Layered steered space-time codes and their capacity

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A capacity analysis of a multifunctional multiple-input multiple-output system that combines the benefits of vertical Bell Labs space-time scheme, space-time block codes and beamforming is presented.

Introduction: Recent information theoretic studies [1] have revealed that employing a multiple-input multiple-output (MIMO) scheme significantly increases the capacity of the system. In [2], Wolniansky *et al.* proposed the popular multilayer MIMO structure, known as the vertical Bell Labs layered space-time (V-BLAST) scheme that is capable of providing a tremendous increase of a specific user's effective bit-rate, since it was designed for achieving a MIMO-aided multiplexing gain. On the other hand, space-time codes (STC) [3] were designed for a high diversity gain. Furthermore, it was proposed in [4] to combine the benefits of these two techniques for the sake of providing both diversity as well as multiplexing gains. Moreover, beamforming [5] constitutes an effective technique for increasing the antenna gain in the desired user's direction and thus minimising the effects of interference in the other users' directions. In this Letter, we propose a new generalised MIMO, which we refer to as layered steered space-time code (LSSTC) that combines the benefits of V-BLAST, STC and beamforming for improving the achievable system performance and deriving the capacity limits for this powerful new scheme.

Layered steered space-time codes: A high-level block diagram of the proposed scheme is shown in Fig. 1. The antenna architecture employed in Fig. 1 has M transmit antenna arrays (AA) spaced sufficiently far apart in order to experience independent fading and hence achieve transmit diversity. A number of elements L of each of the AAs are spaced at a distance of $d=\lambda/2$ for the sake of achieving beamforming. Furthermore, the receiver is equipped with $N \geq M$ antennas. According to Fig. 1, a block of B input information bits is serial-to-parallel converted to K groups of bit streams of length B_1, $B_2, \ldots, B_K$, where $B_1+B_2+\ \ldots\ +B_K=B$. Each group of B_k bits, $k \in [1,\ K]$, is then encoded by a component space-time code STC_k associated with m_k transmit AAs, where $m_1+m_2+\ldots+m_K=M$. We consider transmissions over a correlated narrowband Rayleigh fading channel, associated with a normalised Doppler frequency of $f_D=f_dT_s=0.01$, where f_d is the Doppler frequency and T_s is the symbol duration. The complex additive white Gaussian noise (AWGN) has a zero mean and a variance of $N_0/2$ per dimension.

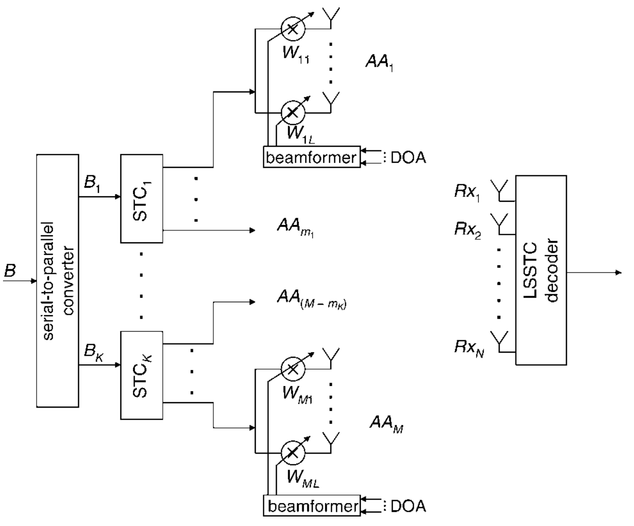

Fig. 1 *Proposed system model*

The L-dimensional spatio-temporal (ST) channel impulse response (CIR) vector spanning the mth transmitter AA, $m \in [1, \ldots, M]$, and the nth receiver antenna, $n \in [1, \ldots, N]$, can be expressed as $\boldsymbol{h}_{nm}(t) = {}_{nm}(t) = \boldsymbol{a}_{nm}(t)\delta(t-\tau_k) = [a_{nm,0}(t), \ldots, a_{nm,(L-1)}(t)\tau_k)$, where τ_k is the signal's delay, $a_{nm,l}(t)$ is the CIR with respect to the mnth link and the lth element of the mth AA. Based on the assumption that the array elements are separated by half a wavelength, we have $\boldsymbol{a}_{nm}(t) = \alpha_{nm}(t) \cdot \boldsymbol{d}_{nm}$, where $\boldsymbol{a}_{nm}(t)$ is a Rayleigh faded envelope, $\boldsymbol{d}_{nm} = [1, \exp(j[\pi\ \sin(\Psi_{nm})]), \ldots,\ \exp(j[(L-1)\pi\ \sin(\Psi_{nm})])]^T$ and Ψ_{nm} is the nmth link's direction of arrival (DOA).

The received baseband data matrix $\boldsymbol{Y}$ can be expressed as $\boldsymbol{Y}=\boldsymbol{HWX}+\boldsymbol{N}$, where $\boldsymbol{N}$ denotes the AWGN matrix and $\boldsymbol{H}$ is an $(N \times M)$ matrix whose entries are $\boldsymbol{h}_{nm}$. Furthermore, $\boldsymbol{W}$ is a diagonal weight matrix, whose diagonal entry $\boldsymbol{w}_{mn}$ is the L-dimensional weight vector for the mth beamformer AA and the nth receive antenna. Let $\boldsymbol{w}_{mn} = \boldsymbol{d}^{\dagger}_{nm}$; then the received signal can be expressed as $\boldsymbol{Y}=L\tilde{\boldsymbol{H}}\boldsymbol{X}+\boldsymbol{N}$, where $\tilde{\boldsymbol{H}}$ is an $(N \times M)$ matrix whose entries are α_{nm}. Moreover, $\boldsymbol{Y}$ can be written as $\boldsymbol{Y}=L\sum_{k=1}^{K}\tilde{\boldsymbol{H}}_k\boldsymbol{x}_k+\boldsymbol{N}$, where $\boldsymbol{x}_k$ represents the component STC used at layer k, with $k \in\ [1, \ldots, K]$. The most beneficial decoding order of the STC layers is determined on the basis of detecting the higher-power layer first for the sake of a higher correct detection probability. For simplicity let us consider the case of $K=2$ STBC layers and that layer 1 is detected first, which allows us to eliminate the interference caused by the signal of layer 2. However, the proposed concept is applicable to arbitrary STCs. For this reason, the decoder of layer 1 has to compute a matrix $\boldsymbol{Q}$, so that we have $\boldsymbol{Q}\ \cdot\ \tilde{\boldsymbol{H}}_2=0$. Therefore, the decoder computes an orthonormal basis for the left null space of $\tilde{\boldsymbol{H}}_2$ and assigns the vectors of the basis to the rows of $\boldsymbol{Q}$. Multiplying $\boldsymbol{Q}$ by $\boldsymbol{Y}$ suppresses the interference of layer 2 originally imposed on layer 1 and generates a signal, which can be decoded using maximum likelihood STBC detection. Then, the decoder subtracts the remodulated contribution of the decoded symbols of layer 1 from the composite twin-layer received signal. Finally, the decoder applies direct STBC decoding to the second layer, since the interference imposed by the first layer has been eliminated. This group-interference cancellation procedure can be generalised to any M and K values.

Capacity analysis: Upon using the decoding order of $(1, 2, \ldots, K)$, group k will have a diversity order of $m_k \times (N-M+m_1+m_2+\ldots+m_k) = m_k \times N_k$. Thus, the LSSTC-decoded signal of layer k can be described as $y_k = L\sum_{\tau=1}^{N_k}\ \sum_{t=1}^{m_k}\ \alpha_{rt}\ \ x_k+\Delta_k=\sum_{r=1}^{N_k}\ \chi^2_{2m_kr}\ \ x_k+\Delta_k$, where $\chi^2_{2m_kr}=L\sum_{t=1}^{m_k}\ \alpha_{rt}$ represents a chi-squared distributed random variable having $2m_k$ degrees of freedom and Δ_k is the AWGN after decoding having a noise variance of $\chi^2_{2m_kr}\ N_0/2$ per dimension. Let $y=(y_1,\ y_2,\ \ldots,\ y_K)$, $x=(x_1,\ x_2, \ldots, x_K)$ and note that with K groups there are $D=F^K$ number of possible F-ary phasor combinations where F-ary signalling is used for transmission. Thus, the achievable capacity of the proposed MIMO system for transmission over the discrete-input continuous-output memoryless channel (DCMC) can be derived from that of the discrete memoryless channel as [6, 7]:

$$C_{DCMC} = \max_{p(x_1),\cdots,p(x_D)} \sum_{d=1}^{D} \int_{-\infty}^{+\infty} p(y/x_d) \cdot p(x_d) \times \log_2\left(\frac{p(y/x_d)}{\sum_{v=1}^{D} p(y/x_v)\cdot p(x_v)}\right) \cdot dy[\text{bit/sym}] \tag{1}$$

where x_d represents the dth phasor out of the D possible phasor combinations, and

$$p(y/x) = \prod_{k=1}^{K} p(y_k/x_k) \text{ and } p(y_k/x_k) = \frac{1}{\pi N_0 \sum_{r=1}^{N_k} \chi^2_{2m_kr}} \exp\left(\sum_{r=1}^{N_k} \frac{-(y_k - \chi^2_{2m_kr}x_k)}{\chi^2_{2m_kr}N_0}\right)$$

Furthermore, C_{DCMC} in (1) is maximised, when the transmitted symbols are equiprobably distributed, i.e. $p(x_d)=1/D$. Finally, (1) can be simplified to

$$C_{DCMC} = \log_2(D) - \frac{1}{D}\sum_{d=1}^{D} E \times \left[\log_2\left(\sum_{v=1}^{D} exp(\Psi_{dv})\right)\Big|x_d\right][\text{bit/sym}] \tag{2}$$

where $E[A|B]$ is the expectation of A conditioned on B and

$$\Psi_{dv} = \sum_{k=1}^{K}\sum_{r=1}^{N_k} - \frac{|\chi^2_{2m_k r}(x_{dk} - x_{vk}) + \Delta_k|^2 + |\Delta_k|^2}{\chi^2_{2m_k r} N_0} \quad (3)$$

where x_{dk} and x_{vk} represent the kth element in the vectors x_d and x_v, respectively. Furthermore, the continuous-input continuous-output memoryless channel (CCMC) capacity of the proposed LSSTC scheme can be expressed as [6, 7]:

$$C_{CCMC} = \sum_{k=1}^{K} E\left[\log_2\left(1 + \sum_{r=1}^{N_k} \chi^2_{2m_k r}\frac{SNR}{m_k}\right)\right][\text{bit/sym}] \quad (4)$$

where we have $SNR = K \cdot R_{STC} \cdot \log_2(F) \cdot E_b/N_0$ and R_{STC} is the rate of the specific STC used.

Results: We consider a system employing $M \times N = 4 \times 4$ antennas and $K = 2$ layers, in order to demonstrate the performance improvements achieved by a downlink (DL) scheme where a BS employing $M = 4$ transmit antennas is communicating with a laptop receiver employing $N = 4$ back plane antennas. The system employs QPSK modulation and considers transmission over a correlated Rayleigh fading channel. Fig. 2 shows the effect of increasing the DL BS beamforming gain by increasing the number of beam-steering elements L in the AA, while maintaining the same number of AAs. As shown in the Figure, when the number of beam-steering elements L increases, the achievable BER performance substantially improves. Fig. 3 quantifies the channel capacity limits of the proposed LSSTC scheme employing QPSK, $M = N = 4$ and a variable value of L.

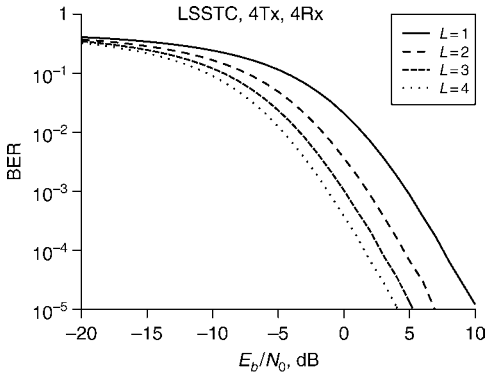

Fig. 2 *BER performance of QPSK modulated $M \times N = 4 \times 4$ LSSTC system for variable L*

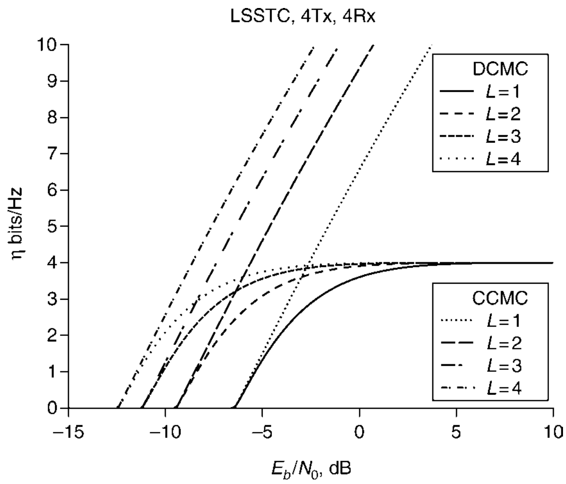

Fig. 3 *Capacity of QPSK modulated $M \times N = 4 \times 4$ LSSTC system for variable L*

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