LUBY TRANSFORM CODING AIDED ITERATIVE DETECTION FOR DOWNLINK SDMA SYSTEMS

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ABSTRACT

A Luby Transform (LT) coded downlink Spatial Division Multiple Access (SDMA) system using iterative detection is proposed, which invokes a low-complexity near-Maximum-Likelihood (ML) Sphere Decoder (SD). The Ethernet-based Internet section of the transmission chain inflicts random packet erasures, which is modelled by the Binary Erasure Channel (BEC), while the wireless downlink imposes both fading and noise. A novel Log-Likelihood Ratio based packet reliability meric is used for identifying the channel-decoded packets, which are likely to be error-infested. Packets having residual errors must not be passed on to the LT decoder for the sake of avoiding LT-decoding-induced error propagation. The proposed scheme is capable of maintaining an infinitesimally low packet error ratio in the downlink of the wireless Internet for E_b/N_0 values in excess of about 3dB.

1. INTRODUCTION

In the conventional Internet, the Transport Control Protocol (TCP) is used for requesting the source to retransmit packets that were lost owing to statistical multiplexing. In addition to the packet-loss events inflicted by statistical multiplexing, in the wireless Internet packets may be corrupted owing to the hostile channel conditions encountered and hence the retransmission overhead may become excessive. A particular problem in hostile wireless channels is that the flood of packet acknowledgements requested by the plethora of Mobile Station (MS) may overwhelm not only the Base Station (BS) but the entire network. This deficiency may severely limit the effective data throughput of the system. Thus the provision of reliable packet transmission – preferably without invoking an acknowledgement-based retransmission mechanism – is vital. In 2002, Luby Transform (LT) codes [1] were proposed for solving this problem in the context of the conventional Internet, where the main cause of packet erasure is statistical multiplexing-induced collisions, rather than fading and co-channel interference. In order to fill the packet erasures, LT codes utilize a certain fraction of redundant packets and, as a benefit, we can dispense with any packet acknowledgements. In other words, given a sufficiently high fraction of redundant packets, the LT decoder becomes capable of detecting all packets without retransmissions.

The efficient design of the down-link wireless transmitter is of paramount importance for the sake of achieving a high throughput. The family of Multiple Input and Multiple Output (MIMO) systems employing multiple antennas at both the transmitter and receiver have the ability to satisfy these challenging requirements, since they exhibit a substantially higher spectral efficiency than conventional single-antenna aided systems. The flexible configuration of a MIMO system's antennas allows us to satisfy a number of potentially contradictory design objectives in terms of the achievable multiplexing and diversity gain [2,3]. In this context Spatial Division Multiple Access (SDMA) constitutes an attractive MIMO subclass, which is capable of achieving a high user capacity by supporting a multiplicity of subscribers within the same frequency bandwidth [4,5].

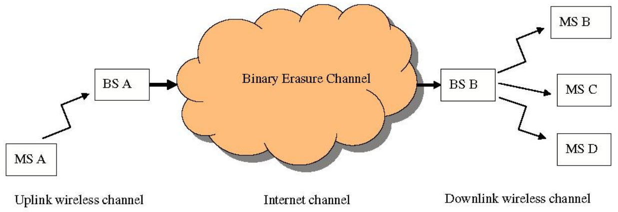

Fig. 1. Data transmission scheme

In this treatise we considered the system configuration illustrated in Figure 1 and studied an LT-coding

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aided Down-Link SDMA system (LT-DL-SDMA). As shown in Figure 1, we considered the scenario, when Mobile Station (MS) B would like to receive data packets from MS A, where BS A and B are connected to each other via either the conventional Ethernet-based Internet or by a dedicated optical fiber backbone. In this study we consider the Ethernet-based Internet aided scenario, which inflicts packets erasures owing to statistical multiplexing-induced packet collision events. First, the data packets are generated either by a conventional Ethernet-based terminal or transmitted from MS A via the wireless uplink channel to BS A. Then the data packets are routed through the statistical multiplexing aided Ethernet-based BS-BS channel, where some packets may be lost. These packet-loss events can be modeled by the Binary Erasure Channel (BEC) [6], which is characterized by a single parameter, namely by its packet dropping probability P_e. Finally, BS B receives the data packets from the Internet and then transmits them via the wireless downlink channel to MS B.

As shown in Figure 2, the LT packets generated by MS A and received by BS A are conveyed first via the Ethernet-based, lossy Internet modeled by the BEC channel, while BS B at the output of BEC channel forwards the LT packets to the target user. Since we focus our attention on the DL transceiver design in this treatise, we assume that the packet source is either a fixed terminal correctly connected to the conventional Ethernet or, alternatively that no impairments are imposed by the uplink wireless channel between MS A and BS A. In our DL transmitter design, the LT packets are forwarded to the convolutional channel encoder of Figure 2 followed by an interleaver, as well as a unity-rate inner encoder [7] and finally to the modulator. The modulated signal is transmitted by the Multi-User DL-SDMA scheme of Figure 3, which employs a spatio-temporal SDMA DL pre-processing technique proposed by Choi and Murch [8], allowing for a specific user to receive his/her dedicated signal, essentially free from Multi-user Interference (MUI) inflicted by other users. In order to maintain a high data throughput for the system, a novel detector suitable for the so-called rank-deficient scenario is considered in the proposed DL-SDMA system [9], where the number of transmitters exceeds the number of receivers. While linear detectors, such as the Minimum Mean Square Error (MMSE) detector [4] were shown to perform unsatisfactorily in high-throughput rank-deficient scenarios, the novel Optimized Hierarchy (OH) RSA-aided ML detection method of [10] exhibits a relatively low complexity even in highly rank-deficient scenarios and thus its employment is meritorious.

In order to further increase the attainable performance, additionally a unity-rate precoder [7] is employed in the iterative decoding aided system, where extrinsic information is exchanged between the precoder's decoder and the convolutional channel decoder of Figure 4. Furthermore, based on the soft-bit outputs of the channel decoder, we introduced a packet reliability metric calculation technique for quantifying the Packet Reliability Information (PRI) forwarded to the LT decoder for informing it as to whether the packet considered is sufficiently reliable for employment by the LT decoder. More explicitly, based on the PRI, the LT decoder erases the unreliable packets, namely those that are likely to contain errors, in order to perfectly recover all the original data packets. By contrast, unreliable packets must not be forwarded to the LT decoder, since they would inflict a high number of further propagated errors, depending on the specific degree-distribution of the LT code [11].

The rest of this paper is structured as follows. In Section 2 we outline the transmission scheme considered. In Section 3 we detail the SDMA DL design and introduce a measure of the normalized system throughput. The LT decoding aided receiver employing iterative detection is summarized in Section 4, followed by the portrayal of the above-mentioned packet reliability calculation algorithm in Section 5. Our performance results are provided in Section 6. Finally, we conclude our discourse in Section 7.

2. SYSTEM OVERVIEW

Again, the system's schematic is outlined in gradually increasing detail in Figures 1 - 4. Let us consider, for example, the transmission of 13 000 LT encoded packets, which were generated from 10 000 source packets by incorporating 30% redundancy. At a packet erasure rate of $P_e = 0.1$ we will lose 1300 randomly positioned LT packets, while the LT packets are routed over the Internet. The LT packets, which do reach BS B will be transmitted via the wireless downlink channel to MS B of Figure 1. The DL transmitter of the BS encodes the bits of the LT-encoded packets using both a convolutional encoder and a unity-rate precoder, as shown in Figure 2. The rationale of using the unity-rate precoder serially concatenated with the convolutional encoder is that it improves the convergence of the iterative receiver, when its decoder exchanges extrinsic information with the convolutional decoder of Figure 4 [7]. When using the system parameters summarized in Table 1, the DL-SDMA system employed the Multi-User Transmission (MUT) scheme of Figure 3 for transmitting the data stream of MS B [9] in Figure 1. Figure 4 details the design of the MS's receiver, utilizing the so-called Optimized Hierarchy Reduced Search Algorithm (OHRSA) of [10], which is suitable for the high-throughput rank-deficient scenarios. Finally, the LT decoder assisted by the PRI provided by the convolutional decoder generates the recovered source packets constituting the original data file. In our forthcoming discussions we will briefly summarize the operation of MUT scheme [9] utilized and outline the corresponding receiver design employing iterative decoding.

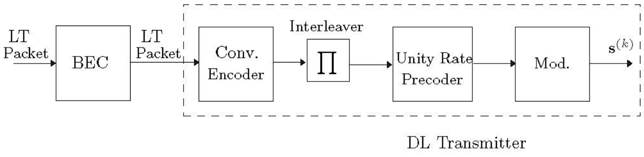

Fig. 2. Transmitter of the k-th user

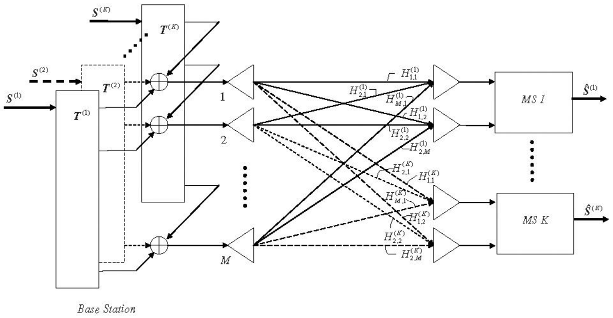

Fig. 3. Multiuser transmission in the LT-DL-SDMA system using the useful single-user transmitter of Figure 2

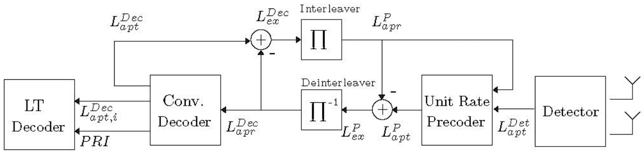

Fig. 4. LT coding aided iteratively decoded MS receiver design

3. MULTI-USER TRANSMISSION SCHEME

Again, the MUT scheme considered is depicted in Figure 3. More specifically, the BS employs M transmit antennas for supporting K MSs, each employing N_k receive antennas.

In this contribution we consider a flat-fading MIMO channel. The MIMO channel corresponding to the k-th user may be described by an $(N_k \times M)$-dimensional complex-valued time-domain channel matrix $\mathbf{H}^{(k)}$. Each element of $\mathbf{H}^{(k)}$ is characterized by a complex-valued scalar channel coefficient $H^{(k)}_{ij}$, which represents the link between the i-th BS transmit antenna and the j-th MS receiver antenna of the k-th user, and each fading MIMO link is modeled by an i.i.d. complex Gaussian random variable having a variance of unity and a mean of zero.

In order to eliminate the MUI, let us now construct the MUT preprocessor $\mathbf{T}_k$ of Figure 3. As outlined in [9], $\mathbf{T}_k$ can be generated based on the null space $\mathbf{V}^{(k)}$ of $\bar{\mathbf{H}}^{(k)}$, calculated using the SVD [12] of $\bar{\mathbf{H}}^{(k)}$ expressed as:

$$\bar{\mathbf{H}}^{(k)} = \begin{pmatrix} \tilde{\mathbf{U}}^{(k)} & \mathbf{U}^{(k)} \end{pmatrix} \cdot \begin{pmatrix} \Sigma & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \cdot \begin{pmatrix} \tilde{\mathbf{V}}^{(k)^H} \\ \mathbf{V}^{(k)^H} \end{pmatrix}, \qquad (1)$$

where

$$\bar{\mathbf{H}}^{(k)} = \begin{pmatrix} \mathbf{H}^{(1)} & \cdots & \mathbf{H}^{(k-1)} & \mathbf{H}^{(k+1)} & \cdots & \mathbf{H}^{(K)} \end{pmatrix}^T. \quad (2)$$

As advocated in [9], we will categorize the potential system design options of an $(M \times N)$-dimensional MIMO system, where M and N refer to the number of transmit and receive antennas. To elaborate a little further, the total number of antennas employed by all the K user terminals is KN_k. Let us now introduce a measure of the normalized *system load* expressed as

$$L_s = \frac{M}{N}. \qquad (3)$$

Consequently, we may distinguish three different scenarios as follows:

1. lightly-loaded scenario, for $L_s < 1$;
2. fully-loaded scenario, for $L_s = 1$;
3. rank-deficient or 'over-loaded' scenario, for $L_s > 1$.

4. LT CODING AIDED RECEIVER USING ITERATIVE DETECTION

First, the detector constituting the first stage of the receiver is the OHRSA [10], which is adapted for employment in our system in order to reduce the computational complexity imposed by the ML detector used by each of the MSs. The derivation of an expression for the low-complexity evaluation of the soft-bit information associated with the bit estimates of the OHRSA detector was also given in [10].

Later, iterative decoding is carried out by exchanging extrinsic information between the unity-rate precoder's decoder and the channel decoder. Figure 4 illustrates the iterative receiver structure, where L represents the LLRs. The super-script *Det* indicates the detector, P denotes the precoder, while *Dec* represents the channel decoder. The subscripts *apr*, *ex* and *apt* indicate *a priori*, extrinsic and *a posteriori* LLRs, respectively. First, the unity-rate precoder's decoder processes the soft-bit output L_{apt}^{Det} of the detector generated in the previous stage and the *a priori* LLR values L_{apr}^{P}, which are appropriately arranged by the interleaver ″ are produced from the extrinsic information L_{ex}^{Dec} of the channel decoder. Then the extrinsic information L_{ex}^{P} to be used by the unity-rate precoder's decoder is obtained from L_{apt}^{P} by subtracting the *a posteriori* LLR values L_{apr}^{P}. Then, as portrayed in Figure 4, the channel decoder processes L_{apr}^{Dec}, which was generated by the deinterleaver ″$^{-1}$ from L_{ex}^{P}, and outputs the *a posteriori* LLRs L_{apt}^{Dec} to be used as a feedback for the next decoding iteration. When the iterations are curtailed, the channel decoder outputs $L_{apt,i}^{Dec}$, which represents the hard decision based data bits.

5. PACKET RELIABILITY CALCULATION

In order to assist the LT decoder in successfully decoding the received packets, the LT decoder requires a minimum number of error-free unerased packets. However, when the packets are transmitted through the hostile wireless channel, some packets may not be correctly decoded. If the LT decoder processes error-infested packets, the error will be propagated to all other packets, which are related to this particular packet, as determined by the LT code's degree distribution [11]. Hence we have to ensure that only error-free packets are used by the LT decoder, and therefore we have to identify and erase the incorrectly decoded packets. For the sake of preventing error propagation, it is standard practice to use Cyclic Redundancy Checking (CRC) for error detection, although this requires a substantial transmission overhead of 16 bits, which may be on the order of 10%. Hence here we introduce an attractive technique of quantifying the reliability of the channel-decoded packets, which is referred to as Packet Reliability Information (PRI). This technique imposes no transmission overhead. Based on this PRI, we identify the channel-decoded packets, which are deemed to be error-infested.

The PRI is calculated based on the Log Likelihood Ratio (LLR) [13] of the bits. Hence we introduced an LLR threshold, denoted as l_t. A channel decoded bit is deemed reliable, if the absolute value of its LLR is higher than l_t , where the threshold l_t has to be carefully chosen. By contrast, if the absolute value of the LLR is lower than l_t, it is deemed unreliable. Therefore, the PRI of a packet is simply calculated as the percentage of the reliable bits in the channel-decoded packet. On this basis we can identify packets, which are likely to contain decoding errors.

Let us assume for example that the total number of the LT packets generated by the LT encoder is 13 000. However, only 11 700 packets are transmitted by the BS to MS B of Figure 1 owing to the 10% packet loss rate of the BEC channel. It was found in our experiments not shown here that the minimum number of packets required for achieving a high probability of successful LT decoding is 11 000. Then we have to consider two decoding scenarios, namely when the number of erroneous packets is higher than 11 700 − 11 000 = 700, as well as when it is lower than 700. In the former specific case, the 11 000 packets chosen for LT decoding may contain some incorrectly decoded LT-encoded packets, which hence result in error propagation. However, in this scenario the best course of action is to still forward the most reliable 11 000 packets for LT decoding based on the PRI. In other words, we remove the 700 most unreliable packets, because the chances are that this way we may indeed succeed in removing all the incorrect packets and hence successfully LT decode all the source packets.

Table 1. System Parameters

LT Packet size	120 bits
LT Distribution	Improved robust distribution [11]
Number of information packets	10, 000
Number of redundancy packets	3, 000
Erasure probability of BEC	0.1
Recursive Systematic Code	RSC(5,7)
Interleaver length	10^5 bits
Modulation	4 QAM
Number of users K	3
Number of transmit antennas M	6 for $L_s = 1$, 8 for $L_s = 1.333$
Number of receive antennas N_k	2
Normalized Doppler frequency	$f_d = f_D \cdot T = 0.001$

6. PER AND BER PERFORMANCE

Here we continue our discourse by characterizing the PER versus packet-length performance of the inner encoder/decoder scheme of Figure 5. As expected, the PER degrades upon increasing the packet-length, since the probability of having residual errors after channel decoding increases upon increasing the packet-length. Further, it can be shown that increasing the packet-length increases the LT-decoding complexity. By contrast, upon reducing the packet-length the relative overhead of using a 16-bit CRC sequence increases. As a compromise, based on the message of Figure 5, we opted for a packet-length of 120 bits. Observe furthermore that as expected, increasing the interleaver length has the beneficial effect of reducing the PER owing to randomizing the bit-errors more efficiently.

In this section, we characterize the achievable performance of the LT-DL-SDMA system assisted by both a 16-bit CRC-based and on the proposed LLR-based PRI estimation scheme. The packet erasure rate P_e of the BEC was set to 0.1 and each. LT packet contained 120 bits. In conjunction with the CRC overhead, a source packet accommodated 136 bits. As seen in Table 1, we employed 4-QAM protected by the half-rate Recursive Systematic Convolutional code RSC(5,7) having a memory of three and using a 10^5-bit interleaver, where (5,7) represent the generator polynomials in octal format. A slow-fading MIMO channel having a normalized Dopper frequency of 0.001 was used. The system configuration studied supported $K = 3$ users and each user terminal employed two receiver antennas. Finally, for the sake of convenience, we assume that each MS is receiving the same number of independent data streams L_t from the BS, which implies that each user has the same data throughput.

Figure 6 portrays the PER performance of the inner decoder scheme of Figure 4 corresponding to the normalized system load of $L_s = 1.0$, i,e. when the number of transmit antennas is $M = 6$ and the number of independent data streams transmitted to each user terminal is $L_t = M - (K-1) \cdot N_k = 2$. In the CRC-assisted scheme, the LT decoder is capable of avoiding the decoding of error-infested packets. As stated above, in the example considered the BEC erases 1300 packets out of 13 000 and hence 11 700 LT-encoded packets will be transmitted via the wireless downlink channel. Recall that since the LT decoder requires a minimum number of 11 000 correct packets, the maximum tolerable number of channel-impaired packets is 700, still allowing the LT decoder to recover the original source packets. This implies that if the PER of the inner decoder is lower than $700/11\,700 \approx 0.06$, the CRC-assisted system is capable of completely recovering all the original source packets with a high probability. The bold horizontal dashed line in Figure 6 characterizes the value of PER=0.06. Observe that in Figure 6 that the system using $I = 4$ iterations exhibits a PER $< 6\%$ for E_b/N_0 values in excess of about 3dB and hence it is highly likely to recover all packets correctly. Furthermore, the LLR-based scheme performs similarly to the CRC-aided arrangement, but the latter suffers from a slight E_b/N_0 loss owing to its CRC overhead of about 13.3%

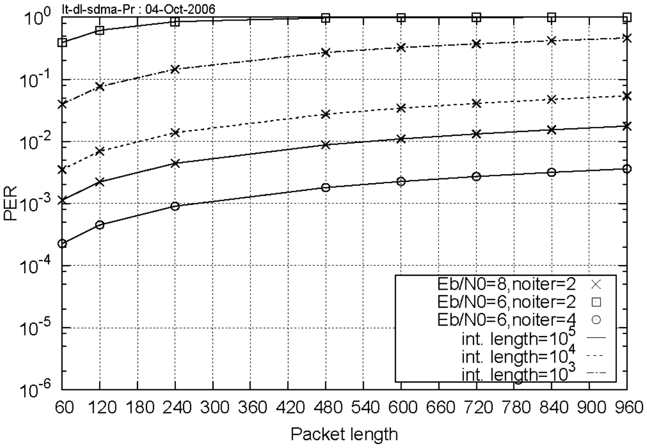

Fig. 5. Inner PER performance of the LT-DL-SDMA scheme having a normalized system load of $L_s = 1.333$ for different E_b/N_0 values, packet sizes and interleaver lengths. Our system supports $K = 3$ users, where each user employs $N_k = 2$ receives antennas.

7. CONCLUSION

In this treatise, we proposed a novel LT-DL-SDMA system using iterative decoding. It was also demonstrated that the LLR-based PRI-aided scheme slightly outperformed its CRC-based counterpart, owing to the E_b/N_0 penalty imposed by the 16-bit CRC overhead. It was shown in Figure 6 that for E_b/N_0 values in excess of about 3dB an infinitesimally low PER may be achieved. Our future research will focus on improving the system's performance by exchanging soft information between the inner decoders and the outer LT decoder.

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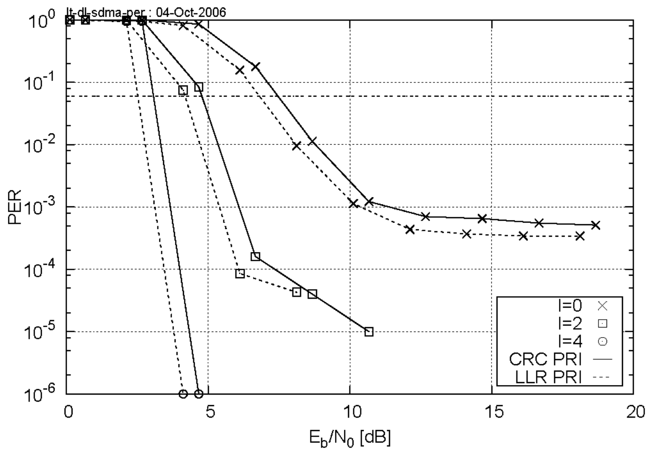

Fig. 6. Inner PER performance of the LT-DL-SDMA scheme system having a normalized system load of 1.0 using either the LLR-based or the CRC-based PRI for different number of iterations. This system supports $K = 3$ users, where each user terminal employs $N_k = 2$ receive antennas. All other system parameters are summarized in Table1.

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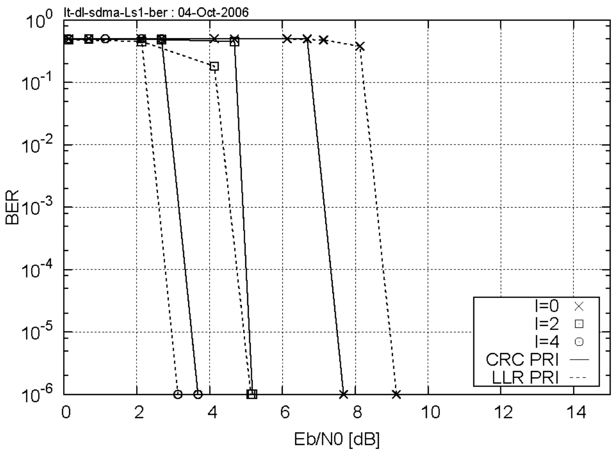

Fig. 7. BER performance of the LT-DL-SDMA system having a normalized system load of 1.0 assisted with the PRI and CRC and compared with different number of iterations. This system supports $K = 3$ users, where each user terminal employs $N_k = 2$ receive antennas. All other system parameters are summarized in Table1.

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