This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2008 proceedings.

Semi-Blind Spatial Equalisation for MIMO Channels with Quadrature Amplitude Modulation

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Abstract— **Semi-blind spatial equalisation is considered for multiple-input multiple-output (MIMO) systems that employ high-throughput quadrature amplitude modulation scheme. A minimum number of training symbols, equal to the number of transmitters, are first utilised to provide a rough least squares channel estimate of the system's MIMO channel matrix for the initialisation of the spatial equalisers' weight vectors. A constant modulus algorithm aided soft decision-directed blind algorithm is then employed to adapt the spatial equalisers. This semi-blind scheme has a very-low computational complexity, and it converges fast to the minimum mean-square-error spatial equalisation solution as demonstrated in our simulation study.**

I. Introduction

Multiple-input multiple-output (MIMO) technologies are capable of substantially improving the achievable system's capacity and/or quality of service [1], [2], [3], [4]. The system's ability to approach the MIMO capacity heavily relies on the channel state information. Accurately estimating a MIMO channel is much more challenging than its single-input single-output (SISO) counterpart. The various MIMO channel estimation methods can be classified into three categories: training-based methods, blind methods and semi-blind methods. Pure training-based schemes are computationally less demanding but a high proportion of training symbols is required in order to obtain a reliable MIMO channel estimate, which considerably reduces the achievable system throughput. The family of blind methods for joint channel estimation and data detection does not require training symbols and hence does not reduce the achievable system throughput, although this is achieved at the expense of high computational complexity. Moreover, blind joint channel estimation and data detection results in unavoidable estimation and decision ambiguities [5], and these ambiguities must be resolved by other means. Semi-blind schemes do not suffer from this ambiguity problem and are computationally simpler than their blind counterparts, at the cost of requiring a few training symbols.

Many semi-blind methods have been developed for MIMO systems. In the schemes of [6], [7], [8], [9], a few training symbols are used to provide an initial MIMO channel estimate, and the channel estimator as well as the data detector iteratively exchange their information, where the channel estimator relies on decision-directed adaptation. In [10], the MIMO channel matrix is decomposed into the product of a whitening matrix and a rotational unitary matrix. The first matrix is estimated blindly while the second is estimated with the aid of training symbols. In contrast to these proposals, recently we have proposed a novel semi-blind scheme for joint maximum likelihood (ML) channel estimation and data detection [11], where the joint ML channel and data estimation optimisation process is decomposed into two levels. At the upper level a global optimisation algorithm searches for an optimal channel estimate, while at the lower level a ML data detector recovers the transmitted data. Joint ML channel estimation and data detection is achieved by iteratively exchanging information between the channel estimator and the data detector. A minimum number of training symbols, equal to the number of transmitters, are used to provide an initial least squares channel estimate (LSCE) [12] for aiding the upper level channel estimator to improve convergence. The employment of a minimum training overhead has an additional benefit in terms of avoiding the ambiguities inherent in pure blind joint channel estimation and data detection.

In the above-mentioned semi-blind methods, the data detector is typically based on the ML detection principle. These semi-blind joint ML schemes are attractive because they are capable of approaching the optimal joint ML solution. However, for MIMO systems that employ high-throughput quadrature amplitude modulation (QAM) [13], these schemes become computationally prohibitive owe to the high complexity of ML data detection. Instead of performing joint channel estimation and data detection, we consider direct spatial filtering or equalisation for MIMO systems that employ high-order QAM schemes. The proposed method is semi-blind as we employ a minimum number of pilots to estimate the MIMO channel matrix via the LSCE. The resulting LSCE is used to initialise the weight vectors of the spatial equalisers. In general, this initialisation is not sufficiently accurate to achieve "eye opening", and therefore it is not safe to carry out decision-directed (DD) adaptation for the spatial equalisers. We propose to use a constant modulus algorithm (CMA) assisted soft DD (SDD) blind adaptive algorithm to adapt the spatial equalisers. The concurrent CMA and SDD algorithm was originally derived for blind equalisation of single-input single-output (SISO) QAM systems [14], and it was extended to single-input multiple-output (SIMO) systems in [15]. This blind adaptive scheme has a very low computational complexity. In the present MIMO application, owing to the initial information provided by the training pilots, the algorithm converges much

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faster than the pure blind adaptation case, and it is capable of approaching the performance of the minimum mean square error (MMSE) spatial equalisers based on the perfect channel knowledge, as will be shown in our simulation study.

To the best of our knowledge, this is the first time that a very low-complexity stochastic gradient adaptive semi-blind spatial equalisation scheme is proposed for MIMO-aided high-order QAM schemes. Recently, we have found one journal paper [16] in which the authors propose to adapt the spatial equaliser by minimising the combined cost function of the training-based sum of the squared errors and a higher-order statistic (HOS) aided criterion using a block-data based gradient algorithm. In terms of computational requirements, the complexity of the block-data based algorithm in [16] is significantly higher than that of our proposed stochastic gradient algorithm. In terms of the achievable equalisation performance, our simpler stochastic gradient scheme actually outperforms the more complex block-data based gradient scheme of [16]. This is because the blind adaptive process in the semi-blind scheme of [16] is based on the HOS (e.g. CMA) criterion, while our blind adaptive process is based on the HOS (CMA) aided SDD criterion. The latter can approach the optimal MMSE solution more accurately and achieve a faster convergence, as a benefit of the fact that SDD adaptation is more like the true training. Furthermore, in [16] the authors make an unnecessary assumption of the known MIMO channel matrix[1].

Throughout our discussions we adopt the following notational conventions. Boldface capitals and lower-case letters stand for matrices and vectors, respectively, while $\mathbf{I}_K$ denotes the $K \times K$ identity matrix. Furthermore, $(\bullet)^T$ and $(\bullet)^H$ are the transpose and Hermitian operators, respectively, while $\| \bullet \|$ and $| \bullet |$ denote the norm and magnitude operators, respectively. $E[\bullet]$ is the expectation operator, while $(\bullet)^*$ denotes the complex conjugate. Finally, $j = \sqrt{-1}$.

II. System Model

We consider a MIMO system consisting of n_T transmitters and n_R receivers, which communicates over flat fading channels. The system is described by the well-known MIMO model

$$\mathbf{x}(k) = \mathbf{H}\,\mathbf{s}(k) + \mathbf{n}(k), \tag{1}$$

where k is the symbol index, $\mathbf{H}$ denotes the $n_R \times n_T$ MIMO channel matrix, $\mathbf{s}(k) = [s_1(k)\ s_2(k) \cdots s_{n_T}(k)]^T$ is the transmitted symbols vector of the n_T transmitters with the symbol energy given by $E\left[|s_m(k)|^2\right] = \sigma_s^2$ for $1 \le m \le n_T$, $\mathbf{x}(k) = [x_1(k)\ x_2(k) \cdots x_{n_R}(k)]^T$ denotes the received signal vector, and $\mathbf{n}(k) = [n_1(k)\ n_2(k) \cdots n_{n_R}(k)]^T$ is the complex-valued Gaussian white noise vector associated with the MIMO channels with $E\left[\mathbf{n}(k)\mathbf{n}^H(k)\right] = 2\sigma_n^2 \mathbf{I}_{n_R}$. We assume that $n_T \le n_R$ and the channels are non-dispersive. Frequency selective channels can be made narrowband using for example the orthogonal frequency division multiplexing technique [17].

[1] If the MIMO channel matrix were known, the MMSE spatial equaliser could be designed directly and there would be no need for any semi-blind adaptation.

Specifically, the narrowband MIMO channel matrix is defined by $\mathbf{H} = [h_{l,m}]$, for $1 \le l \le n_R$ and $1 \le m \le n_T$, where $h_{l,m}$ denotes the non-dispersive channel coefficient linking the m-th transmitter to the l-th receiver. Moreover, the fading is assumed to be sufficiently slow, so that during the time period of a transmission block or frame, all the channel impulse response (CIR) taps $h_{l,m}$ in the MIMO channel matrix $\mathbf{H}$ may be deemed unchanged. From frame to frame, the CIR taps $h_{l,m}$ are independently and identically distributed (i.i.d.) complex-valued Gaussian processes with zero mean and $E\left[|h_{l,m}|^2\right] = 1$. The modulation scheme is the M-QAM and, therefore, the transmitted data symbols $s_m(k)$, $1 \le m \le n_T$, take the values from the M-QAM symbol set

$$\mathcal{S} \stackrel{\triangle}{=} \{s_{i,q} = u_i + j u_q,\ 1 \le i, q \le \sqrt{M}\} \tag{2}$$

with the real-part symbol $\Re[s_{i,q}] = u_i = 2i - \sqrt{M} - 1$ and the imaginary-part symbol $\Im[s_{i,q}] = u_q = 2q - \sqrt{M} - 1$. The average signal-to-noise ratio (SNR) is defined by

$$\text{SNR} = n_T \times \sigma_s^2 / 2\sigma_n^2. \tag{3}$$

A bank of the spatial filters or equalisers

$$y_m(k) = \mathbf{w}_m^H \mathbf{x}(k),\ 1 \le m \le n_T, \tag{4}$$

are used to detect the transmitted symbols $s_m(k)$ for $1 \le m \le n_T$, where $\mathbf{w}_m$ is the $n_R \times 1$ complex-valued weight vector of the m-th spatial equaliser.

III. The Proposed Semi-Blind Algorithm

Let the number of training symbols be K, and denote the available training data as $\mathbf{X}_K = [\mathbf{x}(1)\ \mathbf{x}(2) \cdots \mathbf{x}(K)]$ and $\mathbf{S}_K = [\mathbf{s}(1)\ \mathbf{s}(2) \cdots \mathbf{s}(K)]$. The LSCE of the MIMO channel matrix $\mathbf{H}$ based on $\{\mathbf{S}_K, \mathbf{X}_K\}$ is readily given as

$$\hat{\mathbf{H}} = \mathbf{X}_K \mathbf{S}_K^H \left(\mathbf{S}_K \mathbf{S}_K^H\right)^{-1}. \tag{5}$$

As a byproduct of the LSCE (5), an estimated noise variance is also produced as $2\hat{\sigma}_n^2 = \frac{1}{K \cdot n_R} \|\mathbf{X}_K - \hat{\mathbf{H}}\mathbf{S}_K\|^2$. In order to maintain throughput, the number of training pilots should be as small as possible. A necessary condition for $\mathbf{S}_K \mathbf{S}_K^H$ to have full rank is $K \ge n_T$. We will assume a minimum number of training symbols, namely $K = n_T$. The rough LSCE $\hat{\mathbf{H}}$ is utilised to provide the initialisation of the spatial equalisers' weight vectors via the MMSE solutions

$$\mathbf{w}_m(0) = \left(\hat{\mathbf{H}}\hat{\mathbf{H}}^H + \frac{2\hat{\sigma}_n^2}{\sigma_s^2}\mathbf{I}_{n_R}\right)^{-1} \hat{\mathbf{h}}_m,\ 1 \le m \le n_T, \tag{6}$$

where $\hat{\mathbf{h}}_m$ denotes the m-th column of $\hat{\mathbf{H}}$. Because the training data are insufficient, the weight vectors (6) are not sufficiently accurate to open the eye. Therefore, DD adaptation is generally unsafe. However, we can apply the concurrent CMA and SDD blind scheme [14], [15] to adapt the spatial filters (4) with $\mathbf{w}_m(0)$ of (6) as their initial weight vectors. Let the weight vector of the m-th spatial equaliser be split into two parts, yielding $\mathbf{w}_m = \mathbf{w}_{m,c} + \mathbf{w}_{m,d}$. The initial $\mathbf{w}_{m,c}$ and $\mathbf{w}_{m,d}$ can simply be set to $\mathbf{w}_{m,c}(0) = \mathbf{w}_{m,d}(0) = 0.5\mathbf{w}_m(0)$. Denote the spatial equaliser's output at sample k as $y_m(k) = \mathbf{w}_m^H(k)\mathbf{x}(k)$.

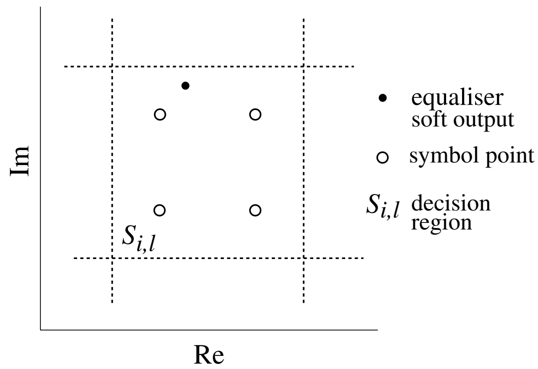

Fig. 1. Illustration of local decision regions for the soft decision-directed adaptation procedure for QAM constellation.

Specifically the weight vector $\mathbf{w}_{m,c}$ is updated using the classical CMA [18], [19]

$$\left.\begin{array}{l} \varepsilon_m(k) = y_m(k)\left(\Delta - |y_m(k)|^2\right), \\ \mathbf{w}_{m,c}(k+1) = \mathbf{w}_{m,c}(k) + \mu_{\text{CMA}}\varepsilon_m^*(k)\mathbf{x}(k), \end{array}\right\} \quad (7)$$

where $\Delta = E\left[|s_i(k)|^4\right]/E\left[|s_i(k)|^2\right]$ and μ_{CMA} is the step size of the CMA. The weight vector $\mathbf{w}_{m,d}$ by contrast is updated using the SDD scheme [14], [15], which has its root in the blind scheme of [20]. The complex phasor plane is divided into the $M/4$ rectangular regions, as illustrated in Fig. 1. Each region $\mathcal{S}_{i,l} = \{s_{p,q},\ p = 2i-1, 2i, q = 2l-1, 2l\}$ contains four symbol points. If the spatial equaliser's output $y_m(k) \in \mathcal{S}_{i,l}$, a local approximation of the marginal probability density function (PDF) of $y_m(k)$ is given by [14], [15]

$$\hat{p}(\mathbf{w}_m, y_m(k)) \approx \sum_{p=2i-1}^{2i} \sum_{q=2l-1}^{2l} \frac{1}{8\pi\rho} e^{-\frac{|y_m(k)-s_{p,q}|^2}{2\rho}}, \quad (8)$$

where ρ defines the cluster width associated with the four clusters of each region $\mathcal{S}_{i,l}$. The SDD algorithm is designed to maximise the log of the local marginal PDF criterion $E[J_{\text{LMAP}}(\mathbf{w}_m, y_m(k))]$, where $J_{\text{LMAP}}(\mathbf{w}_m, y(k)_m) = \rho \log\left(\hat{p}(\mathbf{w}_m, y_m(k))\right)$, via a stochastic gradient optimisation. Specifically, $\mathbf{w}_{m,d}$ is updated according to

$$\mathbf{w}_{m,d}(k+1) = \mathbf{w}_{m,d}(k) + \mu_{\text{SDD}} \frac{\partial J_{\text{LMAP}}(\mathbf{w}_m(k), y_m(k))}{\partial \mathbf{w}_{m,d}}, \quad (9)$$

where μ_{SDD} is the step size of the SDD, and

$$\frac{\partial J_{\text{LMAP}}(\mathbf{w}_m, y_m(k))}{\partial \mathbf{w}_{m,d}} = \frac{1}{Z_N} \sum_{p=2i-1}^{2i} \sum_{q=2l-1}^{2l} e^{-\frac{|y_m(k)-s_{p,q}|^2}{2\rho}} (s_{p,q} - y_m(k))^* \mathbf{x}(k), \quad (10)$$

with the normalisation factor

$$Z_N = \sum_{p=2i-1}^{2i} \sum_{q=2l-1}^{2l} e^{-\frac{|y_m(k)-s_{p,q}|^2}{2\rho}}. \quad (11)$$

The choice of ρ, defined in the context of local PDF (8), should ensure a proper separation of the four clusters of $\mathcal{S}_{i,l}$. As the minimum distance between the two neighbouring constellation points is 2, ρ is typically chosen to be less than 1. More specifically, when the equalisation objective is accomplished, $y_m(k) \approx s_m(k) + e_m(k)$, where $e_m(k)$ is Gaussian distributed with zero mean. Therefore, the value of ρ is related to the variance of $e_m(k)$, which is $2\sigma_n^2 \mathbf{w}_m^H \mathbf{w}_m$. Thus, for high SNR situations, small ρ is desired, while for low SNR cases, large ρ is preferred. Soft decision nature becomes explicit in (10), because rather than committing to a single hard decision $\mathcal{Q}[y_m(k)]$, where $\mathcal{Q}[\bullet]$ denote the quantisation operator, as the hard DD scheme would, alternative decisions are also considered in the local region $\mathcal{S}_{i,l}$ that includes $\mathcal{Q}[y_m(k)]$, and each tentative decision is weighted by an exponential term $e^{\{\bullet\}}$, which is a function of the distance between the equaliser's soft output $y_m(k)$ and the tentative decision $s_{p,q}$. This soft decision nature substantially reduces the risk of error propagation and achieves faster convergence, compared with the hard DD scheme [14], [15].

IV. Simulation Study

The achievable performance was assessed in the simulation using the symbol error rate (SER). The analytical SER for the spatial equaliser (4) is given in [21].

Stationary MIMO system. We considered a fixed MIMO system with $n_T = 4$ and $n_R = 4$, and the modulation scheme was 16-QAM. The simulated stationary 4×4 MIMO channel matrix $\mathbf{H}$ is listed in Table I. The number of pilot symbols used for the semi-blind scheme was $K = 4$. Firstly, training-based spatial filtering was demonstrated. Given K training symbols, the LSCE $\hat{\mathbf{H}}$ was obtained, which was then used to calculate the MMSE solution for the weight vectors of the four spatial equalisers. The average SER performance over all the four spatial equalisers as a function of the training symbols K are depicted in Fig. 2, with the average SER of the true MMSE spatial equalisers calculated based on the true MIMO channel matrix $\mathbf{H}$ as the benchmark. It can be seen from Fig. 2 that the training-based scheme required more than 64 training pilots to closely approach the optimal MMSE performance. For the simulated MIMO system, the 4-th spatial equaliser had the worst SER performance while the 1st spatial equaliser had the best SER performance. Therefore, the average SER

TABLE I
THE SIMULATED STATIONARY 4×4 MIMO SYSTEM

$-1.377 - 0.600j$	$0.474 + 1.105j$	$0.370 - 0.775j$	$-0.569 - 0.298j$
$1.700 - 0.290j$	$1.346 - 0.348j$	$-0.130 - 1.413j$	$-0.532 - 0.494j$
$1.027 + 0.466j$	$-0.580 + 0.833j$	$-0.586 - 0.231j$	$-0.340 + 0.184j$
$1.352 - 1.313j$	$-0.678 + 0.968j$	$0.874 - 0.338j$	$-0.128 + 0.659j$

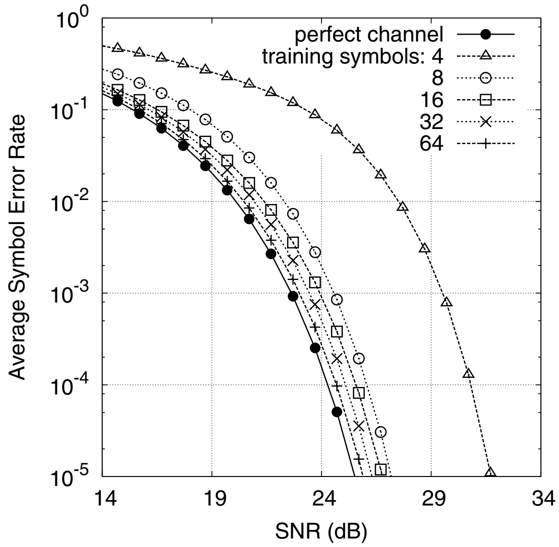

Fig. 2. Average SER performance of the training-based spatial equalisation given different numbers of training symbols, in comparison with the case of perfect channel knowledge.

performance shown in Fig. 2 was dominated by the worst case of the 4-th spatial equaliser.

The proposed semi-blind spatial equalisation scheme was next investigated. Given the average SNR of 21.7 dB, $K = 4$ training pilots were first used to provide the initial weight vectors of the four spatial equalisers according to (6). The appropriate values for the step sizes of the CMA and SDD were found empirically to be $\mu_{\rm CMA} = 0.000005$ and $\mu_{\rm CMA} = 0.0005$. Fig. 3 plots the learning curves of the combined CMA and SDD adaptive algorithm, in terms of the average SER over all the four spatial equalisers and over ten different runs, for the three values of the cluster width ρ. It is observed from Fig. 3 that, aiding by the information provided by the four training pilots, the convergence rate of the concurrent CMA and SDD algorithm was much faster than the pure blind adaptive counterpart of [14], [15]. Furthermore, the proposed semi-blind scheme is capable of approaching the optimal

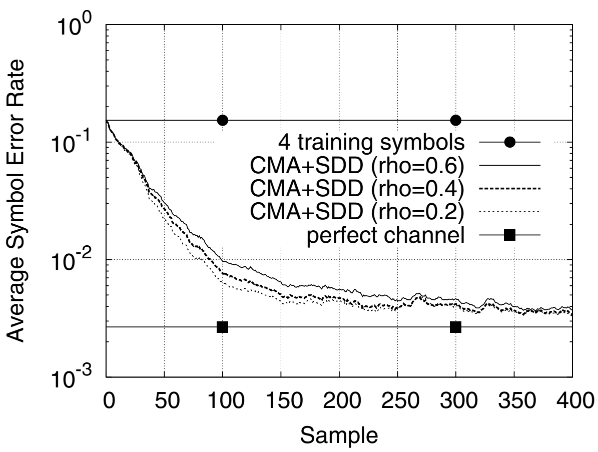

Fig. 3. Learning curves of the concurrent CMA and SDD scheme in terms of the SER average over all the four spatial equalisers and over ten different runs, given SNR of 21.7 dB and three values of ρ.

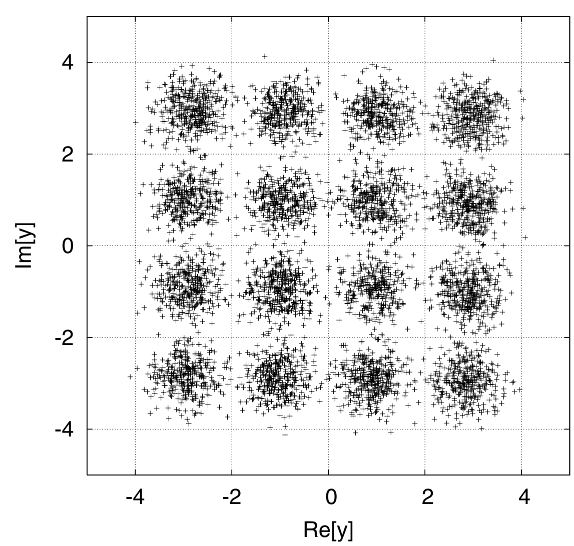

Fig. 4. The 4-th spatial equaliser's output constellation after blind adaptation, given SNR of 21.7 dB.

MMSE solution, as can be seen in Fig. 3.

Given the average SNR of 21.7 dB, $K = 4$ training symbols were generally insufficient for a spatial equaliser to achieve opening-eye. By contrast, the 4-th spatial equaliser's output constellation after blind adaptation is illustrated in Fig. 4, clearly showing that the eye was opened. Finally, the average SER performance achieved by the proposed semi-blind spatial equalisation scheme with assistant of four training pilots is compared with that of the perfect channel knowledge as well as that of the training-based scheme utilising only four training pilots. The results showing in Fig. 5 clearly confirm that the proposed semi-blind spatial equalisation scheme closely approached the optimal MMSE spatial equalisation solution.

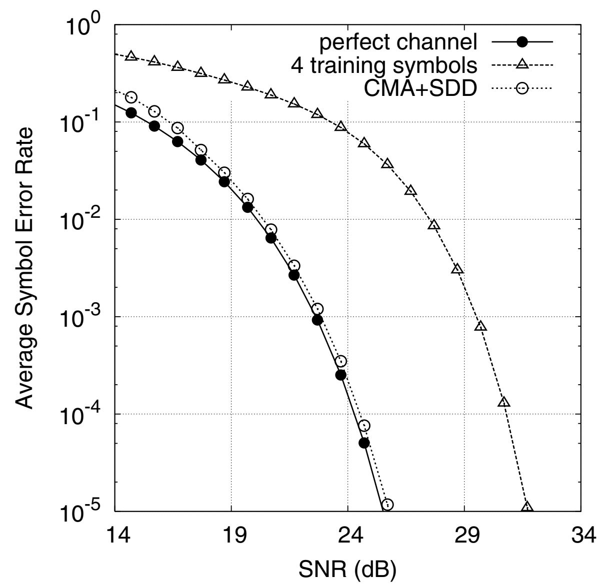

Fig. 5. Average SER performance of the proposed semi-blind spatial equalisation scheme with four training symbols, in comparison with the cases of training only based on four training symbols and perfect channel knowledge.

Flat fading MIMO system. A flat fading MIMO system with $n_T = 4$, $n_R = 5$ and the 16-QAM modulation scheme was simulated, whose CIR taps $h_{l,m}$, $1 \le l \le 5$ and $1 \le m \le 4$, were i.i.d. complex-valued Gaussian processes with zero mean and $E\left[|h_{l,m}|^2\right] = 1$. The number of pilot symbols used for the semi-blind scheme was $K = 5$, and the performance was averaged over 100 channel realisations. The average SER performance over all the four spatial equalisers for the purely training based scheme with 5, 15 and 55 training symbols, respectively, as well as the proposed semi-blind spatial equalisation scheme with aid of 5 training symbols are shown in Fig. 6, in comparison with the achievable performance given the perfect channel knowledge. The step size of the CMA as well as the step size and cluster width of the SDD were empirically set to $\mu_{\mathrm{CMA}} = 2 \times 10^{-6}$, $\mu_{\mathrm{SDD}} = 5 \times 10^{-4}$ and $\rho = 0.5$. The blind adaptive process was observed to achieve convergence typically within 300 samples. It can be seen from Fig. 6 that to achieve a similar performance as the semi-blind CMA-SDD scheme the training based scheme required 55 training symbols.

V. Conclusions

A semi-blind spatial equalisation scheme has been proposed for MIMO systems that employ high throughput QAM signalling. A minimum number of training symbols, equal to the number of transmitters, is used to estimate the MIMO channel matrix and the resulting rough LSCE is utilised for the initialisation of the spatial equalisers. The CMA aided SDD blind adaptive scheme is then adopted to adapt the spatial equalisers. The proposed semi-blind spatial equalisation scheme has a very low computational complexity. Our simulation study has confirmed that this semi-blind concurrent CMA and SDD adaptive algorithm converges much faster than its pure blind counterpart, and it is capable of approaching the optimal MMSE spatial equalisation solution calculated based on the perfect channel knowledge.

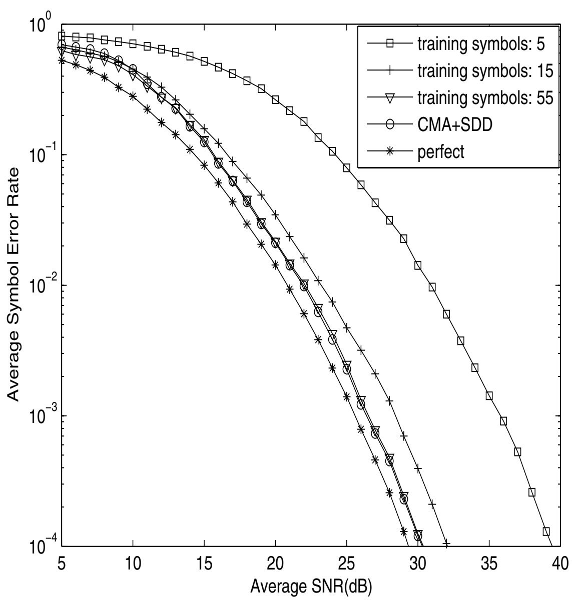

Fig. 6. Average symbol error rate performance of the proposed semi-blind spatial equalisation scheme with five training symbols, in comparison with the cases of training only based on different numbers of training symbols and perfect channel knowledge, averaged over 100 realisations of the flat Rayleigh fading 5×4 16-QAM MIMO system.

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