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SCALARIZING COST-EFFECTIVE MULTIOBJECTIVE OPTIMIZATION ALGORITHMS MADE POSSIBLE WITH KRIGING

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Abstract ***– The use of kriging in cost-effective single-objective optimization is well established, and a wide variety of different criteria now exist for selecting design vectors to evaluate in the search for the global minimum. Additionly, a large number of methods exist for transforming a multi-objective optimization problem to a single-objective problem. With these two facts in mind, this paper discusses the range of kriging assisted algorithms which are possible (and which remain to be explored) for cost-effective multi-objective optimization.***

Selection Criteria with Kriging Models for Single-Objective Optimization

The use of kriging surrogate models in single-objective optimization is now well established (including within the electromagnetic design community [1]) and a wide variety of methods now exist for exploiting statistical information from them for the purpose of selecting a design vector to evaluate in the search for the global minimum [2]. These include (but are not limited to) evaluating the design vectors which maximize: the probability of improvement (POI) [2]; the expectation value of the improvement (EI) [3]; the generalized expected improvement (GEI) [4]; the weighted expected improvement (WEI) [5]; the credibility of a hypothesis (CH) about the location of the minimum [2] (also known as the one-stage approach); and most recently the 'minimizer entropy' (ME) criterion [6] . Due to lack of space, their exact descriptions are omitted; suffice to say several allow the delicate balance between exploration and exploitation to be controlled through 'cooling' schemes; furthermore many offer the facility for selecting multiple design vectors for evaluation each iteration.

Scalarization of Multi-Objective Optimization Problems

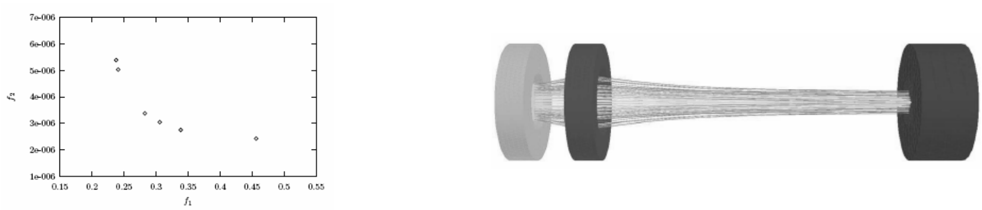
A popular method for solving a multi-objective optimization problem (MOOP) is to transform it to a single-objective optimization problem (SOOP), and then solve the SOOP using a single-objective optimization algorithm. A large number of methods exist for transforming a MOOP to a SOOP including (but again not limited to): ε-constraint (ε-C); weighting method (W); weighted metrics (WM) (including the Tchbycheff metric) method; achievement scalarizing function approach (AF); lexicographic ordering approach (LO); and the value function method (VF), descriptions of all of which may be found in [7]. This wide variety of scalarizing methods, coupled with the large range of selection crietria from single-objective optimization mentioned in the previous paragraph, leads to a plethora of (scalarizing) multi-objective algorithms being possible with kriging; a selection of these are shown in Table 1. Despite this fact, surprisingly few have been explored in the literature: two which have been explored are labelled with their reference.

One of the potential algorithms, marked X in Table 1, was tested on an electromagnetic design problem. The voltage on, and position of, the focus electrode of an electron gun was varied so as to achieve two objectives: to focus the beam of electrons on the center of the anode as much as possible, and to make the electrons hit the anode face as perpendicular as possible. Formally, denoting the voltage on the focus electrode by V Volts, and

Table 1. The family of scalarizing multi-objective optimization algorithms made possible with kriging.

Selection Criteria	**Scalarizing Method**						
	GC	W	ε-C	WM	AF	LO	VF
POI							
EI			[3]	[8]			
GEI							
WEI							
CH							
ME							
Hybrids				X			

its perpendicular distance from the emitting surface by d cm, the objective functions to be minimized are $f_1(V,d) = \int_{\text{anode}} J(r) r^2 \mathrm{d}S$ and $f_2(V,d) = \int_{\text{anode}} \frac{(v_x^2 + v_y^2)}{(v_x^2 + v_y^2 + v_z^2)} \mathrm{d}S$ with $V \in [0,1000]$ and $d \in [4,10]$, where r is the radial distance from the center of the anode surface, $J(r)$ is the current density at r, and the integrals are taken over the surface of the anode. v_x, v_y and v_z are the components of the electron velocities as they hit the surface of the anode, which lies in the xy plane. Each analysis was carried out using the Vector Fields space charge solver, SCALA. The Pareto optimal points found, along with one of the solutions, are shown in Fig.1.



(a) (b)

Fig.1 (a) Pareto optimal front for electron gun problem, and (b) the left-most Pareto-optimal solution (which has the most parallel beam, as measured by objective f_2), as found using algorithm X from Table 1.

Conclusions

Many advances have been made in recent years in kriging-assisted single-objective optimization. Using scalarizing methods, it is possible to also use each in multi-objective optimization. Despite this, relatively few of the possibilities have been explored in the literature. This paper has made it explicit the range of multi-objective algorithms possible with kriging, and demonstrated the use of one of the algorithms on an electromagnetic design problem. Other algorithms from the family of scalarizing algorithms will be explored in the full paper.

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