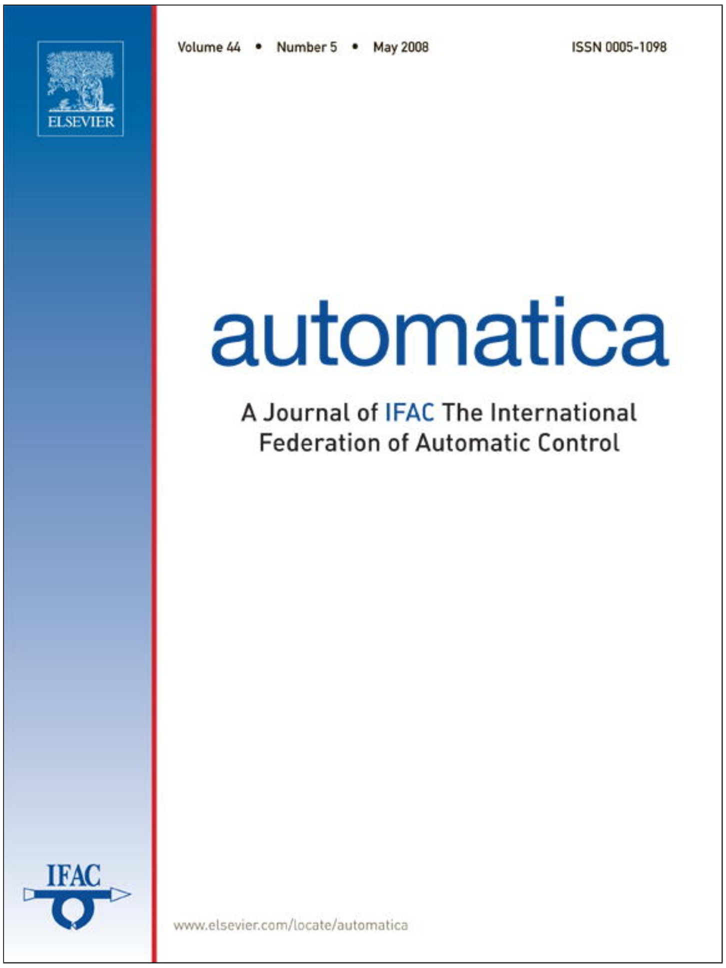
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Technical communique

PI output feedback control of differential linear repetitive processes[☆]

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Abstract

Repetitive processes are characterized by a series of sweeps, termed passes, through a set of dynamics defined over a finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. This can lead to oscillations which increase in amplitude in the pass-to-pass direction and cannot be controlled by standard control laws. Here we give new results on the design of physically based control laws. These are for the sub-class of so-called differential linear repetitive processes which arise in applications areas such as iterative learning control. They show how a form of proportional-integral (PI) control based only on process outputs can be designed to give stability plus performance and disturbance rejection.

Keywords: Repetitive dynamics; Stability; Stabilization; Output controller design; LMI

1. Introduction

The operation of a repetitive process, i.e. a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length can lead to oscillations in the output sequence of pass profiles which increase in amplitude in the pass-to-pass direction. These are caused by the previous pass profile acting as a forcing function on, and hence contributing to, the dynamics of the next pass profile and so on.

Physical examples of these processes include long-wall coal cutting and metal rolling operations. Also in recent years applications have arisen where adopting a repetitive process setting for analysis has distinct advantages over alternatives, e.g. classes of linear iterative learning control (ILC) schemes. For full details in all these cases see the relevant references in Rogers, Gałkowski, and Owens (2007).

Attempts to control these processes using standard algorithms fail, except in a few very restrictive special cases, precisely because such an approach ignores their inherent 2D system structure, i.e. information propagation occurs from pass to pass and along a given pass and also the initial conditions are reset before the start of each new pass. In this paper we consider control in the presence of disturbances for so-called differential linear repetitive processes, where the dynamics in one direction of information propagation is governed by a linear matrix differential equation and in the other by a discrete updating structure. We use $M > 0$ (respectively $M < 0$) to denote a real symmetric positive (respectively negative) definite matrix.

2. Background

Following Rogers et al. (2007) the state-space model of a differential linear repetitive process has the following form over $0 \le t \le \alpha, : k \ge 0$:

$$\dot{x}_{k+1}(t) = Ax_{k+1}(t) + B_0 y_k(t) + Bu_{k+1}(t) + Ew(t)$$
$$y_{k+1}(t) = Cx_{k+1}(t) + D_0 y_k(t) + Du_{k+1}(t) + Fw(t) \qquad (1)$$

where $\alpha < +\infty$ denotes the pass length, and on pass k, $x_k(t) \in \mathbb{R}^n$ is the state vector, $y_k(t) \in \mathbb{R}^m$ is the pass profile vector, $u_k(t) \in \mathbb{R}^r$ is the input vector, and $w(t) \in \mathbb{R}^q$ denotes the

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disturbance vector which is assumed to be constant from pass to pass but can evolve dynamically along the pass. Consequently the disturbances here are periodic with the period equal to the pass length α. This is a restriction (see also the conclusions section of this paper), but in some industrial situations, such as long-wall coal cutting, is not unrealistic since in this application the disturbance signal on the floor thickness measurement device can be adequately modelled in this manner. We also assume, as in most practical applications, that $n \geq m \geq r$. Also the boundary conditions are taken as a given initial pass profile $y_0(t), 0 \leq t \leq \alpha$ and $x_{k+1}(0) = d_{k+1}, : k \geq 0$, where d_{k+1} has known constant entries.

Note 1. To be clear, 'pass to pass' here means information propagation in k and 'along the pass' the same in t.

This state-space model allows for disturbances which affect both the state and pass profile dynamics on each pass. The stability theory (Rogers et al., 2007) for linear repetitive processes is based on an abstract model in a Banach space setting which includes a wide range of such processes as special cases, including those described by (1). In terms of their dynamics it is the pass-to-pass coupling, noting again the unique control problem for them, which is critical. This is of the form $y_{k+1} = L_\alpha y_k$, where $y_k \in E_\alpha$, where E_α a Banach space with norm $\|\cdot\|$, and L_α is a bounded linear operator mapping E_α into itself.

Given the unique control problem, the stability theory for $y_{k+1} = L_\alpha y_k$ requires that the sequence of pass profiles generated converges strongly to zero as $k \to \infty$. Since the pass length α can take any finite value, this holds if and only if there exist numbers $M_\infty > 0$ and $\lambda_\infty \in (0, 1)$ independent of α such that $\|L_\alpha^k\| \leq M_\infty \lambda_\infty^k, : k \geq 0$, where $\|\cdot\|$ also denotes the induced operator norm. In the case when control inputs and/or disturbances are present, stability along the pass results in convergence to a so-called limit profile whose dynamics is uniformly bounded in the along the pass direction.

It is of interest to relate this theory to a physical example in the form of long-wall coal cutting where the pass profile is the thickness, relative to a fixed datum, of the coal left after the cutting machine has moved along the pass length, i.e. the coal face. The stability problem here is caused by the machine's weight as it rests on the previous pass profile during the cutting of the next pass profile. The undulations caused can be very severe and result in productive work having to stop to enable them to be removed. Stability along the pass here means that after a sufficient number of passes have elapsed the profiles produced on each successive pass are the same.

It can be shown from results in Rogers et al. (2007) that stability along the pass holds for (1) if, and only if,

$$\mathcal{C}(s, z_2) = \det(sI - \widehat{A}_1 - z_2\widehat{A}_2) \neq 0 \text{ in } \mathbb{I} \times \overline{U} \tag{2}$$

where $\overline{U} = \{z_2 \in \mathbb{C} : |z_2| \leq 1\}$, $\mathbb{I} = \{s \in \mathbb{C} : Re(s) \geq 0\}$, and

$$\widehat{A}_1 = \begin{bmatrix} A & B_0 \\ 0 & 0 \end{bmatrix}, \qquad \widehat{A}_2 = \begin{bmatrix} 0 & 0 \\ C & D_0 \end{bmatrix}.$$

In this case the resulting limit profile, $y_\infty(t)$, is described by a 1D differential linear system state-space model with state matrix, setting $D = 0$ for simplicity, $A + B_0(I_m - D_0)^{-1}C$, which is also guaranteed to be stable.

In this work, we use the following Linear Matrix Inequality (LMI) based sufficient condition derived from (2) which, unlike all other existing stability tests, leads immediately (see below) to systematic methods for control law design.

Lemma 1 (Gałkowski et al., 2003). *A differential linear repetitive process described by (1) is stable along the pass if $\exists$ matrices $Y > 0$ and $Z > 0$ satisfying the following LMI:*

$$\begin{bmatrix} YA^T + AY & (*) & (*) \\ ZB_0^T & -Z & (*) \\ CY & D_0Z & -Z \end{bmatrix} < 0. \tag{3}$$

In this paper we use $(*)$ to denote off-diagonal symmetric entries in LMIs. See also Rogers et al. (2007) for a detailed treatment of the sufficient only condition in this result.

3. PI control

In terms of acceptable, or desired, performance from a given example, it is clear that stability along the pass must hold and how to ensure this property has been the subject of recent research (Gałkowski et al., 2003). Suppose first that control action is only to be based on current pass action, e.g. a state feedback control law of the form $u_{k+1}(t) = Gx_{k+1}(t)$. Then examples are easily generated where the resulting controlled process cannot be stable along the pass. Likewise for the use of a control law based on previous pass information alone, e.g. $u_{k+1}(t) = Hy_k(t)$ applied to any example where all eigenvalues of the matrix A do not have strictly negative real parts. Hence control laws must be activated by an appropriate combination of current and previous pass information.

If a process described by (1) is stable along the pass then the resulting limit profile $y_\infty(t)$ is, as noted above described by a standard differential linear systems state-space model. In this paper, the design objective is stability along the pass and a resulting $y_\infty(t)$ which has acceptable dynamics despite the presence of the disturbance. More precisely, we want to produce a limit profile which has been pre-specified by interpreting the design specifications. Referring back to the coal cutting example, this would be to drive the coal cutting machine for eventually producing the same floor profile on each successive pass and hence maximizing productive work.

With the additional objective of avoiding undue control law complexity, we now develop a solution based on a repetitive process version of proportional plus integral or PI control.

Define for pass k and 'position' $t \in [0, \alpha]$ along this pass the so-called total tracking error $\chi_k(t)$ as

$$\chi_k(t) = \sum_{j=0}^{k} \left(y_j(t) - y_{\text{ref}}(t) \right),$$

where $y_{\text{ref}}(t), 0 \leq t \leq \alpha$, denotes the required reference signal. Then it follows immediately on using (1) that

$$\chi_{k+1}(t) = \chi_k(t) + Cx_{k+1}(t) + Du_{k+1}(t) + D_0y_k(t) + Fw(t) - y_{\text{ref}}(t). \tag{4}$$

Note here that the pass profile vectors are the process outputs and hence available for measurement. We also assume that these vectors are not subject to undue levels of measurement noise etc. The control law here does not require an observer to reconstruct the current pass state vector (most of the currently available control law design methods assume direct access to all entries in the current pass state vector).

Previous work Sulikowski, Gałkowski, Rogers, and Owens (2006) has considered the design of PI control laws for discrete linear repetitive processes but the results here do not follow by substitution, i.e. simple rearrangement of these previous results. In particular, the resulting LMI is directly related to a Lyapunov function in each case (Rogers et al., 2007). The Lyapunov function here is a quadratic in $x_{k+1}(t)$ (differential updating) plus a quadratic in $y_k(t)$ (discrete updating).

Now, introduce the so-called extended pass profile vector as

$$z_k(t) = \begin{bmatrix} y_k(t)^T & \chi_k(t)^T \end{bmatrix}^T. \tag{5}$$

Then use of the second equation of (1) together with (4) yields the following state-space model of the so-called augmented differential linear repetitive process:

$$\begin{aligned} \dot{x}_{k+1}(t) &= Ax_{k+1}(t) + \hat{B}_0 z_k(t) + Bu_{k+1}(t) + Ew(t) \\ z_{k+1}(t) &= \hat{C}x_{k+1}(t) + \hat{D}_0 z_k(t) + [0 - I]^T y_{\text{ref}}(t) \\ &\quad + \hat{D}u_{k+1}(t) + \begin{bmatrix} F^T F^T \end{bmatrix}^T w(t) \end{aligned} \tag{6}$$

where $\hat{B}_0 = \begin{bmatrix} B_0 & 0 \end{bmatrix}$, $\hat{C} = \begin{bmatrix} C^T & C^T \end{bmatrix}^T$, $\hat{D}_0 = \begin{bmatrix} D_0 & 0 \\ D_0 & I \end{bmatrix}$, $\hat{D} = \begin{bmatrix} D^T & D^T \end{bmatrix}^T$. Suppose that as $k \to \infty$, $x_k(t) \to x_\infty(t)$, $u_k(t) \to u_\infty(t)$ and $y_k(t) \to y_{\text{ref}}(t)$, $\chi_k(t) \to \chi_\infty(t)$ (hence $z_k(t) \to z_\infty(t)$). Define also the following incremental vectors:

$$\begin{aligned} \hat{z}_k(t) &= z_k(t) - z_\infty(t), \qquad \hat{u}_k(t) = u_k(t) - u_\infty(t) \\ \hat{x}_k(t) &= x_k(t) - x_\infty(t). \end{aligned} \tag{7}$$

Then it is straightforward to obtain (by subtracting the extended steady state model equations from (6) and applying (7))

$$\begin{aligned} \dot{\hat{x}}_{k+1}(t) &= A\hat{x}_{k+1}(t) + \hat{B}_0 \hat{z}_k(t) + B\hat{u}_{k+1}(t) \\ \hat{z}_{k+1}(t) &= \hat{C}\hat{x}_{k+1}(t) + \hat{D}_0 \hat{z}_k(t) + \hat{D}\hat{u}_{k+1}(t) \end{aligned} \tag{8}$$

and hence the disturbance term $w(t)$ is completely decoupled from the process dynamics.

The only problem in the above analysis is that (8) is not stable along the pass since $\hat{D}_0$ has at least some eigenvalues of modulus unity and hence (2) cannot hold. This means that no limit profile can exist and hence design can only proceed by application of a control law designed to give stability along the pass.

For the incremental model of (8), define the output (or pass profile) only actuated control law as

$$\begin{aligned} \hat{u}_{k+1}(t) &= \widetilde{K}_1 \hat{z}_{k+1}(t) + \widetilde{K}_2 \hat{z}_k(t) \\ &= \widetilde{K}_{11} \hat{y}_{k+1}(t) + \widetilde{K}_{12} \hat{\chi}_{k+1}(t) + \widetilde{K}_{21} \hat{y}_k(t) + \widetilde{K}_{22} \hat{\chi}_k(t). \end{aligned} \tag{9}$$

Now we have the following result.

Theorem 1. *Suppose that a control law of the form (9) is applied to (8). Then the resulting process is stable along the pass if there exist matrices $\hat{Y} > 0$, $\hat{Z} > 0$, $\hat{X} > 0$, $\hat{M}$ and $\hat{N}$ such that the following LMI holds:*

$$\begin{bmatrix} \hat{Y}A^T + A\hat{Y} + \hat{C}^T \hat{N}^T B^T + B\hat{N}\hat{C} & (*) & (*) \\ \hat{Z}\hat{B}_0^T + \hat{M}^T B^T & -\hat{Z} & (*) \\ \hat{C}\hat{Y} + \hat{D}\hat{N}\hat{C} & \hat{D}_0 \hat{Z} + \hat{D}\hat{M} & -\hat{Z} \end{bmatrix} < 0 \tag{10}$$

with the equation constraint $\hat{X}\hat{C} = \hat{C}\hat{Y}$. If this condition holds, the matrices L_x and K_z are given by

$$L_x = \hat{N}\hat{X}^{-1}, \qquad K_z = \hat{M}\hat{Z}^{-1} \tag{11}$$

and hence in (9)

$$\begin{aligned} \widetilde{K}_1 &= L_x (I + \hat{D}L_x)^{-1} \\ \widetilde{K}_2 &= [I - L_x (I + \hat{D}L_x)^{-1} \hat{D}] K_z - L_x (I + \hat{D}L_x)^{-1} \hat{D}_0. \end{aligned} \tag{12}$$

Proof. Simply note that (8) is of the form (1) (with $w(t) = 0$). Then from (11) we have that $\hat{N} = L_x \hat{X}$ and substitution into the LMI of (10) with $\hat{X}\hat{C} = \hat{C}\hat{Y}$ applied yields

$$\begin{bmatrix} \hat{Y}(A^T + \hat{C}^T L_x^T B^T) + (A + BL_x \hat{C})\hat{Y} & (*) & (*) \\ \hat{Z}\hat{B}_0^T + \hat{M}^T B^T & -\hat{Z} & (*) \\ (\hat{C} + \hat{D}L_x \hat{C})\hat{Y} & \hat{D}_0 \hat{Z} + \hat{D}\hat{M} & -\hat{Z} \end{bmatrix} < 0.$$

Finally, set $L_x \hat{C} = K_x$ to obtain an LMI which is just that of (3) interpreted for the controlled process and the proof is complete. □

The equation constraint in this last result places no restrictions on the results developed here when using, e.g., Scilab LMITOOL but could be a source of difficulty in other cases, e.g. in uncertainty analysis where the resulting robust control problem may not be convex.

To implement (9), note that it can also be written as

$$\begin{aligned} u_{k+1}(t) &= (\widetilde{K}_{11} + \widetilde{K}_{12}) y_{k+1}(t) + \widetilde{K}_{21} y_k(t) \\ &\quad + (\widetilde{K}_{22} + \widetilde{K}_{12}) \chi_k(t) - \widetilde{K}_{12} y_{\text{ref}}(t). \end{aligned} \tag{13}$$

The form of the control law is proportional (arising from the $y_k(t)$) plus integral (arising from the $\chi_k(t)$ terms). Note also that this last result may have use in ILC designs for multivariable plants, a subject which is not yet fully resolved and is left here for further research.

As a numerical example, to also highlight channel interaction effects in the light of the last comment, consider the case when

$$A = \begin{bmatrix} -3.6 & 0.2 & -1.4 & -1.7 \\ 0.1 & -6.3 & 1.6 & 1.9 \\ 2.9 & -0.2 & -4.9 & -5.7 \\ -4.4 & -1.4 & -2.7 & -1.1 \end{bmatrix},$$

$$B = \begin{bmatrix} 1.7 & -2.2 & 1.9 \\ 1.4 & -3.4 & -3.4 \\ 1.3 & -2 & -1.6 \\ 0.5 & -2.1 & 2.8 \end{bmatrix},$$

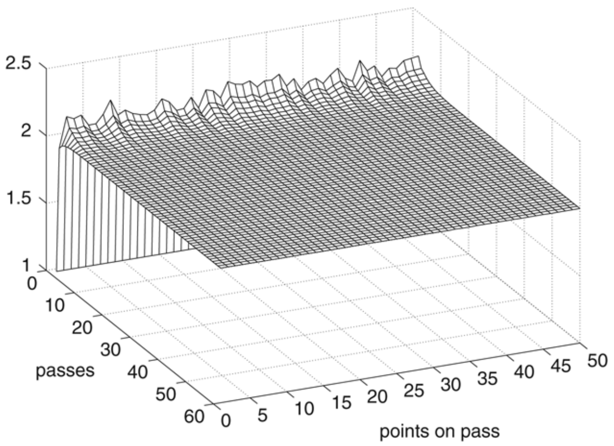

Fig. 1. First channel response.

$$B_0 = \begin{bmatrix} -1 & 1 \\ 0.6 & 0.6 \\ 0.5 & 0.9 \\ -0.3 & 0 \end{bmatrix}, \qquad E = \begin{bmatrix} 1 \\ 0.9 \\ 0.6 \\ -0.1 \end{bmatrix},$$

$$C = \begin{bmatrix} -0.5 & -0.4 & -0.2 & -0.1 \\ 0 & -0.6 & 0.4 & -0.6 \end{bmatrix},$$

$$D_0 = \begin{bmatrix} -1.4 & -0.4 \\ -0.1 & 0 \end{bmatrix},$$

$$D = \begin{bmatrix} -4.3 & -5.7 & -5.5 \\ 4 & 4.7 & -4.7 \end{bmatrix}, \qquad F = \begin{bmatrix} -1.7 \\ 1.3 \end{bmatrix}.$$

over the pass length $\alpha = 50$ when the entries in the disturbance vector $w(t)$ have been randomly generated using a uniform distribution with range ± 1.

In order to assess the quality of the controlled process performance we focus on the fact that the limit profile is described by a standard differential linear system and follow the standard route of using a step signal applied in each of the two channels in turn. Fig. 1 shows the response to the case when $y_{\text{ref}}(t) = [\ 2 \quad 0\ ]^{\text{T}}, 0 \le t \le 50$. Here interaction in the second channel is initially present but critically the process converges relatively quickly to the limit profile which has the required dynamics along the pass and, in particular, the integral term completely kills off the interaction. (The boundary conditions here are $d_{k+1} = [-0.610.36 - 0.390.08]^{\text{T}}$ and $f(t) = [11]^{\text{T}}, 0 \le t \le 50$.)

4. Conclusions

The results in this paper show that a previously known stabilization design based on an LMI setting can be extended to allow the design of the control law to also meet performance specifications for differential linear repetitive processes. The tools used have their origins in robust control theory (see e.g. Scherer, Gahinet, and Chilali (1997)), and yield relatively simple structure control laws and hence onward benefits in terms of implementation costs. Extending the results in Scherer et al. (1997) to the repetitive process setting could allow treatment of disturbances which are not constant in the pass-to-pass direction by developing H_∞/H_2 attenuation results.

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