CALCULATION OF INDUCTED CURRENTS USING EDGE ELEMENTS AND $\boldsymbol{T}$-$\boldsymbol{T}_0$ FORMULATION

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Abstract

The paper discusses methods of calculating induced currents in multiply connected regions containing solid conductors. In particular, the formulation based on edge elements using the electric vector potential has been considered. The equations are explained using the language of circuit theory. It is observed that the edge values of $\boldsymbol{T}_0$ represent the loop currents in the loops surrounding the 'holes'. It is also shown that the iterative solution may be accelerated by over-specifying the number of loop currents in the loops around the 'holes'.

1 Introduction

Many devices operate by utilising conduction currents created by electromotive forces, known as induced currents. Systems using such currents may be categorised as (a) simply connected regions with solid conductors, e.g. the solid part of a magnetic core, (b) multiply connected regions with thin (filament) conductors, e.g. windings composed of stranded conductors, (c) multiply connected regions with solid conductors [1, 5], e.g. a solid core with holes, or windings composed of bars such as in a cage rotor of an induction motor. In (a) we are dealing with eddy currents. In (b) we have both induced and externally enforced currents. Currents imposed by external sources may also exist in category (c) systems, while the induced currents are the eddy currents and/or currents circulating around the 'holes', e.g. currents i_{bi} in the system of Figure 1. Thus (a) and (b) may be treated as special cases of (c) and in this paper we focus on the most general formulation by considering system (c). The application of a vector potential has been explored with the aim of deriving current distributions under the condition of known distribution and time variation of the magnetic flux density.

2 Circuit representation of the $\boldsymbol{T}$-$\boldsymbol{T}_0$ method

The analysis of conduction current distributions in electrical devices may be undertaken using an electric scalar or electric vector potential. Moreover, it is convenient to express the finite element (FE) formulation involving these potentials using the circuit theory analogy [3]. In particular, the FE equations for the scalar potential and nodal elements represent the nodal equations of a conductance network with branches associated with element edges. In contrast, the formulation involving the vector potential and edge elements is equivalent to loop analysis of a resistance network (RN). The nodes of such a network are associated with the volume centres while the branches pass through the facets (see Figure 1); it may therefore be called an electric facet network. The equations for the edge elements of the classical $\boldsymbol{T}$ formulation are equivalent to loop equations for loop currents around element edges [3, 4], designated as i_{mi} in Figure 1. In the case of a machine winding with multiply connected conductors, the fundamental circuits of such a network cannot be formed with these meshes only. In other words, the classical $\boldsymbol{T}$ formulation is not capable of treating such machine windings, a typical example being the common squirrel cage of an induction motor. To resolve this problem, additional loops L_0 with currents around holes must be introduced, e.g. loops with currents i_{bi} and i_{ri} in Figure 1. These loop currents may be considered to be the edge values of a vector potential $\boldsymbol{T}_0$ [2, 4]. Thus the loop equations for loop currents around edges and around 'holes' in the resistance network represent the edge element equations for the $\boldsymbol{T}$-$\boldsymbol{T}_0$ method.

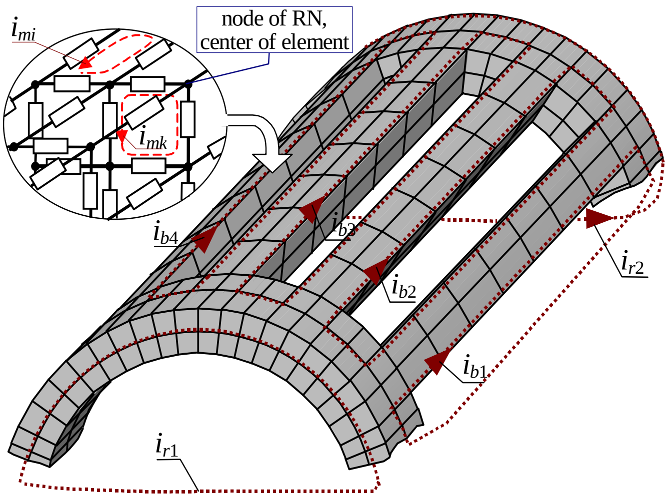

Fig.1. Squirrel cage of an induction motor as a multiply connected conductor and its resistance model.

3 Loops for currents representing edge values of $\boldsymbol{T}_0$

The loops with currents representing the edge values of $\boldsymbol{T}_0$ must be chosen carefully to complement the missing loops in the fundamental set. To achieve this a method put forward in [4] could be used, where a matrix $\boldsymbol{z}_e$ is formed of 'cuts' between the loop surface and the element edges (surface-edge or S-E method), or a method where a loop is defined by a matrix $\boldsymbol{z}_f$ of 'cuts' of the line L_0 with the element facets (line-facet or L-F method). The S-E approach is more universal as it applies to both magnetic scalar Ω and magnetic vector potential $\boldsymbol{A}$ formulations, while L-F is restricted to cases where $\boldsymbol{A}$ is used as the solution potential.

The analysis which follows applies to imposed (known) flux distributions. It is assumed that loop fluxes ϕ_f are known in loops associated with facets, i.e. loops around branches of the resistance network. The product of the vector ϕ_f and the transposed loop matrix $\boldsymbol{k}_e$ for the loops around the edges gives the vector of fluxes that penetrate the loops. For formulations using the matrix $\boldsymbol{z}_f$ the equation describing induced currents reads

$$\begin{bmatrix} \boldsymbol{k}_e^T \boldsymbol{R} \boldsymbol{k}_e & \boldsymbol{k}_e^T \boldsymbol{R} \boldsymbol{z}_f \\ \boldsymbol{z}_f^T \boldsymbol{R} & \boldsymbol{z}_f^T \boldsymbol{R} \boldsymbol{z}_f \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_m \\ \boldsymbol{i}_c \end{bmatrix} = \begin{bmatrix} -\mathrm{p}\, \boldsymbol{k}_e^T \boldsymbol{\phi}_f \\ -\mathrm{p}\, \boldsymbol{z}_f^T \boldsymbol{\phi}_f \end{bmatrix}, \qquad (1)$$

where $\boldsymbol{R}$ is the matrix of branch resistances calculated using the interpolation functions of a facet element, p=d/dt, whereas $\boldsymbol{i}_m$ and $\boldsymbol{i}_c$ are the vectors of loop currents in the loop around edges and holes (edge values of $\boldsymbol{T}$, $\boldsymbol{T}_0$), respectively.

4 Procedures for solving the $\boldsymbol{T}$-$\boldsymbol{T}_0$ equations

Consider iterative methods for solving equation (1) subject to sinusoidal time variation of magnetic flux passing through a thin plate with two holes as depicted by Figure 2. The position and number of additional loops were varied to examine their influence on the convergence of the iterative scheme. A comparison was made of the solution error after a prescribed number of iterations for both the SOR and "Block ICCG" algorithms. In the case of "Block ICCG" algorithm, the loop equations for the $\boldsymbol{T}$ formulation are solved using a conventional ICCG algorithm. The results are substituted to the equations for the $\boldsymbol{T}_0$ potentials which are then solved separately. The resulting edge values of $\boldsymbol{T}_0$ are used in the next step of the $\boldsymbol{T}$ calculations, and so on. The error was estimated as

$$\delta = \sum_{i=1}^{N} \left| i_{k,i} - i_{a,i} \right| \Big/ \sum_{i=1}^{N} \left| i_{a,i} \right| , \qquad (2)$$

where $i_{k,i}$ is the calculated current in the i-th branch after the k-th iteration step, $i_{a,i}$ is the exact value of the current in the i-th branch, and N is the total number of branches (N=292 in the example of Figure 2).

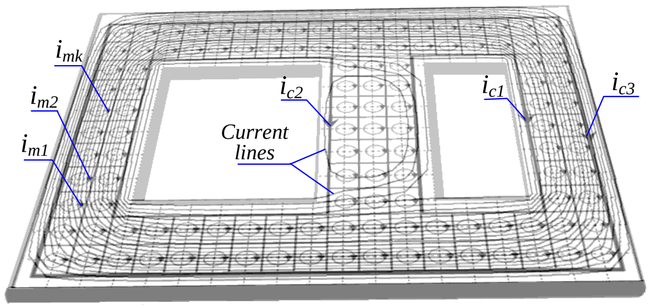

Fig. 2. Example of a conducting plate with 2 holes.

Figure 3 summarises the results showing the errors as iterations progress for the two chosen iterative schemes and three selected cases of additional loops. The case with three loops leads to an over-specified system of equations. It appears that thanks to the increase in the number of loops with current $\boldsymbol{i}_c$, the rate of convergence has been improved significantly. In the case of the "Block ICCG" method, with three additional loops, the calculation error after only 60 iterations is already at the level of rounding errors.

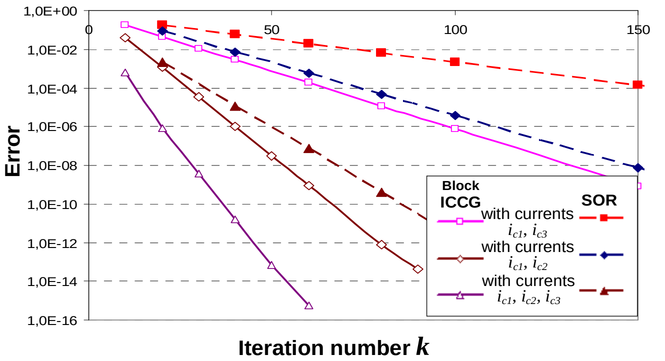

Fig. 3. Errors in current distribution after k iterations.

5 Conclusions

The use of the $\boldsymbol{T}$-$\boldsymbol{T}_0$ formulation has been explored in calculating induced currents in systems containing multiply connected conductors. It has been concluded that the use of the circuit representation of the finite element method is particularly helpful when designing computational algorithms. It has also been shown that – when solving iteratively equations of the $\boldsymbol{T}$-$\boldsymbol{T}_0$ formulation, that is equations describing current distributions in systems containing holes – it is beneficial to introduce superfluous loops. It emerges that by using iterative methods it is possible to solve a system of equations described by a singular matrix of coefficients, in other words obtain one solution of a multi-parameter set. After applying an appropriate iterative scheme (e.g. SOR) one of the solutions is found. It has been established that the process of finding one of available solutions iteratively converges faster than an algorithm seeking one unique solution of a system void of dependant loops.

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