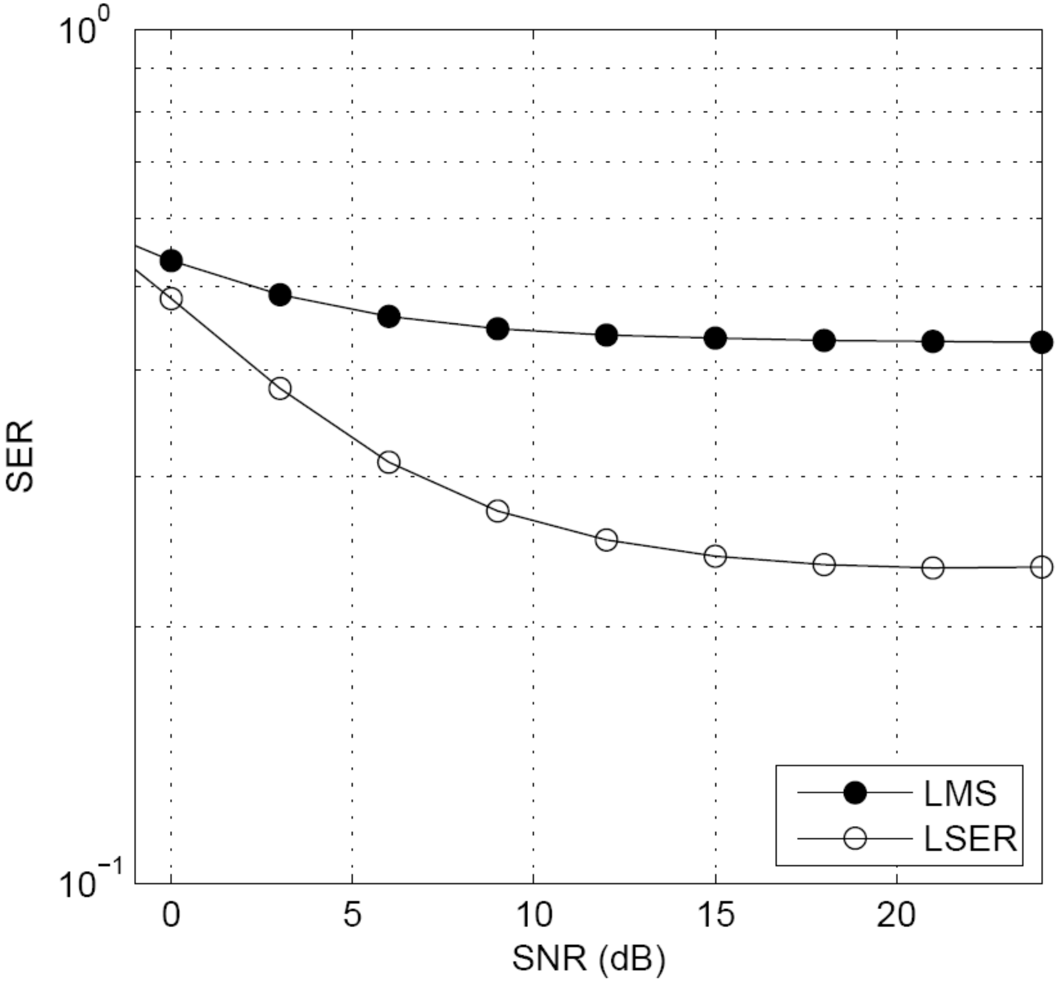


SNR 30dB



- Min. angular separation: $\theta = 44^{\circ}$
- Normalised Doppler frequency $f_D = 10^{-4}$
- Modulation: 64-QAM
- LMS: $\mu = 0.0002$; LSER: $\mu = 0.00005$, $\rho_n = 4\sigma_n$

Results: Averaged SER Performance



- Min. angular separation: θ averaged over [20º, 50º]
- Normalised Doppler frequency

  $f_D = 10^{-3}$
- Modulation: 64-QAM
- LMS: $\mu = 0.0002$
- LSER: $\mu = 0.00005$, $\rho_n = 4\sigma_n$

Conclusions

- LSER is an adaptive implementation of the MSER beamforming solution

- LSER algorithm can operate successfully in
  - Fast fading conditions
  - With bandwidth-efficient M-QAM modulation
  - An SDMA environment with more users than antenna elements

- LSER algorithm consistently outperforms the adaptive LMS algorithm bench marker

- Benefits
  - Higher network capacity
  - Higher data rates
  - Longer range
  - Lower transmit power