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# Robust $\mathscr{H}_\infty$ filtering for uncertain differential linear repetitive processes

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SUMMARY

The unique characteristic of a repetitive process is a series of sweeps or passes through a set of dynamics defined over a finite duration known as the pass length. At the end of each pass, the process is reset and the next time through the output, or pass profile, produced on the previous pass acts as a forcing function on, and hence contributes to, the dynamics of the new pass profile. They are hence a class of systems where a variable must be expressed in terms of two directions of information propagation (from pass-to-pass and along a pass, respectively) where the dynamics over the finite pass length are described by a matrix linear differential equation and from pass to pass by a discrete updating structure. This means that filtering/estimation theory/algorithms for, in particular, 2D discrete linear systems is not applicable. In this paper, we solve a general robust filtering problem with a view towards use in many applications where such an action will be required. Copyright © 2007 John Wiley & Sons, Ltd.

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## 1. INTRODUCTION

The unique characteristic of a repetitive (also termed a multipass process in the early literature) process can be illustrated by considering machining operations where the material or workpiece involved is processed by a sequence of sweeps, termed passes, of the processing tool. To introduce a formal definition, assume that the pass length $\alpha$ (i.e. the duration of a pass of the processing tool), which is finite by definition, has a constant value for each pass. Then in a repetitive process the output vector, or pass profile, $y_k(t),\ 0\leqslant t\leqslant\alpha$ ($t$ being the independent spatial or temporal variable), produced on pass $k$ acts as a forcing function on, and hence contributes to, the dynamics of the new pass profile $y_{k+1}(t),\ 0\leqslant t\leqslant\alpha,\ k\geqslant 0$. This, in turn, can lead to oscillations in the output pass profile sequence $\{y_k\}_k$ which increase in amplitude in the pass-to-pass (i.e. $k$) direction. Examples of such processes include long-wall cutting, metal rolling (see, for example, the references cited in [1]) and there are also many iterative solution algorithms, such as those for nonlinear optimal control/optimization algorithms based on the maximum principle [2], which operate in this manner.

Attempts to analyse these processes using standard (or 1D) systems theory/algorithms fail (except in a few very restrictive special cases) precisely because such an approach ignores their inherent 2D systems structure, i.e. information propagation occurs from pass to pass and along a given pass. Also the initial conditions are reset before the start of each new pass and the structure of these can be somewhat complex. For example, if they are an explicit function of points on the previous pass profile then this alone can destroy the most basic performance specification of stability. In seeking a rigorous foundation on which to develop a control/estimation/filtering theory for these processes, it is natural to attempt to exploit structural links which exist between these processes and other classes of 2D linear systems.

The case of 2D discrete linear systems recursive in the positive quadrant $(i, j): i\geqslant 0,\ j\geqslant 0$ (where $i$ and $j$ denote the directions of information propagation) has been the subject of much research effort over the years using, in the main, the well-known Roesser and Fornasini Marchesini state-space models. More recently, productive research has been reported on $\mathscr{H}_\infty$ and $\mathscr{H}_2$ approaches to filtering and control law design—see, for example, [3, 4]. (Filtering of this general form is, of course, well established in 1D linear systems theory, see, for example, [5, 6]).

As noted above repetitive processes operate over a finite pass length and this is an intrinsic property as opposed to an assumption made to simplify analysis. Also the initial or boundary conditions are reset before the start of each new pass and as noted above the structure of these is crucial in terms of stability and performance. Initial conditions which are a function of points along the previous pass have no 2D Roesser or Fornasini Marchesini model counterparts. Moreover, in this paper we consider the so-called differential linear repetitive processes where information propagation along the pass is governed by a matrix differential equation. Consequently, the existing systems theory for 2D discrete linear systems is not applicable.

The theory to date of linear repetitive processes has been based on the assumption that all signals involved are purely deterministic and hence not corrupted by measurement noise, etc. In a recent work, this has led to the development of algorithms for the design of physically based control laws which can be reliably computed using linear matrix inequalities (LMIs)—see, for example, [7]. In many cases, however, this will not be valid due, for example, to measurement noise, etc. arising from the conditions in which physical examples have to operate, e.g. in long-wall coal cutting and iterative learning control applications, such as chain conveyors used in various process control and other applications areas [8].

There is clearly a need to develop a filtering theory for differential processes which can be (eventually) used to enable the implementation of control laws and/or enable (as one of many possible uses) reliable estimates of key signals to be obtained from measured data. In this paper, the problem solved is the design of a full order filter which gives a stable filter error and has prescribed disturbance attenuation performance as measured by an $\mathscr{H}_\infty$ norm measure. The first major new results in this paper can be summarized as a solution of the filter existence problem expressed in terms of LMIs and hence a computational test. Secondly, it is shown that this solution generalizes to the case when there is uncertainty in the process state-space model and an illustrative numerical example is given.

In $H_\infty$ filtering no *a priori* knowledge of the noise statistics is required—instead, the noise signals are only assumed to have finite energy. Also the estimation criterion for filter design is to minimize the worst possible amplification of the estimation error signal in terms of the modelling errors and additive noise. This approach has found numerous applications in signal processing and control, e.g. [9–11].

It should also be noted that in 2D linear systems/repetitive processes the available results in terms of stability and, in particular, control law design which exploit necessary and sufficient conditions are only applicable to low order synthesis type problems. However, in many areas (such as those referred to above with references for the details) where control laws designed in a repetitive process setting could eventually be applied, the number of state variables alone preclude using such an approach to obtain a control law which can proceed to the next stage, i.e. performance evaluation. Moreover, it is not at all clear how this fundamental difficulty can be removed and hence the use of sufficient only conditions is the only feasible way forward.

Throughout this paper, the null matrix and the identity matrix with appropriate dimensions are denoted by 0 and $I$, respectively and also $\mathrm{sym}(X)$ is used to denote $X + X^{\mathrm{T}}$. Moreover, $M>0$ (respectively $\geqslant 0$) denotes a real symmetric positive definite (respectively semi-definite) matrix. Similarly, $M<0$ (respectively $\leqslant 0$) denotes a real symmetric negative-definite (respectively, semi-definite) matrix. We also use $*$ to denote symmetric block entries in some of the LMIs (which are required to be symmetric). Finally, we require the following signal space definition.

*Definition 1*

Consider a $q \times 1$ vector sequence $\{w_j(t)\}$ defined over the real interval $0 \leqslant t \leqslant \infty$ and the non-negative integers $0 \leqslant j \leqslant \infty$, which is written as $\{[0, \infty], [0, \infty]\}$. Then the $L_2$ norm of this vector sequence is given by

$$\|w\|_2 = \sqrt{\sum_{j=0}^{\infty} \int_0^{\infty} w_j^{\mathrm{T}}(t) w_j(t)\,\mathrm{d}t}$$

and this sequence is said to be a member of $L_2^q\{[0, \infty], [0, \infty]\}$, or $L_2^q$ for short, if $\|w\|_2 < \infty$.

## 2. BACKGROUND

The basic form of the differential linear repetitive processes considered in this paper are described by the following state-space model over $0 \leqslant t \leqslant \alpha$, $k \geqslant 0$:

$$\begin{aligned} \dot{x}_{k+1}(t) &= A x_{k+1}(t) + B_0 y_k(t) + B\omega_{k+1}(t) \\ y_{k+1}(t) &= C x_{k+1}(t) + D_0 y_k(t) + D\omega_{k+1}(t) \end{aligned} \tag{1}$$



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where on pass $k$, $x_k(t)$ is the $n \times 1$ state vector, $y_k(t)$ is the $m \times 1$ pass profile vector, and $\omega_k(t)$ is the $l \times 1$ disturbance (or noise) vector which belongs to $L_2^l$.

As the entries of $\omega_k(t)$ belong to $L_2$ they cannot be Gaussian, for which it is necessary to use the Kalman filter. The key point here is the pass length is finite and at the end of each pass the process (and any disturbances present) is rest to begin the next pass. Hence, the disturbances are random over a finite interval and in this sense can be treated in $L_2$.

To complete the process description, it is necessary to specify the boundary conditions, i.e. the state initial vector on each pass and the initial pass profile (on pass 0). The form of these considered here is

$$\begin{aligned} x_{k+1}(0) &= d_{k+1}, \quad k \geqslant 0 \\ y_0(t) &= f(t) \end{aligned} \tag{2}$$

where the $n \times 1$ vector $d_{k+1}$ has known constant entries and the entries in the $m \times 1$ vector $f(t)$ are known functions of $t$ over $0 \leqslant t \leqslant \alpha$.

The stability theory [1] for linear repetitive processes is based on an abstract model in a Banach space setting which includes a large number of such processes as special cases. In this setting, a bounded linear operator mapping a Banach space into itself describes the contribution of the previous pass dynamics to the current one and the stability conditions are described in terms of properties of this operator. Noting again the unique control problem for these processes, i.e. oscillations that increase in amplitude from pass to pass (the $k$ direction in the notation for variables used here), this theory is based on ensuring that such a response cannot occur by demanding that the output sequence of pass profiles generated $\{y_k\}_k$ has a bounded-input bounded-output stability property defined in terms of the norm on the underlying Banach space.

In actual fact, two distinct forms of stability can be defined in this setting which are termed asymptotic stability and stability along the pass, respectively. The former requires this property with respect to the (finite and fixed) pass length and the latter uniformly, i.e. independent of the pass length. Asymptotic stability guarantees the existence of a so-called limit profile defined as the strong limit as $k \to \infty$ of the sequence $\{y_k\}_k$. For the processes considered here, it can be shown that asymptotic stability always holds and the resulting limit profile is described by a 1D differential linear system with state matrix $A_{lp} := A + B_0 C$.

The finite pass length means that it is possible for asymptotic stability to result in a limit profile which is unstable as a 1D differential linear system, e.g. $A = -1$, $B_0 = 1 + \beta$, $C = 1$, $D = 0$, $D_0 = 0$, where $\beta > 0$ is a real scalar. Stability along the pass prevents this from happening by demanding that the stability property be independent of the pass length, which can be analysed mathematically by letting $\alpha \to \infty$. Clearly, asymptotic stability is a necessary but not sufficient condition for stability along the pass.

It is of interest to relate this theory to a physical example in the form of long-wall coal cutting where the pass profile is the thickness (relative to a fixed datum) of the coal left after the cutting machine has moved along the pass length, i.e. the coal face. The stability problem here is caused by the machine's weight as it rests of the previous pass profile during the cutting of the next pass profile. The undulations caused can be very severe and result in productive work having to stop to enable them to be removed. Asymptotic stability here means that after a sufficient number of passes have elapsed the profile produced on each successive pass is the same, i.e. convergence in the pass to pass (i.e. $k$) direction and this converged value is termed the limit profile.



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However, this limit profile can contain growth along it, i.e. non-convergence in the $t$ direction. Stability along this pass prevents this from happening by demanding convergence in both directions.

Several equivalent sets of conditions for stability along the pass are known but here we use a Lyapunov function approach where the function actually used is

$$V(k,t)=V_1(t,k)+V_2(k,t)=x_{k+1}^{\mathrm{T}}(t)P_1x_{k+1}(t)+y_k^{\mathrm{T}}(t)P_2y_k(t) \tag{3}$$

for some $P_1>0$ and $P_2>0$, and associated increment

$$\Delta V(k,t)=\dot{V}_1(t,k)+\Delta V_2(k,t) \tag{4}$$

where

$$\dot{V}_1(t,k)=\dot{x}_{k+1}^{\mathrm{T}}(t)P_1x_{k+1}(t)+x_{k+1}^{\mathrm{T}}(t)P_1\dot{x}_{k+1}(t), \quad \Delta V_2(k,t)=y_{k+1}^{\mathrm{T}}(t)P_2y_{k+1}(t)-y_k^{\mathrm{T}}(t)P_2y_k(t)$$

*Lemma 1* ([7])
A differential linear repetitive process described by (1) and (2) is stable along the pass if

$$\Delta V(k,t)<0 \tag{5}$$

or in LMI form (and hence computational tests)

*Lemma 2*
A differential linear repetitive process described by (1) and (2) is stable along the pass if there exist matrices $W_1>0$ and $W_2>0$ such that the following LMI holds:

$$\begin{bmatrix} W_1A+A^{\mathrm{T}}W_1 & W_1B_0 & C^{\mathrm{T}}W_2 \\ * & -W_2 & D_0^{\mathrm{T}}W_2 \\ * & * & -W_2 \end{bmatrix}<0 \tag{6}$$

The proof of this last result can be found in, for example, [12].

## 3. FILTERING ANALYSIS

The problem considered in this part of this paper is to estimate the $p\times 1$ signal

$$v_{k+1}(t)=Gx_{k+1}(t)+H_0y_k(t) \tag{7}$$

where $G$ and $H_0$ are known real constant matrices. As the process state $x_{k+1}(t)$ and pass profile $y_k(t)$ vectors may not be fully accessible, we consider the estimation to be based on use of the following $r\times 1$ measured vector:

$$z_{k+1}(t)=Ex_{k+1}(t)+F_0y_k(t)+F\omega_{k+1}(t) \tag{8}$$



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using a linear full-order dynamic filter described by the state-space model

$$\begin{aligned}\dot{\varphi}_{k+1}(t) &= A_f\varphi_{k+1}(t) + B_{0f}\phi_k(t) + B_f z_{k+1}(t)\\ \phi_{k+1}(t) &= C_f\varphi_{k+1}(t) + D_{0f}\phi_k(t) + D_f z_{k+1}(t)\\ \hat{v}_{k+1}(t) &= G_f\varphi_{k+1}(t) + H_{0f}\phi_k(t) + H_f z_{k+1}(t)\\ \varphi_{k+1}(0) &= 0, \quad k\geqslant 0, \quad \phi_0(t)=0, \quad 0\leqslant t\leqslant\alpha\end{aligned} \tag{9}$$

where on pass $k$, $\varphi_k(t)$ is the $n\times 1$ state vector of the filter.

Justification for the use of (7) arises from other work into the control of these processes in, for example, [13]. In particular, this work has shown that a control law for these processes which is only activated by current pass information (state or pass profile) is too weak in all but a restricted number of special cases. Instead, it is often required to use a combination of current pass feedback action plus feedforward from the previous pass. The measured vector here is a corrupted linear combination of the entries in the current pass state vector and the previous pass profile. Hence if the filter is derived for this case, it includes others of particular interest as special cases—for example, if $E=0$ then $z_{k+1}(t)$ is the previous pass profile vector corrupted by an additive term.

In the case of (8), most commonly the pass profile vector $y_k(t)$ is simultaneously the process output, i.e. the measured signal $z_k(t)$. In this case, the matrices $E$ and $F$ of (8) would be zero and $F_0=I_m$. However, in some applications this is a significant limitation and the most general form of the output equation is

$$z_{k+1}(t) = \widehat{E}x_{k+1}(t) + \widehat{F}_0 y_{k+1}(t) + \widehat{F}\omega_{k+1}(t) \tag{10}$$

and (8) can be obtained by substituting for the current pass profile vector from the process state-space model (1).

Simple manipulations now yield the following state-space model of the filter error dynamics:

$$\begin{aligned}\dot{\xi}_{k+1}(t) &= \tilde{A}\xi_{k+1}(t) + \tilde{B}_0\zeta_k(t) + \tilde{B}\omega_{k+1}(t)\\ \zeta_{k+1}(t) &= \tilde{C}\xi_{k+1}(t) + \tilde{D}_0\zeta_k(t) + \tilde{D}\omega_{k+1}(t)\\ e_{k+1}(t) &= \tilde{G}\xi_{k+1}(t) + \tilde{H}_0\zeta_k(t) + \tilde{H}\omega_{k+1}(t)\\ \xi_{k+1}(0) &= 0, \quad k\geqslant 0, \quad \zeta_0(t)=0, \quad 0\leqslant t\leqslant\alpha\end{aligned} \tag{11}$$

where

$$\xi_{k+1}(t) = [x_{k+1}^{\mathrm{T}}(t) \;\; \varphi_{k+1}^{\mathrm{T}}(t)]^{\mathrm{T}}, \quad \zeta_k(t) = [y_k^{\mathrm{T}}(t) \;\; \phi_k^{\mathrm{T}}(t)]^{\mathrm{T}}, \quad e_{k+1}(t) = v_{k+1}(t) - \hat{v}_{k+1}(t)$$

and

$$\begin{aligned}&\tilde{A} = \begin{bmatrix} A & 0\\ B_f E & A_f\end{bmatrix}, \quad \tilde{B}_0 = \begin{bmatrix} B_0 & 0\\ B_f F_0 & B_{0f}\end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B\\ B_f F\end{bmatrix}\\ &\tilde{C} = \begin{bmatrix} C & 0\\ D_f E & C_f\end{bmatrix}, \quad \tilde{D}_0 = \begin{bmatrix} D_0 & 0\\ D_f F_0 & D_{0f}\end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} D\\ D_f F\end{bmatrix}\\ &\tilde{G} = [G - H_f E \;\; -G_f], \quad \tilde{H}_0 = [H_0 - H_f F_0 \;\; -H_{0f}], \quad \tilde{H} = -H_f F\end{aligned} \tag{12}$$



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The objective now is to find the filter model matrices such that for any non-zero $\omega_{k+1}(t) \in L_2^l$ the error filter dynamics are stable along the pass and

$$\|e_{k+1}(t)\|_2 < \gamma \|\omega_{k+1}(t)\|_2 \tag{13}$$

holds where $\gamma > 0$ is a given scalar. In this case, the filter error dynamics is said to have $\mathscr{H}_\infty$ performance level $\gamma > 0$ and the following result gives a sufficient condition for this property in terms of an LMI.

*Theorem 1*

The filtering error process (11) is stable along the pass with prescribed $\mathscr{H}_\infty$ performance level $\gamma > 0$ if there exist matrices $P_1 > 0$ and $P_2 > 0$ such that the following LMI holds:

$$\begin{bmatrix} P_1\tilde{A} + \tilde{A}^{\mathrm{T}} P_1 & P_1 \tilde{B}_0 & P_1 \tilde{B} & \tilde{G}^{\mathrm{T}} & \tilde{C}^{\mathrm{T}} P_2 \\ * & -P_2 & 0 & \tilde{H}_0^{\mathrm{T}} & \tilde{D}_0^{\mathrm{T}} P_2 \\ * & * & -\gamma^2 I & \tilde{H}^{\mathrm{T}} & \tilde{D}^{\mathrm{T}} P_2 \\ * & * & * & -I & 0 \\ * & * & * & * & -P_2 \end{bmatrix} < 0 \tag{14}$$

*Proof*

We first show stability along the pass of the filter error dynamics for which we can set with $\omega_{k+1}(t) = 0$. Consider also the candidate Lyapunov function

$$V(k,t) = V_1(t,k) + V_2(k,t) = \xi_{k+1}^{\mathrm{T}}(t) P_1 \xi_{k+1}(t) + \zeta_k^{\mathrm{T}}(t) P_2 \zeta_k(t) \tag{15}$$

where $P_1 > 0$ and $P_2 > 0$ are matrices to be specified, and associated increment

$$\Delta V(k,t) = \dot{V}_1(t,k) + \Delta V_2(k,t) \tag{16}$$

Also introduce (and recall that we can analyse stability along the pass mathematically by letting $\alpha \to \infty$)

$$\sum_{k=0}^{\infty} \int_0^{\infty} \Delta V(k,t)\,\mathrm{d}t = \int_0^{\infty} \dot{V}_1(t,k)\,\mathrm{d}t + \sum_{k=0}^{\infty} \Delta V_2(k,t) \tag{17}$$

Then along the solution of the filtering error process, we have

$$\dot{V}_1(t,k) = 2\xi_{k+1}^{\mathrm{T}}(t) P_1 \dot{\xi}_{k+1}(t) = 2\xi_{k+1}^{\mathrm{T}}(t) P_1 [\tilde{A}\xi_{k+1}(t) + \tilde{B}_0 \zeta_k(t)] \tag{18}$$

$$\begin{aligned} \Delta V_2(k,t) &= \zeta_{k+1}^{\mathrm{T}}(t) P_2 \zeta_{k+1}(t) - \zeta_k^{\mathrm{T}}(t) P_2 \zeta_k(t) \\ &= [\tilde{C}\xi_{k+1}(t) + \tilde{D}_0 \zeta_k(t)]^{\mathrm{T}} P_2 [\tilde{C}\xi_{k+1}(t) + \tilde{D}_0\zeta_k(t)] - \zeta_k^{\mathrm{T}}(t) P_2 \zeta_k(t) \end{aligned} \tag{19}$$

Hence

$$\begin{aligned} \Delta V(k,t) = {} & 2\xi_{k+1}^{\mathrm{T}}(t) P_1 [\tilde{A}\xi_{k+1}(t) + \tilde{B}_0\zeta_k(t)] + [\tilde{C}\xi_{k+1}(t) + \tilde{D}_0\zeta_k(t)]^{\mathrm{T}} P_2 [\tilde{C}\xi_{k+1}(t) + \tilde{D}_0\zeta_k(t)] \\ & - \zeta_k^{\mathrm{T}}(t) P_2 \zeta_k(t) = \varsigma_k^{\mathrm{T}}(t)(\mathscr{P}_1\mathscr{A} + \mathscr{A}^{\mathrm{T}}\mathscr{P}_1 + \mathscr{C}^{\mathrm{T}}\mathscr{P}_2\mathscr{C} - \mathscr{P}_2)\varsigma_k(t) =: \varsigma_k^{\mathrm{T}}(t)\Psi\varsigma_k(t) \end{aligned} \tag{20}$$



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where

$$\varsigma_k(t)=\begin{bmatrix}\xi_{k+1}(t)\\ \zeta_k(t)\end{bmatrix},\quad \mathscr{A}=\begin{bmatrix}\tilde{A} & \tilde{B}_0\\ 0 & 0\end{bmatrix}$$

$$\mathscr{C}=\begin{bmatrix}0 & 0\\ \tilde{C} & \tilde{D}_0\end{bmatrix},\quad \mathscr{P}_1=\begin{bmatrix}P_1 & 0\\ 0 & 0\end{bmatrix},\quad \mathscr{P}_2=\begin{bmatrix}0 & 0\\ 0 & P_2\end{bmatrix}$$

It now follows immediately that if $\Psi<0$, then for any $\varsigma_k(t)\neq 0$, we have $\Delta V(k,t)<0$ and hence stability along the pass by Lemma 1. A straightforward application of Schur's complement formula to the left-hand side of $\Psi<0$ now gives (14) which is simply the LMI of Lemma 2 applied to this case.

To establish the $\mathscr{H}_\infty$ performance level, set $\xi_{k+1}(0)=0,\ k\geqslant 0$ and consider the following cost function or index:

$$\mathscr{J}=\|e_{k+1}(t)\|_2^2-\gamma^2\|\omega_{k+1}(t)\|_2^2 \tag{21}$$

Then since stability along the pass holds we have that

$$\begin{aligned}\mathscr{J} &\leqslant \|e_{k+1}(t)\|_2^2-\gamma^2\|\omega_{k+1}(t)\|_2^2=\sum_{k=0}^{\infty}\int_0^\infty [e_{k+1}^{\mathrm{T}}(t)e_{k+1}(t)-\gamma^2\omega_{k+1}^{\mathrm{T}}(t)\omega_{k+1}(t)]\,\mathrm{d}t\\ &=\sum_{k=0}^{\infty}\int_0^\infty [e_{k+1}^{\mathrm{T}}(t)e_{k+1}(t)-\gamma^2\omega_{k+1}^{\mathrm{T}}(t)\omega_{k+1}(t)+\Delta V(k,t)]\,\mathrm{d}t\\ &=:\sum_{k=0}^{\infty}\int_0^\infty \eta_k^{\mathrm{T}}(t)\Pi\eta_k(t)\,\mathrm{d}t\end{aligned} \tag{22}$$

where $\eta_k(t)=[\xi_{k+1}^{\mathrm{T}}(t)\ \ \zeta_k^{\mathrm{T}}(t)\ \ \omega_{k+1}^{\mathrm{T}}(t)]^{\mathrm{T}}$
and

$$\Pi=\begin{bmatrix}P_1\tilde{A}+\tilde{A}^{\mathrm{T}}P_1 & P_1\tilde{B}_0 & P_1\tilde{B}\\ * & -P_2 & 0\\ * & * & -\gamma^2 I\end{bmatrix}+\begin{bmatrix}\tilde{C}^{\mathrm{T}}\\ \tilde{D}_0^{\mathrm{T}}\\ \tilde{D}^{\mathrm{T}}\end{bmatrix}P_2\begin{bmatrix}\tilde{C}^{\mathrm{T}}\\ \tilde{D}_0^{\mathrm{T}}\\ \tilde{D}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}+\begin{bmatrix}\tilde{G}^{\mathrm{T}}\\ \tilde{H}_0^{\mathrm{T}}\\ \tilde{H}^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\tilde{G}^{\mathrm{T}}\\ \tilde{H}_0^{\mathrm{T}}\\ \tilde{H}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$$

Application of Schur's complement formula to the right-hand side of this last expression and then applying (14) to this case we have $\Pi<0$ and therefore for all $\eta_k(t)\neq 0$, we have $\mathscr{J}<0$, i.e. $\|e_{k+1}(t)\|_2<\gamma\|\omega_{k+1}(t)\|_2$ for all non-zero $\omega_{k+1}(t)\in L_2^l$ and the proof is complete. $\square$

The following result now gives an algorithm for computing the filter state-space matrices.

*Theorem 2*
Consider a differential linear repetitive processes described by (1) and (2) and let $\gamma>0$ be a given scalar. Suppose also that there exist matrices $\mathscr{U}_1>0$, $\mathscr{V}_1>0$, $\mathscr{U}_2>0$, $\mathscr{V}_2>0$, $\mathscr{A}_f$, $\mathscr{B}_{0f}$, $\mathscr{B}_f$, $\mathscr{C}_f$,



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$\mathscr{D}_{0f}$, $\mathscr{D}_f$, $\mathscr{G}_f$, $\mathscr{H}_{0f}$ and $\mathscr{H}_f$ such that the following LMI holds:

$$\begin{bmatrix} \mathrm{sym}(\mathscr{U}_1 A+\mathscr{B}_f E) & \Pi_{12} & \mathscr{U}_1 B_0+\mathscr{B}_f F_0 & \mathscr{B}_{0f} & \mathscr{U}_1 B+\mathscr{B}_f F & (G-\mathscr{H}_f E)^{\mathrm{T}} & \Pi_{17} & \Pi_{18} \\ * & \mathscr{A}_f+\mathscr{A}_f^{\mathrm{T}} & \mathscr{V}_1 B_0+\mathscr{B}_f F_0 & \mathscr{B}_{0f} & \mathscr{V}_1 B+\mathscr{B}_f F & -\mathscr{G}_f^{\mathrm{T}} & \mathscr{C}_f^{\mathrm{T}} & \mathscr{C}_f^{\mathrm{T}} \\ * & * & -\mathscr{U}_2 & -\mathscr{V}_2 & 0 & (H_0-\mathscr{H}_f F_0)^{\mathrm{T}} & \Pi_{37} & \Pi_{38} \\ * & * & * & -\mathscr{V}_2 & 0 & -\mathscr{H}_{0f}^{\mathrm{T}} & \mathscr{D}_{0f}^{\mathrm{T}} & \mathscr{D}_{0f}^{\mathrm{T}} \\ * & * & * & * & -\gamma^2 I & -F^{\mathrm{T}}\mathscr{H}_f^{\mathrm{T}} & \Pi_{57} & \Pi_{58} \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\mathscr{U}_2 & -\mathscr{V}_2 \\ * & * & * & * & * & * & * & -\mathscr{V}_2 \end{bmatrix} < 0 \tag{23}$$

where

$$\Pi_{12}=\mathscr{A}_f+(\mathscr{V}_1 A+\mathscr{B}_f E)^{\mathrm{T}}, \quad \Pi_{17}=(\mathscr{U}_2 C+\mathscr{D}_f E)^{\mathrm{T}}$$

$$\Pi_{18}=(\mathscr{V}_2 C+\mathscr{D}_f E)^{\mathrm{T}}, \quad \Pi_{37}=(\mathscr{U}_2 D_0+\mathscr{D}_f F_0)^{\mathrm{T}}, \quad \Pi_{38}=(\mathscr{V}_2 D_0+\mathscr{D}_f F_0)^{\mathrm{T}}$$

$$\Pi_{57}=(\mathscr{U}_2 D+\mathscr{D}_f F)^{\mathrm{T}}, \quad \Pi_{58}=(\mathscr{V}_2 D+\mathscr{D}_f F)^{\mathrm{T}}$$

Then there exists a full-order filter of the form of (9) such that the filtering error dynamics are stable along the pass and the prescribed $\mathscr{H}_\infty$ performance level $\gamma$ is achieved. This $\mathscr{H}_\infty$ filter can be computed from

$$\begin{bmatrix} A_f & B_{0f} & B_f \\ C_f & D_{0F} & D_F \\ G_f & H_{0F} & H_F \end{bmatrix} = \begin{bmatrix} \mathscr{V}_1^{-1} & 0 & 0 \\ 0 & \mathscr{V}_2^{-1} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \mathscr{A}_f & \mathscr{B}_{0f} & \mathscr{B}_f \\ \mathscr{C}_f & \mathscr{D}_{0f} & \mathscr{D}_f \\ \mathscr{G}_f & \mathscr{H}_{0f} & \mathscr{H}_f \end{bmatrix} \tag{24}$$

*Proof*

By Theorem 1, $P_1$ and $P_2$ are non-singular if (14) holds and partition them as

$$P_1=\begin{bmatrix} P_{11} & P_{12} \\ P_{12}^{\mathrm{T}} & P_{13} \end{bmatrix}, \quad P_2=\begin{bmatrix} P_{21} & P_{22} \\ P_{22}^{\mathrm{T}} & P_{23} \end{bmatrix} \tag{25}$$

and, without loss of generality, we assume that $P_{12}$ and $P_{22}$ are non-singular. (If this is not the case for, say, $P_1$, then a perturbation matrix $\Delta P_{12}$ of sufficiently small norm can be used to make $P_{12}+\Delta P_{12}$ non-singular. Then we can use (14) with this substitution and likewise for the other cases which could arise here.) Also, introduce

$$\Gamma_1=\begin{bmatrix} I & 0 \\ 0 & P_{13}^{-1}P_{12}^{\mathrm{T}} \end{bmatrix}, \quad \Gamma_2=\begin{bmatrix} I & 0 \\ 0 & P_{23}^{-1}P_{22}^{\mathrm{T}} \end{bmatrix}, \quad \mathscr{U}_1=P_{11}$$
$$\mathscr{V}_1=P_{12}P_{13}^{-1}P_{12}^{\mathrm{T}}, \quad \mathscr{U}_2=P_{21}, \quad \mathscr{V}_2=P_{22}P_{23}^{-1}P_{22}^{\mathrm{T}} \tag{26}$$



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and

$$\begin{bmatrix} \mathscr{A}_f & \mathscr{B}_{0f} & \mathscr{B}_f \\ \mathscr{C}_f & \mathscr{D}_{0f} & \mathscr{D}_f \\ \mathscr{G}_f & \mathscr{H}_{0f} & \mathscr{H}_f \end{bmatrix} = \begin{bmatrix} P_{12} & 0 & 0 \\ 0 & P_{22} & I \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} A_f & B_{0f} & B_f \\ C_f & D_{0f} & D_f \\ G_f & H_{0f} & H_f \end{bmatrix} \begin{bmatrix} P_{13}^{-1} P_{12}^{\mathrm{T}} & 0 & 0 \\ 0 & P_{23}^{-1} P_{22}^{\mathrm{T}} & 0 \\ 0 & 0 & I \end{bmatrix} \tag{27}$$

Next, pre- and post-multiply (14) by the matrix diag $(\Gamma_1, \Gamma_2, I, I, \Gamma_2)$ to yield

$$\begin{bmatrix} \mathscr{X} & \Gamma_1^{\mathrm{T}} P_1 \tilde{B}_0 \Gamma_2 & \Gamma_1^{\mathrm{T}} P_1 \tilde{B} & \tilde{G}^{\mathrm{T}} & \Gamma_1^{\mathrm{T}} \tilde{C}^{\mathrm{T}} P_2 \Gamma_2 \\ * & -\Gamma_2^{\mathrm{T}} P_2 \Gamma_2 & 0 & \Gamma_2^{\mathrm{T}} \tilde{H}_0^{\mathrm{T}} & \Gamma_2^{\mathrm{T}} \tilde{D}_0^{\mathrm{T}} P_2 \Gamma_2 \\ * & * & -\gamma^2 I & \tilde{H}^{\mathrm{T}} & \tilde{D}^{\mathrm{T}} P_2 \Gamma_2 \\ * & * & * & -I & 0 \\ * & * & * & * & -\Gamma_2^{\mathrm{T}} P_2 \Gamma_2 \end{bmatrix} < 0 \tag{28}$$

where $\mathscr{X} = \mathrm{sym}(\Gamma_1^{\mathrm{T}} P_1 \tilde{A} \Gamma_1)$ and

$$\Gamma_1^{\mathrm{T}} P_1 \tilde{A} \Gamma_1 = \begin{bmatrix} \mathscr{X}_1 & P_{12} A_f P_{13}^{-1} P_{12}^{\mathrm{T}} \\ \mathscr{X}_2 & P_{12} A_f P_{13}^{-1} P_{12}^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \mathscr{U}_1 A + \mathscr{B}_f E & \mathscr{A}_f \\ \mathscr{V}_1 A + \mathscr{B}_f E & \mathscr{A}_f \end{bmatrix} \tag{29}$$

$$\mathscr{X}_1 = P_{11} A + P_{12} B_f E, \quad \mathscr{X}_2 = P_{12} P_{13}^{-1} P_{12}^{\mathrm{T}} A + P_{12} B_f E$$

$$\Gamma_1^{\mathrm{T}} P_1 \tilde{B}_0 \Gamma_2 = \begin{bmatrix} \mathscr{Y}_1 & P_{12} B_{0f} P_{23}^{-1} P_{22}^{\mathrm{T}} \\ \mathscr{Y}_2 & P_{12} B_{0f} P_{23}^{-1} P_{22}^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \mathscr{U}_1 B_0 + \mathscr{B}_f F_0 & \mathscr{B}_{0f} \\ \mathscr{V}_1 B_0 + \mathscr{B}_f F_0 & \mathscr{B}_{0f} \end{bmatrix} \tag{30}$$

$$\mathscr{Y}_1 = P_{11} B_0 + P_{12} B_f F_0, \quad \mathscr{Y}_2 = P_{12} P_{13}^{-1} P_{12}^{\mathrm{T}} B_0 + P_{12} B_f F_0$$

$$\Gamma_1^{\mathrm{T}} P_1 \tilde{B} = \begin{bmatrix} P_{11} B + P_{12} B_f F \\ P_{12} P_{13}^{-1} P_{12}^{\mathrm{T}} B + P_{12} B_f F \end{bmatrix} = \begin{bmatrix} \mathscr{U}_1 B + \mathscr{B}_f F \\ \mathscr{V}_1 B + \mathscr{B}_f F \end{bmatrix} \tag{31}$$

$$\Gamma_2^{\mathrm{T}} P_2 \tilde{C} \Gamma_1 = \begin{bmatrix} \mathscr{Z}_1 & P_{22} C_f P_{13}^{-1} P_{12}^{\mathrm{T}} \\ \mathscr{Z}_2 & P_{22} C_f P_{13}^{-1} P_{12}^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \mathscr{U}_2 C + \mathscr{D}_f E & \mathscr{C}_f \\ \mathscr{V}_2 C + \mathscr{D}_f E & \mathscr{C}_f \end{bmatrix} \tag{32}$$

$$\mathscr{Z}_1 = P_{21} C + P_{22} D_f E, \quad \mathscr{Z}_2 = P_{22} P_{23}^{-1} P_{22}^{\mathrm{T}} C + P_{22} D_f E$$

$$\Gamma_2^{\mathrm{T}} P_2 \tilde{D}_0 \Gamma_2 = \begin{bmatrix} \mathscr{U}_1 & P_{22} D_{0f} P_{23}^{-1} P_{22}^{\mathrm{T}} \\ \mathscr{U}_2 & P_{22} D_{0f} P_{23}^{-1} P_{22}^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \mathscr{U}_2 D_0 + \mathscr{D}_f F_0 & \mathscr{D}_{0f} \\ \mathscr{V}_2 D_0 + \mathscr{D}_f F_0 & \mathscr{D}_{0f} \end{bmatrix} \tag{33}$$

$$\mathscr{U}_1 = P_{21} D_0 + P_{22} D_f F_0, \quad \mathscr{U}_2 = P_{22} P_{23}^{-1} P_{22}^{\mathrm{T}} D_0 + P_{22} D_f F_0$$

$$\Gamma_2^{\mathrm{T}} P_2 \tilde{D} = \begin{bmatrix} P_{21} D + P_{22} D_f F \\ P_{22} P_{23}^{-1} P_{22}^{\mathrm{T}} D + P_{22} D_f F \end{bmatrix} = \begin{bmatrix} \mathscr{U}_2 D + \mathscr{D}_f F \\ \mathscr{V}_2 D + \mathscr{D}_f F \end{bmatrix} \tag{34}$$

$$\Gamma_2^{\mathrm{T}} P_2 \Gamma_2 = \begin{bmatrix} P_{21} & P_{22} P_{23}^{-1} P_{22}^{\mathrm{T}} \\ P_{22} P_{23}^{-1} P_{22}^{\mathrm{T}} & P_{22} P_{23}^{-1} P_{22}^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \mathscr{U}_2 & \mathscr{V}_2 \\ \mathscr{V}_2 & \mathscr{V}_2 \end{bmatrix} \tag{35}$$

$$\tilde{G}\Gamma_1 = [G - H_f E \quad -G_f P_{13}^{-1} P_{12}^{\mathrm{T}}] = [G - \mathscr{H}_f E \quad -\mathscr{G}_f] \tag{36}$$

$$\tilde{H}_0\Gamma_2 = [H_0 - H_f F_0 \quad -H_{0f} P_{23}^{-1} P_{22}^{\mathrm{T}}] = [H_0 - \mathscr{H}_f F_0 \quad -\mathscr{H}_{0f}] \tag{37}$$

$$\tilde{H} = -H_f F = -\mathscr{H}_f F \tag{38}$$

Substituting (25)–(27) and (29)–(38) into (28) now gives (23).

Also, (27) is equivalent to

$$\begin{aligned}\begin{bmatrix} A_f & B_{0f} & B_f \\ C_f & D_{0f} & D_f \\ G_f & H_{0f} & H_f \end{bmatrix} &= \begin{bmatrix} P_{12}^{-1} & 0 & \\ 0 & P_{12}^{-1} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \mathscr{A}_f & \mathscr{B}_{0f} & \mathscr{B}_f \\ \mathscr{C}_f & \mathscr{D}_{0f} & \mathscr{D}_f \\ \mathscr{G}_f & \mathscr{H}_{0f} & \mathscr{H}_f \end{bmatrix} \\ &\quad \times \begin{bmatrix} P_{12}^{-\mathrm{T}} P_{13} & 0 & 0 \\ 0 & P_{22}^{-\mathrm{T}} P_{23} & 0 \\ 0 & 0 & I \end{bmatrix} \\ &= \begin{bmatrix} \mathscr{W}_1^{-1} \mathscr{V}_1^{-1} & 0 & 0 \\ 0 & \mathscr{W}_2^{-1} \mathscr{V}_2^{-1} 0 & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \mathscr{A}_f & \mathscr{B}_{0f} & \mathscr{B}_f \\ \mathscr{C}_f & \mathscr{D}_{0f} & \mathscr{D}_f \\ \mathscr{G}_f & \mathscr{H}_{0f} & \mathscr{H}_f \end{bmatrix} \\ &\quad \times \begin{bmatrix} \mathscr{W}_1 & 0 & 0 \\ 0 & \mathscr{W}_1 & 0 \\ 0 & 0 & I \end{bmatrix}\end{aligned} \tag{39}$$

where $\mathscr{W}_1 = P_{12}^{-\mathrm{T}} P_{13}$ and $\mathscr{W}_2 = P_{22}^{-\mathrm{T}} P_{23}$ Note that the filter matrices of (9) can be written in the form (39) and hence diag $(\mathscr{W}_1, \mathscr{W}_2, I)$ can be viewed as a similarity transformation on the state-space realization of the filter and, as such, has no effect on the filter mapping from $z_{k+1}(t)$ to $\hat{v}_{k+1}(t)$. Without loss of generality, we can set $\mathscr{W}_1 = \mathscr{W}_2 = I$, to obtain (24). Therefore, we conclude that the filter (9) can be constructed using (24) and the proof is complete. □

*Remark 1*

Note that Theorem 2 provides a sufficient condition for solvability of the $\mathscr{H}_\infty$ filter problem and, since the resulting condition is in LMI form, a filter which minimizes the $\mathscr{H}_\infty$ performance level (i.e. maximize the level of noise removal) can be determined by solving the following convex optimization problem:

$$\text{Minimize } \delta \quad \text{subject to (23) where } \delta = \gamma^2 \tag{40}$$

with $\mathscr{U}_1 > 0$, $\mathscr{V}_1 > 0$, $\mathscr{U}_2 > 0$, $\mathscr{V}_2 > 0$, $\mathscr{A}_f$, $\mathscr{B}_{0f}$, $\mathscr{B}_f$, $\mathscr{C}_f$, $\mathscr{D}_{0f}$, $\mathscr{D}_f$, $\mathscr{G}_f$, $\mathscr{H}_{0f}$ and $\mathscr{H}_f$.



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## 4. ROBUST $\mathscr{H}_\infty$ FILTERING

In this section, we extend the analysis to the case when there is uncertainty associated with the process model. As in the 1D linear systems case, we assume that the uncertainty satisfies two model structures, polytopic and norm-bounded, respectively.

### *4.1. Polytopic uncertainty*

Here we assume that the matrices which define the filtering problem of the previous section are not known exactly but lie within a given polytope. In particular, we assume that

$$\Omega := (A, B_0, B, C, D_0, D, E, F_0, F) \in \chi$$

where $\chi$ is a given convex bounded polyhedral domain described by $s$ vertices, i.e.

$$\chi = \left\{ \chi(\lambda) \left| \chi(\lambda) = \sum_{i=1}^{s} \lambda_j \chi_j, \ \sum_{i=1}^{s} \lambda_j = 1, \lambda_j \geqslant 0 \right. \right\}$$

where

$$\chi_j = (A_j, B_{0j}, B_j, C_j, D_{0j}, D_j, E_j, F_{0j}, F_j)$$

denotes the $j$th vertex of the polytope $\chi$.

The following result can now be given and its proof is omitted since it follows in a similar manner to that of Theorem 2.

*Theorem 3*

Consider a differential linear repetitive process described by (1) and (2) in the presence of uncertainty in the process state-space model which satisfies the polytopic model given above and let $\gamma > 0$ be a given scalar. Suppose also that there exist matrices $\mathscr{U}_1 > 0$, $\mathscr{V}_1 > 0$, $\mathscr{U}_2 > 0$, $\mathscr{V}_2 > 0$, $\mathscr{A}_f$, $\mathscr{B}_{0f}$, $\mathscr{B}_f$, $\mathscr{C}_f$, $\mathscr{D}_{0f}$, $\mathscr{D}_f$, $\mathscr{G}_f$, $\mathscr{H}_{0f}$ and $\mathscr{H}_f$ such that, for $j = 1, 2, \ldots, s$, the following LMI holds:

$$\begin{bmatrix} \mathrm{sym}(\mathscr{U}_1 A_j + \mathscr{B}_f E_j) & \hat{\Pi}_{12} & \mathscr{U}_1 B_{0j} + \mathscr{B}_f F_{0j} & \mathscr{B}_{0f} & \mathscr{U}_1 B_j + \mathscr{B}_f F_j & (G - \mathscr{H}_f E_j)^{\mathrm{T}} & \hat{\Pi}_{17} & \hat{\Pi}_{18} \\ * & \mathscr{A}_f + \mathscr{A}_f^{\mathrm{T}} & \mathscr{V}_1 B_{0j} + \mathscr{B}_f F_{0j} & \mathscr{B}_{0f} & \mathscr{V}_1 B_j + \mathscr{B}_f F_j & -\mathscr{G}_f^{\mathrm{T}} & \mathscr{C}_f^{\mathrm{T}} & \mathscr{C}_f^{\mathrm{T}} \\ * & * & -\mathscr{U}_2 & -\mathscr{V}_2 & 0 & (H_0 - \mathscr{H}_f F_{0j})^{\mathrm{T}} & \hat{\Pi}_{37} & \hat{\Pi}_{38} \\ * & * & * & -\mathscr{V}_2 & 0 & -\mathscr{H}_{0f}^{\mathrm{T}} & \mathscr{D}_{0f}^{\mathrm{T}} & \hat{\mathscr{D}}_{0f}^{\mathrm{T}} \\ * & * & * & * & -\gamma^2 I & -F_j^{\mathrm{T}} \mathscr{H}_f^{\mathrm{T}} & \hat{\Pi}_{57} & \hat{\Pi}_{58} \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\mathscr{U}_2 & -\mathscr{V}_2 \\ * & * & * & * & * & * & * & -\mathscr{V}_2 \end{bmatrix} < 0 \tag{41}$$



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where

$$\hat{\Pi}_{12}=\mathscr{A}_f+(\mathscr{V}_1A_j+\mathscr{B}_fE_j)^{\mathrm{T}},\quad \hat{\Pi}_{17}=(\mathscr{U}_2C_j+\mathscr{D}_fE_j)^{\mathrm{T}}$$
$$\hat{\Pi}_{18}=(\mathscr{V}_2C_j+\mathscr{D}_fE_j)^{\mathrm{T}},\quad \hat{\Pi}_{37}=(\mathscr{U}_2D_{0j}+\mathscr{D}_fF_{0j})^{\mathrm{T}},\quad \hat{\Pi}_{38}=(\mathscr{V}_2D_{0j}+\mathscr{D}_fF_{0j})^{\mathrm{T}}$$
$$\hat{\Pi}_{57}=(\mathscr{U}_2D_j+\mathscr{D}_fF_j)^{\mathrm{T}},\quad \hat{\Pi}_{58}=(\mathscr{V}_2D_j+\mathscr{D}_fF_j)^{\mathrm{T}}$$

Then there exists a full-order filter of the form (9) for which the filtering error is stable along the pass and the prescribed $\mathscr{H}_\infty$ performance level $\gamma$ is achieved. This $\mathscr{H}_\infty$ filter can be computed from

$$\begin{bmatrix} A_f & B_{0f} & B_f \\ C_f & D_{0F} & D_F \\ G_f & H_{0F} & H_F \end{bmatrix}=\begin{bmatrix} \mathscr{V}_1^{-1} & 0 & 0 \\ 0 & \mathscr{V}_2^{-1} & 0 \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} \mathscr{A}_f & \mathscr{B}_{0f} & \mathscr{B}_f \\ \mathscr{C}_f & \mathscr{D}_{0f} & \mathscr{D}_f \\ \mathscr{G}_f & \mathscr{H}_{0f} & \mathscr{H}_f \end{bmatrix} \tag{42}$$

### *4.2. Norm-bounded uncertainty*

Here we assume that the matrices defining the basic filtering problem of Section 2 satisfy

$$\begin{aligned} &A=\breve{A}+\Delta A,\quad B_0=\breve{B}_0+\Delta B_0,\quad B=\breve{B}+\Delta B\\ &C=\breve{C}+\Delta C,\quad D_0=\breve{D}_0+\Delta D_0,\quad D=\breve{D}+\Delta D\\ &E=\breve{E}+\Delta E,\quad F_0=\breve{F}_0+\Delta F_0,\quad F=\breve{F}+\Delta F \end{aligned} \tag{43}$$

where $\breve{A}$, etc. are real constant matrices; $\Delta A$, etc. are real-valued time-varying matrix functions representing norm-bounded parameter uncertainties which are assumed to satisfy

$$\begin{bmatrix} \Delta A & \Delta B_0 & \Delta B \\ \Delta C & \Delta D_0 & \Delta C \\ \Delta E & \Delta F_0 & \Delta F \end{bmatrix}=\begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix}\mathscr{F}[N_1\ \ N_2\ \ N_3] \tag{44}$$

where $\mathscr{F}$ has unknown elements but satisfies $\mathscr{F}^{\mathrm{T}}\mathscr{F}\leqslant I$, and $M_1$, etc. are known real constant matrices of compatible dimensions.

The following result will required in the proof of the next theorem.

*Lemma 3* ([*14*])
Let $\Sigma_1$, $\Sigma_2$ be real matrices of appropriate dimensions. Then for any matrix $\Delta$ satisfying $\Delta^{\mathrm{T}}\Delta\leqslant I$ and a scalar $\varepsilon>0$

$$\Sigma_1\Delta\Sigma_2+\Sigma_2^{\mathrm{T}}\Delta^{\mathrm{T}}\Sigma_1^{\mathrm{T}}\leqslant\varepsilon^{-1}\Sigma_1\Sigma_1^{\mathrm{T}}+\varepsilon\Sigma_2^{\mathrm{T}}\Sigma_2 \tag{45}$$

*Theorem 4*
Consider a differential linear repetitive process described by (1) and (2) in the presence of uncertainty in the process state-space model which satisfies the norm-bounded model given above and let $\gamma>0$ be a given scalar. Suppose also that there exist matrices $\mathscr{U}_1>0$, $\mathscr{V}_1>0$, $\mathscr{U}_2>0$, $\mathscr{V}_2>0$, $\mathscr{A}_f$,

 
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$\mathscr{B}_{0f}$, $\mathscr{B}_f$, $\mathscr{C}_f$, $\mathscr{D}_{0f}$, $\mathscr{D}_f$, $\mathscr{G}_f$, $\mathscr{H}_{0f}$, $\mathscr{H}_f$ and a scalar $\varepsilon>0$ such that the following LMI holds:

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \mathscr{B}_{0f} & \Upsilon_{15} & (G-\mathscr{H}_f\breve{E})^{\mathrm{T}} & \Upsilon_{17} & \Upsilon_{18} & \mathscr{U}_1 M_1+\mathscr{B}_f M_3 \\ * & \mathscr{A}_f+\mathscr{A}_f^{\mathrm{T}} & \mathscr{V}_1\breve{B}_0+\mathscr{B}_f\breve{F}_0 & \mathscr{B}_{0f} & \mathscr{V}_1\breve{B}+\mathscr{B}_f\breve{F} & -\mathscr{G}_f^{\mathrm{T}} & \mathscr{C}_f^{\mathrm{T}} & \mathscr{C}_f^{\mathrm{T}} & \mathscr{V}_1 M_1+\mathscr{B}_f M_3 \\ * & * & -\mathscr{U}_2+\varepsilon N_2^{\mathrm{T}} N_2 & -\mathscr{V}_2 & \varepsilon N_2^{\mathrm{T}} N_3 & (H_0-\mathscr{H}_f\breve{F}_0)^{\mathrm{T}} & \Upsilon_{37} & \Upsilon_{38} & 0 \\ * & * & * & -\mathscr{V}_2 & 0 & -\mathscr{H}_{0f}^{\mathrm{T}} & \mathscr{D}_{0f}^{\mathrm{T}} & \mathscr{D}_{0f}^{\mathrm{T}} & 0 \\ * & * & * & * & -\gamma^2 I+\varepsilon N_3^{\mathrm{T}} N_3 & -\breve{F}^{\mathrm{T}}\mathscr{H}_f^{\mathrm{T}} & \Upsilon_{57} & \Upsilon_{58} & 0 \\ * & * & * & * & * & -I & * & * & -\mathscr{H}_f M_3 \\ * & * & * & * & * & * & -\mathscr{U}_2 & -\mathscr{V}_2 & \mathscr{U}_2 M_2+\mathscr{D}_f M_3 \\ * & * & * & * & * & * & * & -\mathscr{V}_2 & \mathscr{V}_2 M_2+\mathscr{D}_f M_3 \\ * & * & * & * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{46}$$

where

$$\begin{aligned} &\Upsilon_{11}=\mathscr{U}_1\breve{A}+\mathscr{B}_f\breve{E}+(\mathscr{U}_1\breve{A}+\mathscr{B}_f\breve{E})^{\mathrm{T}}+\varepsilon N_1^{\mathrm{T}} N_1, \quad \Upsilon_{12}=\mathscr{A}_f+(\mathscr{V}_1\breve{A}+\mathscr{B}_f\breve{E})^{\mathrm{T}} \\ &\Upsilon_{13}=\mathscr{U}_1\breve{B}_0+\mathscr{B}_f\breve{F}_0+\varepsilon N_1^{\mathrm{T}} N_2, \quad \Upsilon_{15}=\mathscr{U}_1\breve{B}+\mathscr{B}_f\breve{F}+\varepsilon N_1^{\mathrm{T}} N_3 \\ &\Upsilon_{17}=(\mathscr{U}_2\breve{C}+\mathscr{D}_f\breve{E})^{\mathrm{T}}, \quad \Upsilon_{18}=(\mathscr{V}_2\breve{C}+\mathscr{D}_f\breve{E})^{\mathrm{T}} \\ &\Upsilon_{37}=(\mathscr{U}_2\breve{D}_0+\mathscr{D}_f\breve{F}_0)^{\mathrm{T}}, \quad \Upsilon_{38}=(\mathscr{V}_2\breve{D}_0+\mathscr{D}_f\breve{F}_0)^{\mathrm{T}} \\ &\Upsilon_{57}=(\mathscr{U}_2\breve{D}+\mathscr{D}_f\breve{F})^{\mathrm{T}}, \quad \Upsilon_{58}=(\mathscr{V}_2\breve{D}+\mathscr{D}_f\breve{F})^{\mathrm{T}} \end{aligned} \tag{47}$$

Then there exists a full-order filter of the form (9) for which the filtering error is stable along the pass and prescribed $\mathscr{H}_\infty$ performance level $\gamma$ is achieved. This $\mathscr{H}_\infty$ filter can be computed from

$$\begin{bmatrix} A_f & B_{0f} & B_f \\ C_f & D_{0F} & D_F \\ G_f & H_{0F} & H_F \end{bmatrix} = \begin{bmatrix} \mathscr{V}_1^{-1} & 0 & 0 \\ 0 & \mathscr{V}_2^{-1} & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \mathscr{A}_f & \mathscr{B}_{0f} & \mathscr{B}_f \\ \mathscr{C}_f & \mathscr{D}_{0f} & \mathscr{D}_f \\ \mathscr{G}_f & \mathscr{H}_{0f} & \mathscr{H}_f \end{bmatrix} \tag{48}$$

*Proof*

Given Theorem 2, the first step here is to substitute (43) into (41) to obtain

$$\Sigma+\Sigma_1\Delta\Sigma_2+\Sigma_2^{\mathrm{T}}\Delta^{\mathrm{T}}\Sigma_1^{\mathrm{T}}<0 \tag{49}$$



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where

$$\Sigma = \begin{bmatrix} \mathrm{sym}(\mathscr{U}_1\breve{A}+\mathscr{B}_f\breve{E}) & \Upsilon_{12} & \mathscr{U}_1\breve{B}_0+\mathscr{B}_f\breve{F}_0 & \mathscr{B}_{0f} & (G-\mathscr{H}_f\breve{E})^{\mathrm{T}} & & \Upsilon_{17} & \Upsilon_{18} \\ * & \mathscr{A}_f+\mathscr{A}_f^{\mathrm{T}} & \mathscr{V}_1\breve{B}_0+\mathscr{B}_f\breve{F}_0 & \mathscr{B}_{0f} & \mathscr{U}_1\breve{B}+\mathscr{B}_f\breve{F} & -\mathscr{G}_f^{\mathrm{T}} & \mathscr{C}_f^{\mathrm{T}} & \mathscr{C}_f^{\mathrm{T}} \\ * & * & -\mathscr{U}_2 & -\mathscr{V}_2 & 0 & (H_0-\mathscr{H}_f\breve{F}_0)^{\mathrm{T}} & \Upsilon_{37} & \Upsilon_{38} \\ * & * & * & -\mathscr{V}_2 & 0 & -\mathscr{H}_{0f}^{\mathrm{T}} & \mathscr{D}_{0f}^{\mathrm{T}} & \mathscr{D}_{0f}^{\mathrm{T}} \\ * & * & * & * & -\gamma^2 I & -\breve{F}^{\mathrm{T}}\mathscr{H}_f^{\mathrm{T}} & \Upsilon_{57} & \Upsilon_{58} \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -\mathscr{U}_2 & -\mathscr{V}_2 \\ * & * & * & * & * & * & * & -\mathscr{V}_2 \end{bmatrix} < 0$$

$$\Sigma_1 = \begin{bmatrix} \mathscr{U}_1 M_1 + \mathscr{B}_f M_3 \\ \mathscr{V}_1 M_1 + \mathscr{B}_f M_3 \\ 0 \\ 0 \\ 0 \\ -\mathscr{H}_f M_3 \\ \mathscr{U}_2 M_2 + \mathscr{D}_f M_3 \\ \mathscr{V}_2 M_2 + \mathscr{D}_f M_3 \end{bmatrix}, \quad \Sigma_2 = [N_1 \;\; 0 \;\; N_2 \;\; 0 \;\; N_3 \;\; 0 \;\; 0 \;\; 0], \quad \Delta = \mathscr{F}$$

and $\Upsilon_{12}$, $\Upsilon_{17}$, $\Upsilon_{18}$, $\Upsilon_{37}$, $\Upsilon_{38}$, $\Upsilon_{57}$, $\Upsilon_{58}$ have been defined in (47). Use of Lemma 2 together with the Schur's complement formula now shows that (49) holds if (46) holds, and the proof is complete. □

*Remark 2*

The $\mathscr{H}_\infty$ performance level $\gamma$ here can be minimized by solving the following convex optimization problem:

$$\text{Minimize } \delta \quad \text{subject to (46) where } \delta = \gamma^2$$

with $\mathscr{U}_1>0$, $\mathscr{V}_1>0$, $\mathscr{U}_2>0$, $\mathscr{V}_2>0$, $\mathscr{A}_f$, $\mathscr{B}_{0f}$, $\mathscr{B}_f$, $\mathscr{C}_f$, $\mathscr{D}_{0f}$, $\mathscr{D}_f$, $\mathscr{G}_f$, $\mathscr{H}_{0f}$, $\mathscr{H}_f$ and $\varepsilon>0$.

Overall, it is important to note that the results developed here are sufficient but not necessary and hence the design will, in general, be conservative. An obvious area for further research is to see ways of reducing this by using parameter-dependent Lyapunov functions and auxiliary slack matrix variables. Note also that of the currently available methods for repetitive process/2D linear systems control law design, it is only the LMI route which leads to algorithms which can actually be numerically evaluated for all but low-order examples and hence the possibility to move to the stage of performance evaluation. It is by no means clear how necessary and sufficient algorithms with this key feature can be developed.



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## 5. A NUMERICAL EXAMPLE

Consider the case when

$$A=\begin{bmatrix}-1.45+0.01\delta & 0.64 & -0.40+0.01\delta\\ -0.60 & -1.41 & 0.00\\ 0.30+0.01\delta & -0.20 & -0.70+0.01\delta\end{bmatrix},\quad B_0=\begin{bmatrix}1.3+0.01\delta & 0.10+0.01\delta\\ -0.20 & -0.90\\ 0.20+0.01\delta & -0.40+0.01\delta\end{bmatrix}$$

$$B=\begin{bmatrix}0.60+0.01\delta\\ -1.20\\ 0.20+0.01\delta\end{bmatrix},\quad D=\begin{bmatrix}1.20+0.01\delta\\ 1.00+0.01\delta\end{bmatrix}$$

$$C=\begin{bmatrix}1.30+0.01\delta & -0.60 & -0.10+0.01\delta\\ 0.30+0.01\delta & -0.20 & 0.60+0.01\delta\end{bmatrix},\quad D_0=\begin{bmatrix}-0.60+0.01\delta & 0.10+0.01\delta\\ 0.01\delta & -0.60+0.01\delta\end{bmatrix}$$

$$E=\begin{bmatrix}-0.80+0.01\delta & 0.40 & 0.20+0.01\delta\end{bmatrix},\quad F_0=[-0.30+0.01\delta \;\; 0.20+0.01\delta]$$

$$G=[-1.00 \;\; 0.60 \;\; 0.30],\quad H_0=[-0.40 \;\; 0.30]$$

$$F=-0.10+0.01\delta$$

Consider also the case when the process matrices are perfectly known, i.e. $\delta=0$. Then using Lemma 1, we have that this process is stable along the pass and solving the LMI-based conditions of Theorem 2 in Matlab we obtain that the minimum $\gamma$ as $\gamma^*=0.2225$ and also

$$A_f=\begin{bmatrix}-0.0314 & 0.0139 & 0.0016\\ -0.0068 & -0.0017 & 0.0009\\ -0.0002 & 0.0001 & -0.0031\end{bmatrix},\quad B_{0f}=\begin{bmatrix}0.0046 & 0.0052\\ -0.0013 & -0.0015\\ -0.0015 & -0.0016\end{bmatrix}$$

$$B_f=\begin{bmatrix}0.0222\\ 0.0051\\ 0.0047\end{bmatrix},\quad C_f=\begin{bmatrix}0.0011 & -0.0006 & 0.0003\\ 0.0012 & -0.0006 & 0.0004\end{bmatrix}$$

$$D_{0f}=1.0\times10^{-3}\cdot\begin{bmatrix}-0.3350 & -0.3768\\ -0.3769 & -0.4239\end{bmatrix},\quad D_f=1.0\times10^{-3}\cdot\begin{bmatrix}-0.1145\\ -0.1288\end{bmatrix}$$

$$G_f=[-0.1163 \;\; -0.0419 \;\; -0.0209],\quad H_{0f}=[-0.0186 \;\; -0.0209]$$

$$H_f=1.3953$$

Now we consider the case when $\delta$ is non-zero and satisfies $|\delta|\leqslant 1$. Then in the polytopic uncertainty model for this case the uncertainties in the parameters are represented by a two-vertex polytope and we take the vertices to be at $\delta=1$ and $-1$, respectively. Applying Theorem 3, the minimum



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$\gamma$ obtained is $\gamma^* = 0.2750$, and the corresponding filter matrices are

$$A_f = \begin{bmatrix} -0.0646 & 0.0244 & 0.0043 \\ -0.0111 & -0.0023 & 0.0016 \\ 0.0016 & 0.0012 & -0.0074 \end{bmatrix}, \quad B_{0f} = \begin{bmatrix} 0.0039 & 0.0091 \\ -0.0009 & -0.0021 \\ -0.0012 & -0.0029 \end{bmatrix}$$

$$B_f = \begin{bmatrix} 0.0519 \\ 0.0058 \\ 0.0074 \end{bmatrix}, \quad C_f = \begin{bmatrix} 0.0005 & -0.0004 & 0.0005 \\ 0.0012 & -0.0009 & 0.0011 \end{bmatrix}$$

$$D_{0f} = 1.0 \times 10^{-3} \cdot \begin{bmatrix} -0.1264 & -0.2957 \\ -0.2957 & -0.6917 \end{bmatrix}, \quad D_f = 1.0 \times 10^{-3} \cdot \begin{bmatrix} 0.3932 \\ 0.9199 \end{bmatrix}$$

$$G_f = [-0.1313 \quad -0.0341 \quad -0.0208], \quad H_{0f} = [-0.0078 \quad -0.0183]$$

$$H_f = 1.3798$$

Finally, we consider the norm-bounded uncertainty case when

$$\breve{A} = \begin{bmatrix} -1.45 & 0.64 & -0.40 \\ -0.60 & -1.41 & 0.00 \\ 0.30 & -0.20 & -0.70 \end{bmatrix}, \quad \breve{B} = \begin{bmatrix} 0.60 \\ -1.20 \\ 0.20 \end{bmatrix}, \quad \breve{B}_0 = \begin{bmatrix} 1.30 & 0.10 \\ -0.20 & -0.90 \\ 0.20 & -0.40 \end{bmatrix}$$

$$\breve{C} = \begin{bmatrix} 1.30 & -0.60 & -0.10 \\ 0.30 & -0.20 & 0.60 \end{bmatrix}, \quad \breve{D}_0 = \begin{bmatrix} -0.60 & 0.10 \\ 0 & -0.60 \end{bmatrix}, \quad D = \begin{bmatrix} 1.20 \\ 1.00 \end{bmatrix}$$

$$\breve{F} = -0.10, \quad \breve{E} = [-0.80 \ \ 0.40 \ \ 0.20], \quad \breve{F}_0 = [-0.30 \ \ 0.20]$$

$$M_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad M_3 = 1$$

$$N_3 = 0.01, \quad N_1 = [0.01 \ \ 0 \ \ 0.01], \quad N_2 = [0.01 \ \ 0.01]$$

Using Theorem 4, the minimum $\gamma$ is $\gamma^* = 0.2750$, and the filter state-space model matrices are

$$A_f = \begin{bmatrix} -0.0515 & 0.0232 & 0.0029 \\ -0.0118 & -0.0023 & 0.0016 \\ 0.0015 & 0.0012 & -0.0073 \end{bmatrix}, \quad B_{0f} = \begin{bmatrix} 0.0035 & 0.0082 \\ -0.0009 & -0.0021 \\ -0.0012 & -0.0029 \end{bmatrix}$$

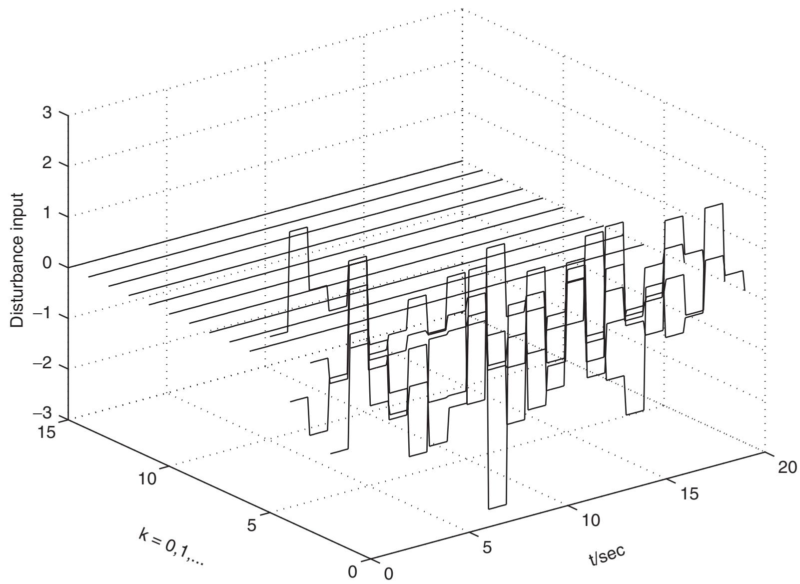


Figure 1. Disturbance $\omega_{k+1}(t)$.

$$B_f = \begin{bmatrix} 0.0416 \\ 0.0064 \\ 0.0074 \end{bmatrix}, \quad C_f = \begin{bmatrix} 0.0006 & -0.0004 & 0.0005 \\ 0.0014 & -0.0009 & 0.0011 \end{bmatrix}$$

$$D_{0f} = 1.0 \times 10^{-3} \cdot \begin{bmatrix} -0.1301 & -0.3044 \\ -0.3044 & -0.7120 \end{bmatrix}, \quad D_f = 1.0 \times 10^{-3} \cdot \begin{bmatrix} -0.0632 \\ -0.1480 \end{bmatrix}$$

$$G_f = [-0.1079 \quad -0.0362 \quad -0.0232], \quad H_{0f} = [-0.0085 \quad -0.0199]$$

$$H_f = 1.3615$$

Now consider the case when the disturbance input $\omega_k(t)$ be

$$\omega_k(t) = \begin{cases} \vartheta(k,\tau), & 0 \leqslant k \leqslant 4 \\ & \tau \leqslant t < \tau + 1, \quad \tau = 0, 1, \ldots, 19 \\ 0 & \text{otherwise} \end{cases}$$

where $\vartheta(k,t)$ is a random variable drawn from a normal distribution with zero mean and unit variance. Suppose also that the boundary conditions are zero, that is $x_{k+1}(0) = 0,\ k \geqslant 0$ and $y_0(t) = 0,\ 0 \leqslant t \leqslant 20$. Figure 1 shows the random disturbance. Figures 2 and 3 show, respectively, the signal to be estimated $v_{k+1}(t)$ and its estimate $\hat{v}_{k+1}(t)$ and Figure 4 the filtering error $e_{k+1}(t)$. (We chose this form of the random variable to illustrate that the filtering error will eventually decay to zero.)

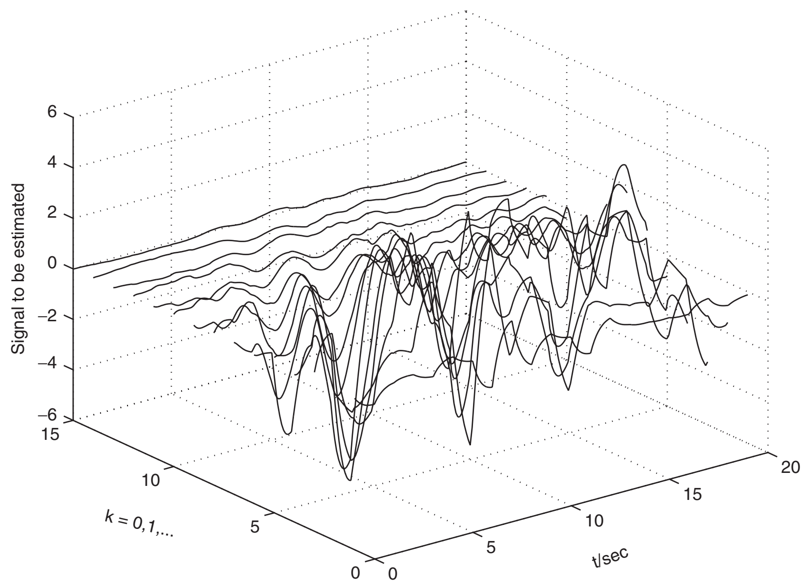


Figure 2. Signal $v_{k+1}(t)$.

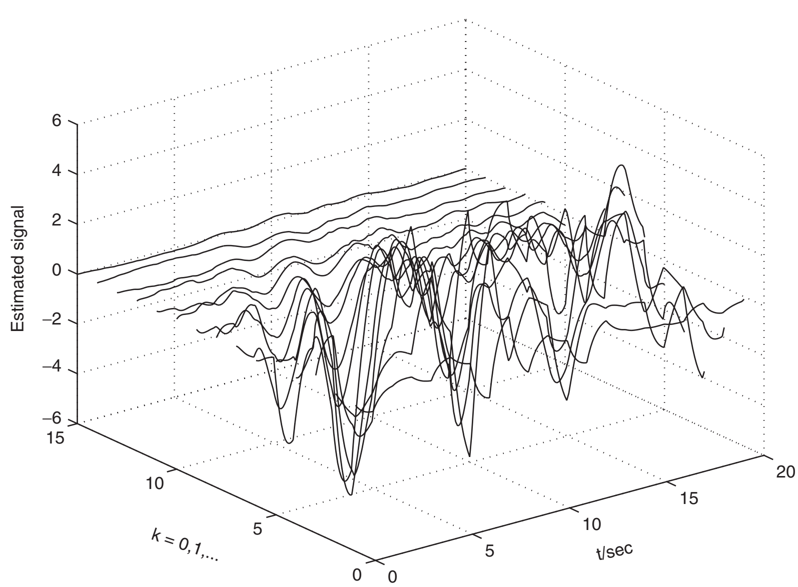


Figure 3. Estimated signal $\hat{v}_{k+1}(t)$.



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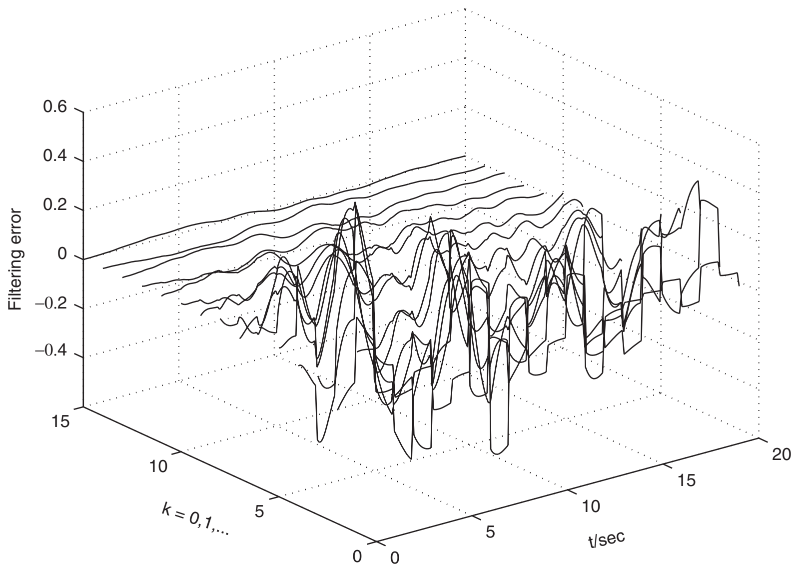


Figure 4. Filtering error $e_{k+1}(t)$.

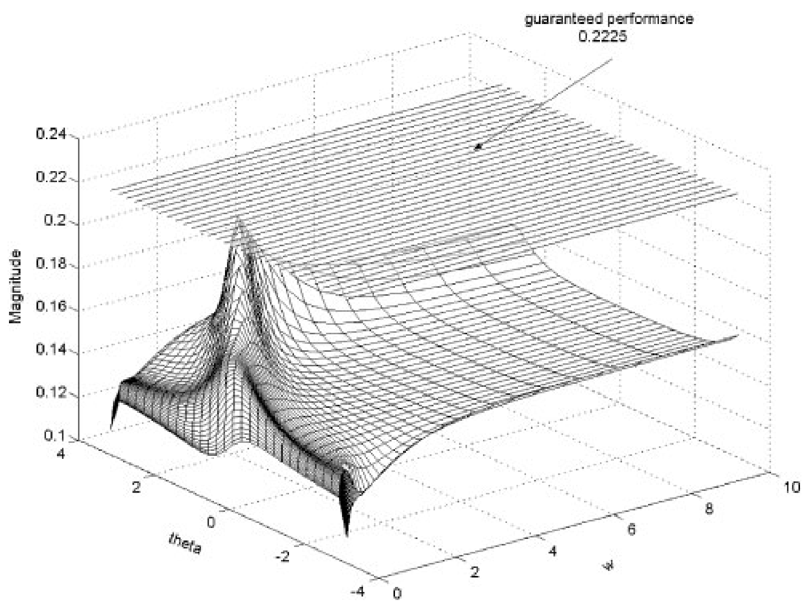


Figure 5. Maximum singular values of the filtering error process.

Figure 5 gives the maximum singular values of the filtering error 2D transfer function

$$T(s,z)=[\tilde{G}\ \ \tilde{H}_0]\begin{bmatrix} sI-\tilde{A} & -\tilde{B}_0 \\ -z\tilde{C} & I-z\tilde{D}_0 \end{bmatrix}^{-1}\begin{bmatrix} \tilde{B} \\ \tilde{D} \end{bmatrix}+\tilde{H}$$



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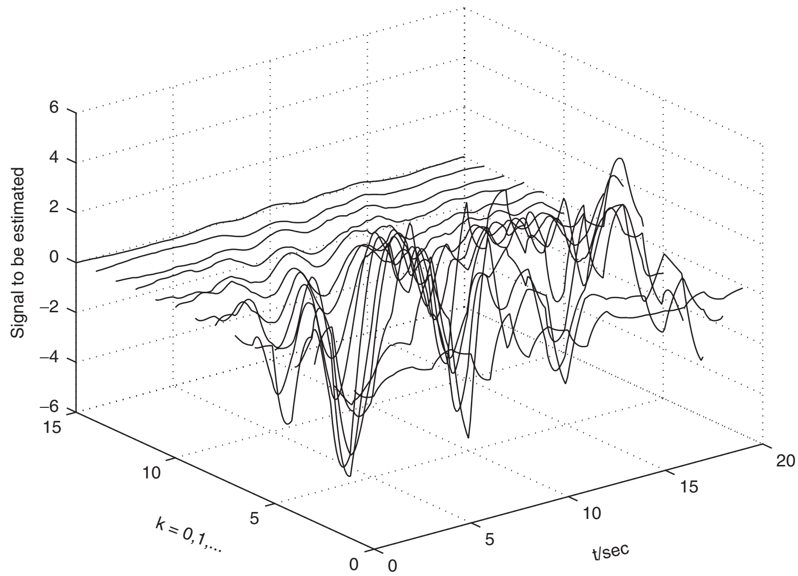


Figure 6. Signal $v_{k+1}(t)$.

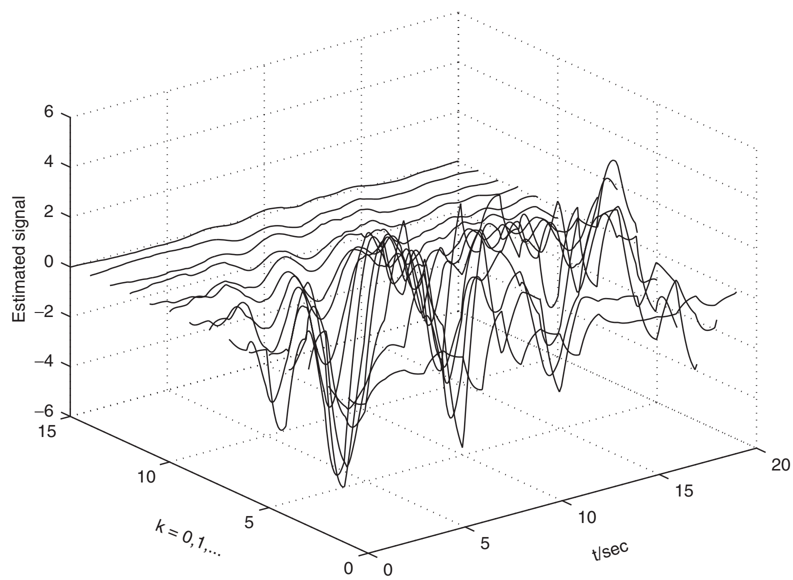


Figure 7. Estimated signal $\hat{v}_{k+1}(t)$.

where $s = \mathrm{j}\omega$, $\omega \in [0, 10]$ and $z = \exp(\mathrm{j}\theta)$, $\theta \in [-\pi, \pi]$, from which we see that the maximum singular value is below its guaranteed $\mathscr{H}_\infty$ performance $\gamma^* = 0.2225$ (the actual achieved maximum singular value is 0.2202).



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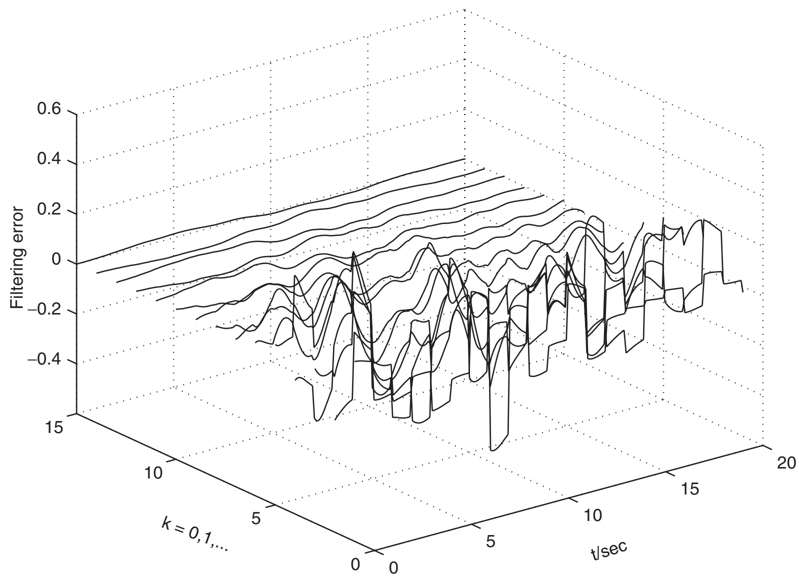


Figure 8. Filtering error $e_{k+1}(t)$.

The corresponding responses for the polytopic uncertainty design (for $\delta = 1$) are shown in Figures 6–8, respectively.

## 6. CONCLUSION

This paper has solved the full-order $\mathscr{H}_\infty$ filtering problem for differential linear repetitive processes, including cases when uncertainty is present in the defining state-space model which is assumed to satisfy one or other of two uncertainty models. These uncertainty models are the repetitive process counterparts of well-known 1D linear systems uncertainty models. The conditions for the existence of the filters are expressed in terms of LMIs and hence are easily computed and lead directly to the filter state-space model matrices. These results are the first on filtering for differential linear repetitive processes and are an important step in extending previously reported control law design algorithms to the (more practically relevant) case when measurements will be corrupted by noise. Further work is required to assess their full usability in filtering and control. Also there is a need to develop alternative filters based, for example, on $\mathscr{H}_2$ or mixed $\mathscr{H}_2/\mathscr{H}_\infty$ settings. (This problem has been solved for 2D discrete linear systems [3], but these results are not applicable here (see also the discussion in the first section of this paper) due to the differential dynamics along the pass and the resetting action before the start of each new pass.) Finally, further research is required on the possibility of reducing any conservativeness in the computations of the results given here and some possible ways forward have been noted in Section 4.

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