Towards Agents Participating in Realistic Multi-Unit Sealed-Bid Auctions

(Short Paper)

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ABSTRACT

When autonomous agents decide on their bidding strategies in real world auctions, they have a number of concerns that go beyond the models that are normally analyzed in traditional auction theory. Oftentimes, the agents have budget constraints and the auctions have a reserve price, both of which restrict the bids the agents can place. In addition, their attitude need not be risk-neutral and they may have uncertainty about the value of the goods they are buying. Some of these issues have been examined individually for single-unit sealed-bid auctions. However, here, we work towards extending this analysis to the multi-unit case, and also analyzing the multi-unit sealed-bid auctions in which a combination of these issues are present. In this paper, we present the initial results of this work. More specifically, we present the equilibria that exist in multi-unit sealed-bid auctions, when either the agents can have any risk attitude, or the auction has a reserve price.

Categories and Subject Descriptors

I.2.11 [**ARTIFICIAL INTELLIGENCE**]: Multiagent Systems; I.2.11 [**ARTIFICIAL INTELLIGENCE**]: Intelligent Agents

General Terms

Theory, Economics

Keywords

game theory, bidding strategies, equilibrium analysis

1. INTRODUCTION

Auctions have become commonplace; they are used to trade all kinds of commodity, from flowers and food to industrial commodities and keyword targeted advertisement slots, from bonds and securities to spectrum rights and gold bullion. Once the preserve of governments and large companies, the advent of online auctions has opened up auctions to millions of private individuals and small commercial ventures. Given this, it is desirable to develop autonomous agents that will let the masses participate effectively in such settings, even though they do not possess professional expertise in this area. To achieve this, however, these agents should account for the features of real-world auctions that expert bidders take into consideration when determining their bidding strategies.

While game theory is widely used in multi-agent systems as a way to model and predict the interactions between rational agents in auctions, the models that are canonically analyzed are rather limited. As discussed below, some work has been done towards extending these models to incorporate features that are important in real auctions, but this work invariably looks at each feature separately; additionally the cases examined are almost all instances of single-unit auctions. While this is useful for economists and perhaps expert bidders, who can integrate the lessons learned using human intuition and imagination, an automated agent cannot do this. It is therefore necessary to analyze the strategic behavior in multi-unit (m^{th} and $(m+1)^{th}$ price) auction models that incorporate all the relevant features. To this end, we have looked at a number of auctions, ranging in scope from the eBay auctions (held mainly between individuals) to B2B auctions (used by businesses to procure materials and commodities), with various different rules, ranging from the traditional English auction to the position auction used by Google Adwords. Despite their differences, a number of common features are present. We list the most important of these below and highlight what is already known about each of them.

First, *budget constraints* are very important, whenever businesses and individuals place bids, because they limit the upper range of these bids. Here, we will assume that the available budget constitutes an absolute spending limit. Now, this case has been examined for single-unit auctions [2], but not for multi-unit ones; it has also been proven that the revenue generated by a 1^{st} price auction is always higher than that of the equivalent 2^{nd} price one.

Second, bidders may adopt different *attitudes towards risk*. Essentially, this indicates whether bidders are conservative or not, and their willingness to take risk in order to gain additional profit. Normally bidders are assumed to be "risk-neutral", meaning their utility equals their profit. However, they can also be "risk-averse", "risk-seeking", or even have a more complicated risk attitude. The equilibrium strategy of a risk-averse agent participating in a 1^{st} price auction has been analyzed in [6].

Third, setting a *reserve price* (i.e. a minimum transaction price) in the auction is a common way for the seller to increase her profit. This case has been examined for single-unit sealed-bid auctions in [7, 9].

Fourth, there may be *uncertainty in the bidders' valuation* of the offered commodity. For example, when businesses bid in the Google Adwords keyword auction, they can't precisely know the

Cite as: Title (Short Paper), Author(s), *Proc. of 7th Int. Conf. on Autonomous Agents and Multiagent Systems (AAMAS 2008)*, Padgham, Parkes, Müller and Parsons (eds.), May, 12-16., 2008, Estoril, Portugal, pp. XXX-XXX.

additional revenue that advertising in this way will bring them, and therefore they can't evaluate the actual economic value of the ad. Nevertheless, it can be assumed that the agent has some idea about his own value and this can be represented by a probability distribution. In the literature, this problem has been mostly looked at from the point of view of having a cost for introspection, which allows the agent to determine his valuation more precisely [5, 10]. However, in many practical settings, introspection is simply not possible, because of the lack of further relevant data, or excessive costs that cannot be justified by the increased accuracy.[1]

The last important feature is considering a bidder's desire to *purchase multiple items*, with a different valuation for each. In this case, it is known that bidders should shade their bids, compared to the case when only one item is desired, even to the point of bidding for less items than desired, in order to gain more profit (strategic demand reduction) [13]. To date, however, an optimal strategy is not known for this feature; it is open problem. This is the reason why we make the usual assumption that each agent wishes to buy only one unit, like e.g. in [11].

This paper is organized as follows:

- In section 2, we give the multi-unit auction model that will be used in our analysis.
- We then proceed to derive novel equilibria for the multi-unit m^{th} price sealed-bid auction case, for the individual cases of reserve prices (in section 3), and of agents having any risk attitude (in section 4). From this analysis, we go on to discuss how these features affect the bidding strategies of the agents in each of these cases.
- Finally, in section 5, we discuss about the extention of this work, in order to examine the remaining cases that are of interest, and conclude.

2. THE MULTI-UNIT AUCTION SETTING

In this section we formally describe the auction setting to be analyzed and define the objective function that the agents wish to maximize. We also give the notation that we use.

In particular, we will compute and analyze the symmetric Bayes-Nash equilibria[2] for sealed-bid auctions where $m \geq 1$ identical items are being sold; these equilibria are defined by a strategy, which maps the agents' valuations v_i to bids b_i. The two most common settings in this context are the m^{th} and $(m+1)^{th}$ price auctions, in which the top m bidders win one item each at a price equal to the m^{th} and $(m+1)^{th}$ highest bid respectively. We assume that there is a reserve price $r \geq 0$ in our setting; this means that bidders, who wish to participate in the auction, must place bids $b_i \geq r$.

We assume that N indistinguishable bidders (where $N \geq m$) participate in the auction and they have a private valuation (utility) v_i for acquiring any one of the traded items; these valuations are assumed to be i.i.d. from a distribution with cumulative distribution function (cdf) $F(u)$, which is the same for all bidders. In the case that there is uncertainty about the valuation v_i, the agent knows that it is drawn from distribution $G_i(v_i)$, but not the precise value.

We also assume that each bidder has a certain budget c_i, which is known only to himself and which limits the maximum bid that he can place in the auction. The available budgets of the agents are i.i.d. drawn from a known distribution with cdf $H(c)$.

According to utility theory, every rational agent has a strictly monotonically increasing utility function $u()$ that maps profit into utility; the alternative with the highest expected utility is the preferred outcome. This function determines the agent's risk attitude. In this paper, we will be initially using the function used in [6], which is $u(x) = -\gamma^x, \gamma \in [0,1]$; this function is used to indicate risk-averse agents, but here we extend it to also indicate risk-seeking bidders:

$$u(x) = sign(\gamma - 1) \cdot \gamma^x, \forall \gamma \geq 0 \quad (1)$$

where $sign(x)$ is the sign function, which returns $+1$, when $x > 0$, -1 when $x < 0$ and 0 when $x = 0$. Other functions $u(x)$ used widely in economics are: $u(x) = x^\alpha, \alpha \in (0,1)$ (CRRA), and $u(x) = 1 - exp(-\alpha \cdot x), \alpha > 0$ (CARA), both of which indicate risk-averse bidders.

We also use the following additional notation in the proofs: $Z(x)$ is the probability distribution of any opponent's bid b_j. Thus $Z(x) = Prob[b_j \leq x]$, and $B^{(k)}$ is the k^{th} order statistic of these bids of the opponents. Since there are $(N-1)$ opponents for each agent, the distribution of $B^{(k)}$ is $\Phi_k(x) = Prob[B^{(k)} \leq x]$. It can be computed as [8]:

$$\Phi_k(x) = \sum_{i=0}^{k-1} C(N-1,i) \cdot (Z(x))^{N-1-i} \cdot (1 - Z(x))^i \quad (2)$$

where the notation $C(n,k)$ is the total number of possible combinations of k items chosen from n.

As shown in [11], for all N and m, such that $N \geq m$ the following equation holds:

$$\Phi'_m(x) = (N-m) \cdot \big(\Phi_m(x) - \Phi_{m-1}(x)\big) \cdot \frac{Z'(x)}{Z(x)} \quad (3)$$

The equilibria that we compute in this paper, as well as the equilibria that exist in the more general cases, which will be examined in the continuation of this work, are the solutions of differential equations of the form described by the following theorem [1]:

THEOREM 1. *Let $f(x,z)$ and $\frac{\partial f(x,z)}{\partial z}$ be continuous functions of x and z at all points (x,z) in some neighborhood of the initial point (x_0, Y_0). Then there is a unique function $Y(x)$ defined on some interval $[x_0 - \alpha, x_0 + \beta]$, satisfying:*

$$Y'(x) = f(x, Y(x)), \forall x : x_0 - \alpha \leq x \leq x_0 + \beta \quad (4)$$

with boundary condition: $Y(x_0) = Y_0$

This theorem *guarantees the existence and uniqueness of the equilibria* we compute in the next sections.

3. EQUILIBRIA IN THE PRESENCE OF RESERVE PRICES

In this section we examine the equilibria that exist in the case that the reserve price of the auction is $r \geq 0$. Here we assume the bidders have no budget constraints and they are risk-neutral.

THEOREM 2. *In the case of an m^{th} price sealed-bid auction, with reserve price $r \geq 0$, with N participating risk-neutral bidders, in which each bidder i is interested in purchasing one unit of the*

[1]In most of the above settings, the dominant strategy in the case of a 2^{nd} price auction is some variation of truth-telling [4]. This result is generalized trivially to the multi-unit variant (the $(m+1)^{th}$ price auction).

[2]The Bayes-Nash equilibrium is the standard solution used in game theory to analyze auctions. The equilibria being symmetric means that all agents use the same bidding strategy. This is a common assumption made in game theory, in order to restrict the space of strategies that we examine. It is likely that in addition to the symmetric equilibria we compute there are also asymmetric ones.

good for sale with inherent utility (valuation) for that item equal to v_i, where v_i are i.i.d. drawn from $F(v)$, the following bidding strategy constitutes a symmetric Bayes-Nash equilibrium:

$$g(u) = u - (F(v))^{-(N-m)} \cdot \int_r^v (F(z))^{N-m} \cdot dz \quad (5)$$

PROOF. Because of the reserve price r, there is a chance that an agent will not be able to participate in the auction, because his valuation for the item is $v_i < r$.[3] We therefore begin by analyzing the case when exactly $n \leq N$ agents can participate in the auction; these agents have $c_i \geq r$ and $v_i \geq r$. The probability that a particular agent participates in the auction is: $Prob[v_i \geq r] = (1-F(r))$. The probability that exactly n (out of the N total) agents participate in this auction is thus:

$$\pi_n = C(N-1, n-1) \cdot (1-F(r))^{n-1} \cdot (F(r))^{N-n} \quad (6)$$

The distribution $F_r(v)$ from which the participating agents' valuations v_i are drawn, is the initial distribution $F()$, conditional on the fact that $v_i \geq r$. Thus it is:
$F_r(v) = \frac{F(v)-F(r)}{1-F(r)},\ if\ v \geq r\ \&\ F_r(v) = 0,\ if\ v < r.$

The distribution, from which the opponents' bids b_j are drawn, is:

$$Z_r(x) = \frac{F(g^{-1}(x)) - F(r)}{1-F(r)} \quad (7)$$

The distribution of the k^{th} highest opponent bid $B^{(k)}$ is:

$$\Phi_k^{n,r}(x) = \sum_{i=0}^{k-1} C(n-1, i) \cdot (Z_r(x))^{n-1-i} \cdot (1 - Z_r(x))^i \quad (8)$$

To analyze the expected profit of a bidder who places a bid b_i in the auction, we distinguish the following cases:

- If $b_i < B^{(m)}$, then bidder i is outbid and doesn't win any items, therefore his utility is $u_i = 0$.
- If $B^{(m)} \leq b_i \leq B^{(m-1)}$, then bidder i has placed the last winning bid. Thus the payment equals his bid and his utility is $u_i = v_i - b_i$. This happens with probability: $Prob[B^{(m)} \leq b_i \leq B^{(m-1)}] = \Phi_m^{n,r}(b_i) - \Phi_{m-1}^{n,r}(b_i)$.
- If $B^{(m-1)} < b_i$, then bidder i is a winner, the payment is equal to bid $B^{(m-1)}$ and his utility is $u_i = v_i - B^{(m-1)}$. Note that: $Prob[B^{(m-1)} \leq \omega] = \Phi_{m-1}^{n,r}(\omega)$.

Therefore the expected utility of bidder i, when he places a bid equal to b_i, is equal to:

$$Eu_i^{n,r}(b_i) = (v_i - b_i) \cdot \Phi_m^{n,r}(b_i) + \int_r^{b_i} \Phi_{m-1}^{n,r}(\omega) \cdot d\omega \quad (9)$$

From Bayes' rule, we know that the expected utility that bidder i gets, by placing bid b_i, for any possible numbers of total participating agents, is: $Eu_i(b_i) = \sum_{n=1}^{N} \pi_n \cdot Eu_i^{n,r}(b_i)$. Then using equations 6, 7, 8 and 9, this becomes:

$$Eu_i(b_i) = (v_i - b_i) \cdot \Phi_m(b_i) + \int_r^{b_i} \Phi_{m-1}(\omega) \cdot d\omega \quad (10)$$

where the terms $\Phi_k(x)$ (for $k = m-1$ and $k = m$) are:

$$\Phi_k(x) = \sum_{i=0}^{k-1} C(N-1, i)(F(g^{-1}(x)))^{N-1-i}(1-F(g^{-1}(x)))^i \quad (11)$$

[3]Note that, as part of the work we did in [12], we looked at the expected utility of an agent who participates in an auction with a non-zero starting price; this proof borrows elements from that work.

To find the bid which maximizes the expected utility, we set $\frac{dEu_i}{db_i} = 0$. Using equation 3, and the fact $b_i = g(v_i)$, since it is the bid b_i that maximizes the expected utility, we substitute this in the equation to get:

$$g'(v_i) = \frac{(N-m) \cdot (v_i - g(v_i)) \cdot F'(v_i)}{F(v_i)} \quad (12)$$

As the boundary condition is $g(r) = r$, the solution of this differential equation is equation 5. $\square$

From equation 5, it is evident that, for the same valuation v_i, the bid b_i increases with each increase of the reserve price.

In the case of an $(m+1)^{th}$ price auction, the optimal strategy is [4]:

THEOREM 3. *In an $(m+1)^{th}$ price auction, with reserve price r, where the bidders are risk-neutral, have valuations v_i and no budget constraints, it is a dominant strategy to bid truthfully: $b_i = v_i$, if $v_i \geq r$, and not to participate otherwise.*

4. EQUILIBRIA IN THE CASE OF VARYING RISK ATTITUDES

In this section we examine the equilibria that exist in the case that agents are not risk-neutral, but rather have a utility function $u()$ that maps profit into utility. If this function is concave, the agents are risk-averse; if it is convex, they are risk-seeking. The bidders have no budget constraints and the reserve price of the auction is $r = 0$.

THEOREM 4. *In the case of an m^{th} price sealed-bid auction with N participating bidders, in which each bidder i is interested in purchasing one unit of the good for sale with inherent utility (valuation) for that item equal to v_i, where v_i are i.i.d. drawn from $F(v)$, the bidders have no budget constraints and they have a risk attitude which is described by utility function $u()$, the bidding strategy $g(v)$, which constitutes a symmetric Bayes-Nash equilibrium, is the solution of the differential equation:*

$$g'(v_i) = \frac{u(v_i - g(v_i)) - u(0)}{u'(v_i - g(v_i))} \cdot (N-m) \cdot \frac{F'(v_i)}{F(v_i)} \quad (13)$$

with boundary condition $g(0) = 0$.

PROOF. Once again we assume that the equilibrium strategy is described by a function $g()$ which maps valuations to bids. We consider any bidder i, who places a bid b_i in the auction. The distribution $Z(x)$ of the bid b_j, that any opponent j ($j \neq i$) of agent i places, is:

$$Z(x) = F(g^{-1}(x)) \quad (14)$$

The k^{th} order statistic of these bids $B^{(k)}$ is drawn from distribution $\Phi_k(x)$, described by equation 2.

Depending on the value of b_i, the following three cases are possible:

- If $b_i < B^{(m)}$, then bidder i is outbid and doesn't win any items, therefore his utility is $u_i = u(0)$.[4] The probability of this case happening is: $Prob[b_i \leq B^{(m)}] = 1 - \Phi_m(b_i)$.
- If $B^{(m)} \leq b_i \leq B^{(m-1)}$, then bidder i has placed the last winning bid. Thus the payment equals his bid, his profit is $(v_i - b_i)$, and his utility is $u_i = u(v_i - b_i)$. The probability of this case happening is: $Prob[B^{(m)} \leq b_i \leq B^{(m-1)}] = \Phi_m(b_i) - \Phi_{m-1}(b_i)$.

[4]Note that profit 0 does not necessarily mean that the utility is 0; it depends on the form of the utility function $u()$.

- If $B^{(m-1)} < b_i$, then bidder i is a winner, the payment is equal to bid $B^{(m-1)}$, his profit is equal to $(v_i - B^{(m-1)})$ and his utility is $u_i = u(v_i - B^{(m-1)})$. Note that the probability: $Prob[B^{(m-1)} \le \omega] = \Phi_{m-1}(\omega)$.

The expected utility of bidder i, who places bid b_i, is:

$$Eu_i(b_i) = u(0) \cdot (1 - \Phi_m(b_i)) + u(v_i - b_i) \cdot (\Phi_m(b_i) - \Phi_{m-1}(b_i)) + \int_0^{b_i} u(v_i - \omega) \cdot \frac{d}{d\omega}\Phi_{m-1}(\omega) \cdot d\omega \quad (15)$$

The bid which maximizes this expected utility, is found by setting: $\frac{dEu_i}{db_i} = 0$. This becomes:

$$(u(v_i - b_i) - u(0)) \cdot \Phi'_m(b_i) = u'(v_i - b_i) \cdot (\Phi_m(b_i) - \Phi_{m-1}(b_i))$$

Using equation 3, to simplify this equation, we derive:

$$(u(v_i - b_i) - u(0)) \cdot (N - m) \cdot \frac{F'(g^{-1}(b_i))}{g'(g^{-1}(b_i)) \cdot F(g^{-1}(b_i))} = u'(v_i - b_i)$$

This value b_i is equal to $b_i = g(v_i)$, since it maximizes the expected utility $Eu_i(b_i)$. Using this substitution, we derive the differential equation 13.

The boundary condition is $g(0) = 0$, because an agent with valuation $v_i = 0$ will bid $b_i = 0$. □

If we use the function $u(x)$ from equation 1, we can solve equation 13, to get the following equilibrium strategy:

$$g(v_i) = v_i - \log_\gamma \left[1 + \frac{\ln \gamma}{F(v_i)^{N-m}} \cdot \int_0^{v_i} F(\omega)^{N-m} \cdot \gamma^{v_i - \omega} \cdot d\omega\right] \quad (16)$$

In [6], the authors take the limits of this equation as γ approaches 1 and 0, which represent the cases when the agent becomes risk-neutral and very risk-averse respectively. Here, we do the same, and also compute the limit as γ approaches ∞, which represents the case when the agent becomes very risk-seeking:

$$\lim_{\gamma \to 0} g(v_i) = v_i \quad (17)$$

$$\lim_{\gamma \to 1} g(v_i) = v_i - \frac{\int_0^{v_i} F(\omega)^{N-m} \cdot d\omega}{F(v_i)^{N-m}} \quad (18)$$

$$\lim_{\gamma \to \infty} g(v_i) = 0 \quad (19)$$

We observe that, when $\gamma \to 1$, i.e. the agents tend to become risk-neutral, equation 16 gives the same solution as the one known from the literature, for the case when agents just maximize their profit (risk-neutral agents) [4]. When $\gamma \to 0$, i.e. the agents become very risk-averse, they bid truthfully, because they are worried too much about losing no matter how small this possibility is.[5] When $\gamma \to \infty$, i.e. the agents become very risk-seeking, they bid 0 (or $\epsilon > 0$, if zero bids are not allowed), gambling on the unlikely chance that there is no competition and they receive the item for free. These results can be generalized to any family of utility functions; the agents bid according to equations 17, 18 and 19, when they are respectively very risk-averse, risk-neutral and very risk-seeking.

In the case of an $(m+1)^{th}$ price auction, the agents submit truthful bids [4]:

THEOREM 5. *In an $(m+1)^{th}$ price auction, where the bidders have valuations v_i, they have no budget constraints and they have a risk attitude described by utility function $u()$, it is a dominant strategy to bid truthfully: $b_i = v_i$*

[5]Both these results are consistent with those reported in [6] for the case of single-unit auctions.

5. DISCUSSION

In this paper, we examined the behavior of agents participating in multi-unit sealed-bid auctions, when the auction has a reserve price, or the agents have varying risk attitudes. In the future, we aim to continue this work to include budget constraints, and valuation uncertainty. Not only will we derive equilibria for the remaining cases, but we will present the dominant strategy for the case of uncertainty in the valuation that bidders have, when the bidders are not risk-neutral, in the setting of the $(m+1)^{th}$ price auction. Furthermore, we will combine all the features in our analysis and derive the equilibrium strategies for both the m^{th} and the $(m+1)^{th}$ price auction, in the presence of budget constraints, reserve prices and any possible bidder risk attitude. Then we will also include the uncertainty of bidders' valuation for the case of the $(m+1)^{th}$ price auction and present the dominant strategy. Finally, we will use simulations to show that this analysis is useful, in practice, both for the bidding agents in order to maximize their utility, and also for the seller in order to select the correct reserve price and thus maximize her revenue.

Other directions for future work include enriching our model and then analyzing the m^{th} price auction equilibria, in the presence of valuation uncertainty. In addition, we would like to extend this work by examining the case of identical items being sold in *multiple concurrent auctions* [3], and the case of competition between the agents [11].

6. ACKNOWLEDGMENTS

This research was undertaken as part of the ALADDIN (Autonomous Learning Agents for Decentralised Data and Information Systems) project which is jointly funded by a BAE Systems and EPSRC (Engineering and Physical Research Council) strategic partnership (EP/C548051/1).

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