A Sensitivity Approach to Force Calculation in Electrostatic MEMS Devices

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Abstract — This paper presents a novel method for the computation of force in an electrostatically operated MEMS device. The approach is based on Continuum Design Senstivity Analysis (CDSA) and can be used with any analysis system. The method, unlike that of Maxwell Stresses, does not require an airgap surrounding the body. The method is applied to the calculation of the tilt angle of a MEMS micro-mirror.

I. INTRODUCTION

Electrostatically operated MEMS devices are being used in many applications from video projectors to sensors in automobiles to variable RF tuning capacitors. In all these applications the mechanical movement is a key part of the function of the device. In a micro-mirror, the mirror is attached to a beam which bends under the influence of an electric field which results from the application of a voltage between the beam and an electrode. The bending of the beam results in a deflection of a ray of light being reflected by the mirror. Thus such a structure can be used to optically switch signals whether they are part of an optical communications system or a video projector. The accurate computation of the forces involved is crucial to the analysis of these devices. Because of the manner in which MEMS devices are constructed and, in particular, the structure and gimballing of the mirror, ideally, the force algorithm used should be able to compute the forces on one body in physical contact with another body.

In recent papers [1], [2], a new approach to force calculation for magnetic fields based on Continuum Design Sensitivity Analysis (CDSA) has been described. In a sense this is a development of the standard virtual work approach developed in [3] but with the difference that the analysis method used to determine the fields is irrelevant. This approach also has the interesting property that, unlike the Maxwell Stress method [4], it is not necessary for there to be an airgap surrounding the body

The intention of this paper is to derive an electrostatic force equation based on CDSA for use with MEMS devices. The approach is then applied to a micro-mirror system.

II. CDSA BASED FORCE CALCULATION

In order to derive the energy sensitivity formula, first, an objective function W is mathematically expressed as

$$W = \int_\Omega g(\varphi(\mathbf{p}))\, d\Omega \tag{1}$$

where g represents an energy function of the electrostatic system, differentiable with respect to the electric scalar potential, φ, that is an implicit function of the design variable vector **p.** To deduce the sensitivity formula and the adjoint system equation systematically, the variational form of Poisson's equation, referred to as the primary system, is added to (1) using the augmented Lagrangian method

$$\begin{aligned}\overline{W} &= \int_\Omega g(\varphi)\, d\Omega \\ &+ \int_\Omega \lambda \{-\varepsilon \nabla\varphi \cdot \nabla\lambda + \rho\}\, d\Omega\end{aligned} \tag{2}$$

where λ is the Lagrange multiplier and interpreted, in this case, as the adjoint variable.

To obtain an explicit expression for the deformation of the interface boundary between different materials, Ω_1 and Ω_2, and accordingly for the variation of the permittivity distribution inside the materials, the second integral on the right-hand side of (2) is split into the two regions. Then the material derivative on both sides of (2) is taken as

$$\begin{aligned}\dot{\overline{W}} &= \int_{\Omega_1} g_\varphi \cdot \overline{\lambda}_1\, d\Omega \\ &- \int_{\Omega_1} \varepsilon_1 \{\nabla\varphi_1 \cdot \nabla\overline{\lambda}_1 + \nabla\lambda_1 \cdot \nabla\overline{\varphi}_1\}\, d\Omega \\ &- \int_{\Omega_2} \varepsilon_2 \{\nabla\varphi_2 \cdot \nabla\overline{\lambda}_2 + \nabla\lambda_2 \cdot \nabla\overline{\varphi}_2\}\, d\Omega \\ &+ \int_{\Omega_1} \{\rho_1\overline{\lambda}_1\}\, d\Omega + + \int_{\Omega_2} \{\rho_2\overline{\lambda}_2\}\, d\Omega \\ &+ \int_\gamma \{\varepsilon_1 \nabla\varphi_1 \cdot \nabla\lambda_1 - \varepsilon_2 \nabla\varphi_2 \cdot \nabla\lambda_2\} V_n d\gamma \\ &+ \int_\gamma \{\rho_2\lambda_2 - \rho_1\lambda_1\} V_n d\gamma\end{aligned} \tag{3}$$

where $g_\varphi \equiv \partial g/\partial\varphi$, $\overline{\varphi} \equiv (\dot{\lambda} - \nabla\lambda \cdot \mathbf{V})$, $\overline{\lambda} \equiv (\dot{\varphi} - \nabla\varphi \cdot \mathbf{V})$ and **V** denotes a design velocity vector, i.e. the direction of boundary movement, and γ denotes the part of the interface boundary that is allowed to move.

Finally, the energy sensitivity formula applicable to electrostatic problems is given by

$$\begin{aligned}\frac{d\overline{W}}{d\mathbf{p}} &= \int_\gamma \{(\varepsilon_1 - \varepsilon_2)[\frac{\varepsilon_1}{\varepsilon_2}(\nabla\varphi_1)_n(\nabla\lambda_1)_n + (\nabla\varphi_1)_t(\nabla\lambda_1)_t]\} V_n d\gamma \\ &+ \int_\gamma \{(\rho_2 - \rho_1)\lambda_1\} V_n d\gamma\end{aligned} \tag{4}$$

where the surface integral represents the variation of the stored total electric energy experienced over the interface. When dealing with the objective functions related to the system energy, the dual system consisting of the primary and the adjoint systems is self-adjoint. In other words, the variational of the adjoint system is the same as that of the primary system. Thus $\varphi = \lambda$ and there is no need to solve the adjoint problem.

From the energy sensitivity (4), the mechanical force **F** acting on the interface γ between two different electric materials of ε_1 and ε_2, can be written as

$$\mathbf{F} = \int_\gamma \mathbf{f}_s \, d\gamma = \int_\gamma \{\frac{1}{2}(\varepsilon_1 - \varepsilon_2)[\frac{\varepsilon_1}{\varepsilon_2}(\nabla\varphi_1)_n(\nabla\lambda_1)_n + (\nabla\varphi_1)_t(\nabla\lambda_1)_t]\}\mathbf{n}d\gamma \quad (5)$$

where $\mathbf{f}_s$ is surface force distribution, and the direction of each surface force is decided by an arbitrary design velocity **V**. In this case, **V** is set to the same direction as a unit normal vector **n** outward to γ.

The force acting on the volume charge density is given as

$$\mathbf{F}_\rho = \int_\gamma \mathbf{f}_s \, d\gamma = \int_\gamma \{(\rho_2 - \rho_1)\lambda_1\}\mathbf{n}d\gamma \quad (6)$$

III. A MICROMIRROR EXAMPLE

Fig. 1 shows a plan view (from below) of a simple micro-mirror structure constructed using the MUMPS process [5] (the mirrors are above the electrodes). All the electrodes and the mirror structures are constructed from polysilicon with a relative permittivity of 11. The central mirror is connected by a set of torsion springs to an outer gimbal which in turn is connected to a fixed frame (all shown in light grey in the figure).

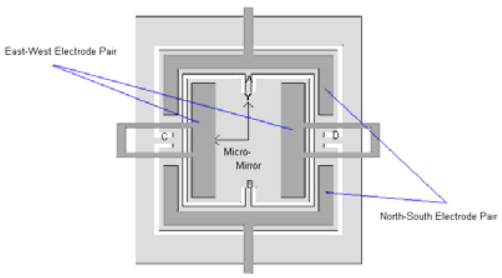

Fig.1 Basic structure of Micromirror.

The micromirror itself is a square of side 128µm; the first frame around it has an outer dimension of 188µm. The thickness of the mirror layer is 2µm and the airgap between the electrodes and the mirror is 2µm. Beams A and B act as torsion springs for E-W rotation, while C and D provide N-S rotation.

The system is energized by two pairs of electrodes under the mirror structure (shown in dark grey in the figure)

While the model is truly three-dimensional, the first analysis has been performed on a two-dimensional cross-section of the model with the EW (horizontal) electrode pair excited. The reaction torque due to the bending of the suspension (torsion springs) was computed from a simple analytical model and the stable position of at each applied voltage was computed. The CDSA calculation was implemented using results generated by the ElecNet [6] finite element based code although the force calculation system is independent of the analysis method used.

IV. RESULTS

Measurements were made on the micro-mirror in terms of the tilt angle of the mirror versus the applied voltage. Fig. 2 shows the computed tilt angles based on the CDSA force calculation compared with the measured angles.

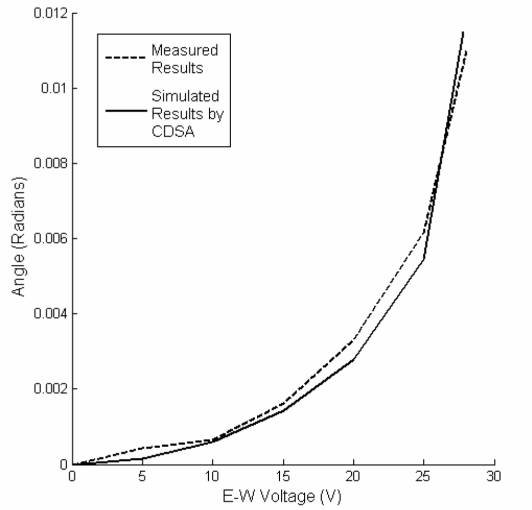

Fig. 2. Tilt angle on N-S axis by exciting the E-W electrodes. Measured vs computed.

V. CONCLUSIONS

The paper has described an electrostatic force calculation system which can be applied to a body which is in physical contact with another structure. The effectiveness of the method has been shown in a two-dimensional analysis of a micro-mirror system. The full paper will provide results of a three-dimensional analysis with two axis rotations of the mirror. This is the first time CDSA has been applied to force calculation in an electrostatic system.

VI. REFERENCES

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