Application of the Front-Fixing Method for Numerical Modelling of Field Diffusion in Non-linear Systems

Igor O. Golosnoy and Jan K. Sykulski, Senior Member IEEE
School of Electronics and Computer Science, University of Southampton, Southampton, SO17 1BJ, UK
ig@ecs.soton.ac.uk, jks@soton.ac.uk

Abstract — Application of a finite difference front fixing method for modelling magnetic field and associated power loss in high temperature superconductors or other strongly non-linear phenomena is considered. Advantages of the scheme are discussed and implementation problems highlighted. Particular attention is paid to conservation properties of the algorithm and accurate solutions close to the transition boundaries. The algorithm is tested using a well-known solution of the spherical diffusion problem with complex conditions at the moving interface.

I. Introduction

It was suggested to model AC losses in high-temperature superconducting (HTS) tapes as a diffusion process [1, 2]. Extreme variations in material properties with the field result in computational complications when conventional techniques are used due to extreme non-linearity. For example, an explicit scheme requires small time steps [2]. For a multi-dimensional case the mesh structure is also affected by stability criteria [1]. Rhyner [3] found that front-type solutions exist in the system. This was confirmed later by numerical simulations [1, 2]. Since field variation at the front is severe, an effective algorithm should take into account the front motion explicitly.

This paper deals with modelling of diffusion in an abstract system with a moving front boundary. The motion is assumed to be a complex function of the solution itself. Many numerical methods developed previously, such as front tracking or re-meshing techniques, are often not able to cope with such a strong coupling [4]. On the other hand, the transformations proposed by Landau [4] introduce a co-ordinate system in which all of the spatial boundaries are fixed. Under the transformation the new computational domains remain the same with an additional non-linear equation for the boundary motion. This allows keeping nodes close to the interface independent of the motion, which gives higher accuracy for the same number of nodes used. The main challenges are the implementation of conservation laws at the moving boundaries and an effective mesh refinement. These issues are addressed here. The solution quality is assessed using a simplified system with a known analytical solution.

II. Front-fixing method for diffusion problems

A diffusion-controlled phase change is an example of a non-linear, coupled diffusion-motion problem with a discontinuity at the moving interface, i.e. sharp front. The well known set of coupled non-linear differential equations may be used to model the system [4]. Landau transformation and further discretisation of the problem is discussed in [5]. The fact that the equations form a non-linear system means that the method could be potentially very demanding in terms of computing times. Nevertheless it is possible to implement an efficient iterative algorithm based on de-coupling the problem, with only 3 to 4 iterations needed to reach a consistent solution.

It is confirmed that the numerical scheme has conservative properties. The question of whether the solution is accurate remains. For particular initial and boundary conditions, analytical solutions are available [4]. This case is presented in Fig. 1, the results correspond to calculations completed using both irregular and regular meshes. It is clear that, for a given number of discretisation points, it is possible to find more accurate solutions by using irregular meshes. In this way errors can be reduced without requiring extra computational effort.

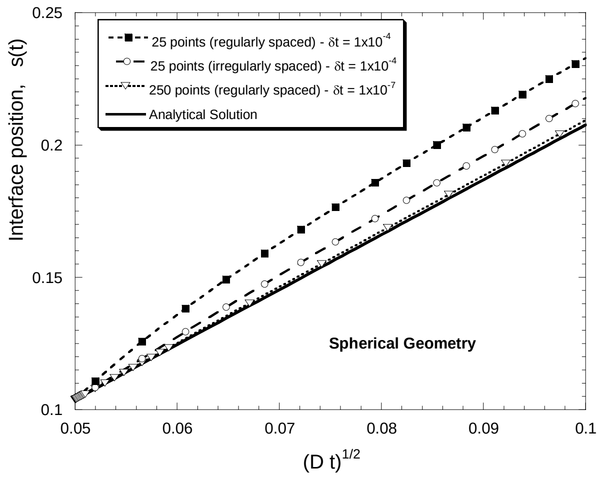

Fig. 1. Predictions for interface position: mesh size and time step effects [5].

III. Conclusions

The use of a front-fixing method for modelling non-linear diffusion processes is demonstrated. Potential problems with implementation of conservation laws and complex boundary conditions are considered and solutions suggested. It is shown that high accuracy can be achieved on a coarse irregular mesh since the interface is fixed in new coordinates.

IV. References

1. J. K. Sykulski, M. Rotaru, and R. L. Stoll, "2D Modelling of Field Diffusion and AC Losses in High Temperature Superconducting Tapes," *IEEE Transactions on Magnetics*, vol.36, no. 4, pp. 1178-1182, 2000.
2. J. K. Sykulski, R L. Stoll, and A. E. Mahdi, "Modelling HTc Superconductors for AC Power Loss Estimation," *IEEE Transactions on Magnetics*, vol.33, no. 2, pp. 1568-1571, 1997.
3. J. Rhyner, "Magnetic properties and AC-losses of superconductors with power law current-voltage characteristics," *Physica C*, vol. 212, no., pp. 292-300, 1993.
4. J. Crank, *Free and Moving Boundary Problems*. Clarendon Press, 1984.
5. T. C. Illingworth and I. O. Golosnoy, "Numerical Solutions of Diffusion-Controlled Moving Boundary Problems which Conserve Solute," *Journal of Computational Physics*, vol. 209, no. 1, pp. 207-225, 2005.