SMOOTH BOUNDARY TOPOLOGY OPTIMIZATION APPLIED TO AN ELECTROSTATIC ACTUATOR

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Keywords: Topology optimisation, design sensitivity, electromechanical design

Abstract

Smooth boundary topology optimization (SBTO) avoids many of the problems encountered by conventional cell based systems coupled with material homogenization or density method. Shape optimization becomes part of topology optimization. The effectiveness of the method is demonstrated through the design of an electrostatic MEMS actuator to generate maximum torque in a predefined area.

1 Introduction

Over the past decade, there has been a growing interest in systems which provide a form of topology optimization for low frequency electromagnetic devices. Unlike pure boundary or shape optimization systems which have received considerable attention in the literature, topology optimization provides the possibility of developing new geometries to meet design specifications. The work is closely coupled to the work on design sensitivity analysis and, indeed, this is a fundamental requirement for efficient topological design. Much of the published research to date has used Discrete Design Sensitivity Analysis (DDSA) as the basis and the topological variations have been achieved through modifying the material properties in a series of rectangular cells. Sensitivity analysis is used to identify how to change the material properties of each cell in order to achieve the desired performance. However, this approach has two problems. The first is that the results are very “granular” since the design space is divided into rectangular cells. The second problem is that the intermediate states between two materials are present. Hence penalization methods are additionally needed, in order to retrieve a realistic material, that are apt to lead to a local optimum solution in practical problems. Recently, an alternate approach has been proposed which uses a continuum version of design sensitivity analysis (CDSA) and, rather than working with rectangular cells, grows areas having smooth boundaries and particular material properties (SBTO). It is the intention of this paper to extend the work in SBTO previously reported to a real application of an electrostatic actuator (a MEMS device) and compare the results with those described in an earlier paper using the cell approach [1].

2 Smooth Boundary Topology Optimization

The boundary shape of an object is described in terms of a B-Spline curve and the SBTO modifies this and introduces new and disjoint boundary structures such that a cost function is optimised. The CDSA is used to determine a sensitivity of the topology (TS) to boundary and geometry changes. Depending on the TS, a small piece of air/material is introduced inside a material/air filled region, and can grow and change shape as the optimization proceeds. Several regions can be created simultaneously and may coalesce into single objects.

At any point during optimization, the device may benefit from the introduction of a small, circular material filled region ($B(r,d)$ with radius d, centered at r) in the design domain Ω. The “topological gradient”, $G(r)$, provides a measure of improvement of the objective function:

$$G(r) = \lim_{d \to 0} \frac{\Psi_{obj}(\Omega \backslash B(r,d)) - \Psi_{obj}(\Omega)}{\delta(\Omega)} \quad (1)$$

where Ψ_{obj} is the objective function, $\Omega \backslash B(r,d)$ is the domain Ω with the material region $B(r,d)$, and $\delta(\Omega)$ is the area difference after and before the small region occurs.

The TS and classical shape sensitivity can be shown to be linked and the derivation is given in [2]. Briefly, consider a scalar function, J, expressed with Ψ_{obj} and d.

$$J(d) = \Psi_{obj}(\Omega \setminus B(r,d)) \quad (2)$$

The classical shape sensitivity of (2), i.e. J', can be expressed in terms of the electric fields and the adjoint fields based on the shape gradient information [3]:

$$\begin{aligned} J'(d) &= -\int_{\gamma_d} L(\phi_1, \lambda_1) d\gamma \\ L(\phi_1, \lambda_1) &= (\varepsilon_1 - \varepsilon_2)[\frac{\varepsilon_1}{\varepsilon_2} p(\phi_1) p(\lambda_1) + q(\phi_1) q(\lambda_1)] \end{aligned} \quad (3)$$

where $p \equiv \partial/\partial n$, $q \equiv \partial/\partial t$, ϕ is the electrical potential, ε_1 and ε_2 refer to either side of the material boundary, while λ denotes the adjoint variable. Using a local expansion of ϕ and λ along the circumference of $B(r,d)$ yields

$$J'(d) = -4\pi d L(\phi_1(r), \lambda_1(r)) + o(d) \quad (4)$$

Finally the difference of the objective function after and before inserting the material is:

$$J(d) - J(0) = \int_0^d J'(\rho) d\rho = -2\pi d^2 L(\phi_1(r), \lambda_1(r)) + o(d^2) \quad (5)$$

Comparing (5) and (1) the TS gives $G(r)=2L(\phi_1(r),\lambda_1(r))$, where the shape sensitivity is $L(\phi_1(r),\lambda_1(r))$. Hence the topological and the shape gradients differ by a factor 2.

3 A design of an Electrostatic Actuator

An electrostatic actuator [1] is shown in Fig. 1.

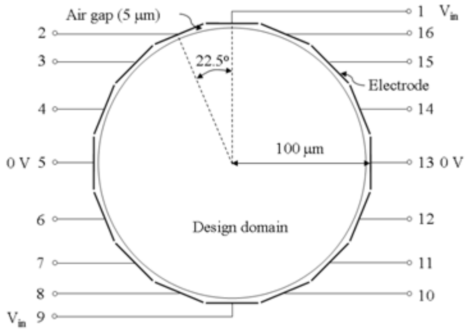

Fig. 1. An actuator layout and design domain [2].

The design goal is to create a dielectric rotor producing as large a torque as possible. The outline of the maximum size of the rotor is depicted in Fig. 1, but this architecture will, of course, generate no torque. We aim to minimise

$$\Psi_{obj} = (W_A - W_{Ao})^2 + (W_B - W_{Bo})^2 \tag{6}$$

where positions A and B activating each electrode pair are 22.5 degrees apart; W_{Ao} is the energy when the initial design space is fully occupied by air; W_{Bo} refers to the area completely filled with dielectric. The dielectric area is confined to 45% of the design domain.

At each iteration, the system decides whether a new material region should be created or whether just a change to the boundary shape will be sufficient. This information is derived from the TS. Fig. 2 shows the evolution of the rotor as the design iterations proceed.

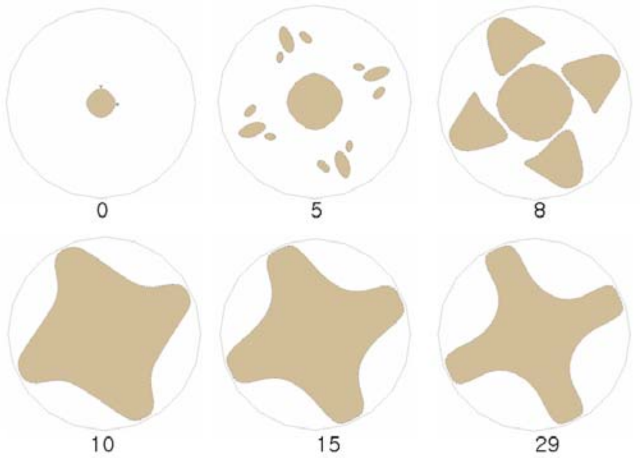

Fig. 2. Insertion and evolution of material regions at design iterations 0, 5, 8, 10, 15 and 29 while maximizing the objective function.

4 Results

Figure 3 compares the final optimised shape of the rotor achieved using SBTO and the density method [1]. The shapes are quite similar; however, the density method produced the "staircase" effect on the boundary which the SBTO does not exhibit. Moreover, it was reported in [1] that the final topology depends strongly on the initial design, suggesting possible local minima traps, which has not been identified as a problem in the SBTO. Finally, it appears that fewer iterations are needed to achieve convergence in SBTO. Figure 4 shows the torque profile of the proposed design. Further comparisons will be made in the extended version.

(a) (b) (c)

Fig 3. Comparison of optimised rotors between two methods. a): after 29 iterations using SBTO with dielectric area constraint b): after 17 iterations using SBTO without dielectric area constraint c): after 100 iterations using the density method of [1].

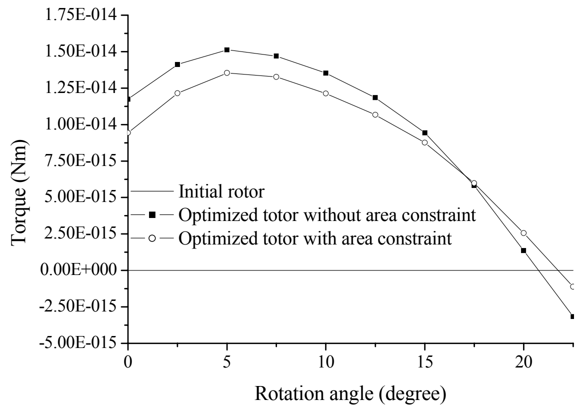

Fig 4. Torque profile before and after optimization.

5 Conclusions

The SBTO process is both efficient and avoids the "staircase" effects that are encountered in the cell-based process. The topology finally generated compares well with that developed in [1].

References

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