Network Representation of Conducting Regions in 3D Finite Element Description of Electrical Machines

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Abstract — The paper introduces a network description of conducting regions in electrical machines. Resistance models are considered, where loop equations are equivalent to an edge element formulation (EEM) using electric vector potential *T*, as well as conductance models, for which the nodal equations refer to a nodal element description (NEM) by means of scalar potential *V*. Network models for multiply connected regions are derived for both Ω-*T*-T_0 and *A*-*T*-T_0 formulations. A network representation of the edge value of potential T_0 is suggested.

I. Introduction

Design and analysis of electrical machines increasingly exploit numerical field simulations. By far the most popular is the finite element method (FEM), although equivalent magnetic and electric circuits continue to be useful as they provide good physical insight and aid understanding of complicated electromagnetic phenomena. It was shown previously [1]–[3] that FEM formulations may be considered as analogous to loop or nodal descriptions of equivalent electric or magnetic circuits (networks). Thus models established using FEM approach may be treated as network models. The number of branches in such models equals the number of edges or facets of the discretising mesh. This paper builds on previous publications and extends the treatment by focussing on network description of regions with conduction currents. The aim is to facilitate the connection between field equations due to such currents and the equations of the supplying circuitry. Coupling between magnetic and electric networks is also considered for models of electric windings.

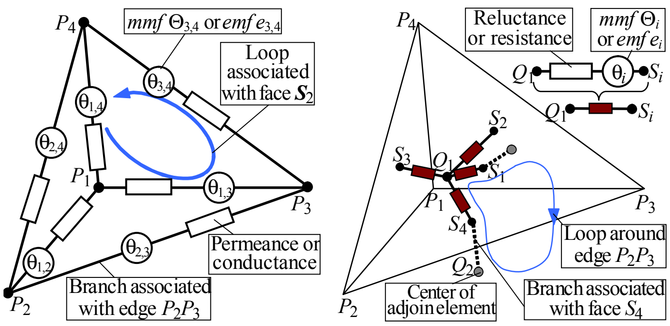
II. Network representation of FE models

It was shown in [1] that finite element formulations using potentials may be seen as equivalent to network models of either edge elements (EN), with branches coinciding with element edges, or facet elements (FN), where the branches connecting the nodes are associated with the facets, while the nodes are positioned in the middle of the volumes. Fig. 1 depicts the edge and facet models of a tetrahedron. The nodal equations of EN are equivalent to the nodal element description (NEM) of the scalar potential formulation, while the loop equations of FN correspond to the edge element formulation (EEM) using vector potential. The edge values of the vector potentials *A* and *T* represent the loop fluxes and currents in loops around the edges, respectively [1]. In regions with conduction currents, a conductance network (CN) may be created from an electric edge model, whereas a resistance network (RN) stems from an electric facet model. The CN conductances may be established from the interpolating functions of the edge element, while resistances of the RN from those of the facet element. In the networks arising from the FE method, coupling between the branches may occur, i.e. mutual conducatnces and resistances may be present, which distinguishes such networks from classical circuits. The voltage across a conductance of the *i*th branch may force a current in the *j*th branch of the CN; similarly, a current in the *i*th branch of the RN may create a voltage in the *j*th branch.

III. Electromotive and magnetomotive forces

The task of describing conductors in the FE domain necessitates defining the *mmf*s set up by currents in conducting regions and *emf*s due to changing magnetic flux. The edge elements (EN) are analysed using the nodal method, thus branch sources need to be introduced. On the other hand, a loop method is applied to evaluate the facet elements (FN), hence either branch or loop sources may be used.

In the models under consideration, the branch *mmf*s and *emf*s are established from loop currents and fluxes. In the case of EN, currents and fluxes ‘around’ the edge are relevant, whereas for the FN case currents and fluxes of the loops associated with facets need to be used (see Fig. 1). Loop *mmf*s in EN represent facet values of ***J***, whereas loop *emf*s arise from time derivatives of facet values of ***B***. The loop sources of FN, on the other hand, may be defined using branch currents and fluxes corresponding to element edges. Table I collects expressions for branch and loop currents of electric networks. Fluxes in magnetic networks may be defined in a similar way by substituting ***J*** with ***B***.



(a) (b)

Fig. 1. Edge (a) and facet (b) model of a tetrahedron

TABLE I. Branch and loop currents associated with an element

Branch currents		Loop currents	
Branch associated with edge P_iP_j	Branch associated with face S_i	Loop around edge P_iP_j	Loop associated with face S_i
$i_{bN_{i,j}}=\iiint_{V_e} \boldsymbol{w}_{eN_{i,j}} \boldsymbol{J}\,\mathrm{d}v$	$i_{si}=\iint_{S_i} \boldsymbol{J}\,\mathrm{d}\boldsymbol{s}$	$i_{RN_{i,j}}=\int_{P_i}^{P_j} \boldsymbol{T}\,\mathrm{d}\boldsymbol{l}$	$i_{Ci}=\iiint_{V_e} \boldsymbol{w}_{fi} \boldsymbol{T}\,\mathrm{d}v$
Comment: $\boldsymbol{w}_{eNi,j}$ is the interpolation function of edge element for edge P_iP_j, V_e is the element volume, $\boldsymbol{w}_{fi}$ is the interpolation function of facet element for face S_i			

IV. Simply connected conducting regions

In field analysis of simply connected conducting regions, e.g. solid parts of a core with no ‘holes’, it is possible to use the ***A***–*V* combination of potentials, as well as Ω–***T*** or ***A***–***T***. The FEM formulation using ***A***–*V* is equivalent to equations of the magnetic FN and electric EN [3]. The less popular application of Ω-***T*** and ***A***-***T*** formulations in the FEM description leads to analogies with representative resistance networks coupled with magnetic EN or magnetic FN, respectively.

V. Multiply Connected Conductors – Windings

In analysis of windings of electrical machines two cases are considered: (a) thin (filament) conductors of cross section less than the facet area of the elements, (b) solid conductors, such as in a cage rotor of an induction motor, whose cross section is larger than the element facet area. In a special case the two areas may actually coincide (Fig. 2) and it is convenient to use this case to explain the differences between the resistance model (***T*** potential) and the conductance model (*V* potential). The FEM equations for the classical ***T*** formulation refer to loops around the element edges. From Fig. 2a it is transparent that all loops around the edge are 'open' and the classical ***T*** solution gives incorrect zero result [4]. It is thus necessary to introduce an additional equation describing the loop current i_c flowing around the 'hole' – Fig. 2a. This current is a circuit representation of the edge value of $\boldsymbol{T}_0$ introduced in [5], [6]. The equation for i_c uniquely describes the current flow in Fig. 2a. A more accurate model of a single winding turn may be obtained by using a conductance network. However, a large number of nodal equations will result, even if skin effect is neglected and a condition is imposed that nodes on the surface of the conductor cross section are shorted, e.g. nodes P_i, P_j, P_k. It may therefore be concluded that for systems with thin conductors the use of potential $\boldsymbol{T}_0$ is to be recommended.

A mixed approach linking the above description with equations based on a scalar potential Ω (equations of magnetic EN) and a vector potential ***A*** (equations of magnetic FN) is now considered, whereby classical formulations involving potential ***T*** may be added, i.e. equations for loops containing eddy currents (the methods Ω-$\boldsymbol{T}$-$\boldsymbol{T}_0$ and $\boldsymbol{A}$-$\boldsymbol{T}$-$\boldsymbol{T}_0$). It has been assumed that the 'loop' *L* shown if Figs. 3 and 4 is a part of a winding consisting of filament conductors or represents a loop around the hole of a solid conductor, e.g. a loop of Fig. 2.

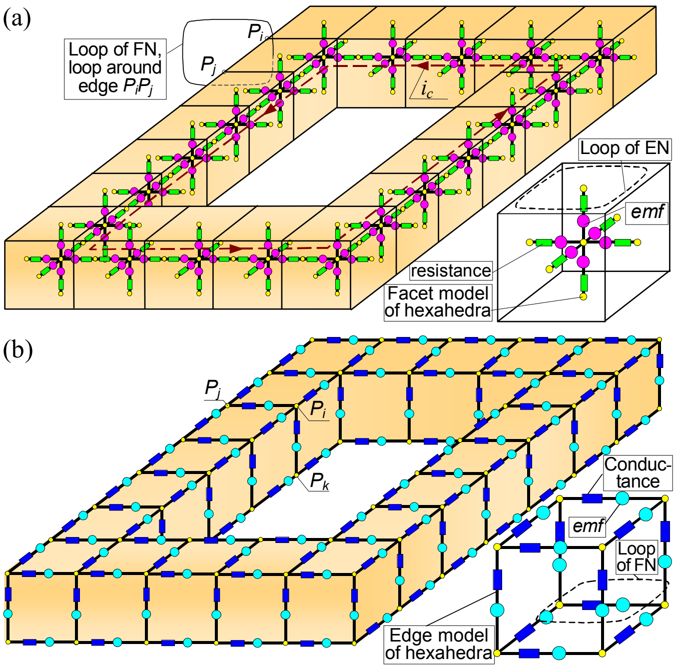


Fig. 2. Models of a turn divided into hexahedrons: resistance (a), conductance (b)

It follows that to define *mmf*s in EN it is necessary to form loops around edges. The *L* loop must therefore be replaced by these loops, as in Fig. 3 (loops with current i_{Rk}). A matrix $\boldsymbol{z}_e$ is being formed transposing a current in *L* into a vector of currents i_{Rk} representing branch *mmf*s in EN. Multiplying the transposed matrix $\boldsymbol{z}_e$ by the vector of fluxes associated with edges yields the flux linkage with *L* as shown in Fig. 3.

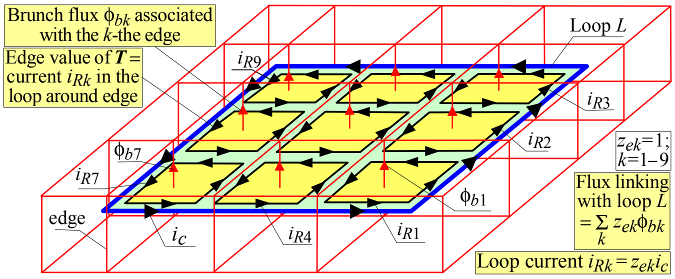


Fig.3. Loop with current in space of edge magnetic network

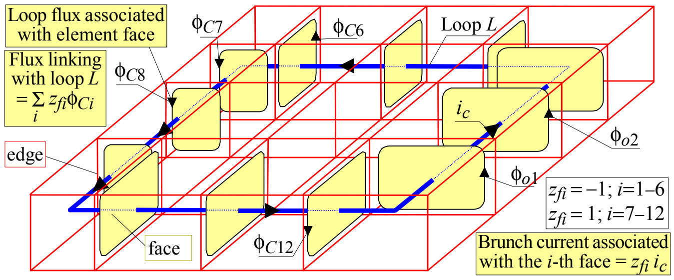


Fig.4. A loop with current in space of a facet magnetic network

Since currents i_{Rk} are edge values of potential $\boldsymbol{T}_0$, by using the interpolation functions of the edge element the values of $\boldsymbol{T}_0$ may be established and – via the relationships of Table I – the currents i_{Ci}, which represent branch *mmf*s in the magnetic FN. When solving the loop equations of FN it is sufficient to define loop *mmf*s, i.e. currents $i_{bN_{i,j}}$, in branches associated with edges. The currents $i_{bN_{i,j}}$ may be found from a relationship in Table I by using the interpolation functions and after first finding the current ***J*** on the basis of edge values, i.e. by using currents i_{si} in branches associated with facets. The matrix of currents i_{si} may be written as a product of the current in *L* and the matrix $\boldsymbol{z}_f$ of cuts of *L* with the edges (Fig. 4). The product of the transposed matrix $\boldsymbol{z}_f$ and the vector of fluxes ϕ_{Ci} through loops associated with a facet FN yields the flux linking with *L* (Fig. 4). The fluxes ϕ_{Ci} are found from loop fluxes FN by applying a similar procedure as used when defining currents i_{Ci} from prescribed currents i_{Rk}.

V. Conclusion

A network description of FEM equations has been derived for systems containing conducting regions. An interpretation of the edge value of potential $\boldsymbol{T}_0$ has been put forward where this potential is related to a current in a loop. Ways of describing a loop with a current in the FE domain have been proposed using both potentials Ω and ***A***. The method is applicable to multiply connected regions, including cage rotors and windings connected to external circuits and sources.

References

[1] A. Demenko and J. K. Sykulski, "Network equivalents of nodal and edge elements in electromagnetics," *IEEE Trans. on Magnetics*, 38 (2), pp. 1305-08, 2002.

[2] A. Demenko, J. K. Sykulski, "Magneto-electric network models in electromagnetism", *COMPEL*, 25 (3), pp.581-588, 2006.

[3] A. Demenko, "Three dimensional eddy current calculation using reluctance-conductance network formed by means of FE method," *IEEE Trans. on Magnetics,* 36 (4), pp. 741-45, 2000.

[4] O. Biro et al, "Performance of different vector potential formulations in solving multiply connected 3D eddy current problems." *IEEE Trans. on Magnetics*, 26, (2), pp.438-41, 1990.

[5] S. Bouissou and F. Piriou, "Numerical simulation of a power transformer using finite element method coupled to circuit equation," *IEEE Trans. on Magnetics*, 30 (5), pp. 3224-27, 1994.

[6] V. P. Bui, Y. Le Floch, G. Meunier and J. L. Coulomb, "A new three-dimensional (3-D) scalar finite element method to compute T_0," *IEEE Trans. on Magnetics*, Vol. 42, No.4, pp. 1035-1038, 2006.