Semi-Blind Spatial Equalisation for MIMO Channels with Quadrature Amplitude Modulation

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Outline

- Motivations for semi-blind detection of quadrature amplitude modulation MIMO
- MIMO signal model and proposed semi-blind spatial equalisation scheme
- Simulation investigation and performance comparison
Motivations

- Knowledge of **channel state information** is critical to achieve capacity enhancement promised by MIMO, but perfect CSI is often unavailable.

- Estimating MIMO channel matrix is a tough job, and **training**-based channel estimation is simple but it reduces achievable throughput.

- **Blind** joint channel estimation and data detection does not reduce achievable throughput but is computationally complex.

- To resolve **ambiguities** in channel estimation and symbol detection, a few pilot symbols, i.e. some training, are necessary.

  ⇒ Various **semi-blind** joint maximum likelihood (ML) channel estimation and data detection schemes.
Motivations (continue)

- Semi-blind iterative least squares channel estimation (LSCE) and ML data detection has attract much attention
  \(\downarrow\) difficult to extend to high-order quadrature amplitude modulation MIMO systems

- Semi-blind **spatial equalisation** offers potentially low-complexity scheme for such MIMO systems

Existing work (Ding, Ratnarajah & Cowan, 2008, TSP)

- We propose a semi-blind spatial equalisation based on **constant modulus algorithm** assisted **soft decision directed** scheme
  \(\uparrow\) low-complexity **high-performance** \(\rightarrow\) approaches **minimum mean square error** solution based on perfect channel state information
Signal Model

- MIMO system of $n_T$ transmitters/$n_R$ receivers, flat fading channels

$$x(k) = Hs(k) + n(k)$$

Transmitted symbol vector $s(k) = [s_1(k) \ s_2(k) \cdots s_{n_T}(k)]^T$, received signal vector $x(k) = [x_1(k) \ x_2(k) \cdots x_{n_R}(k)]^T$, channel AWGN vector $n(k) = [n_1(k) \ n_2(k) \cdots n_{n_R}(k)]^T$, $n_T \leq n_R$

- $n_R \times n_T$ channel matrix $H = [h_{p,m}]$, $1 \leq p \leq n_R$ and $1 \leq m \leq n_T$

  $h_{p,m}$ is a complex Gaussian process with zero mean and $E[|h_{p,m}|^2] = 1$

  **Block fading**, where $h_{p,m}$ is kept constant over small block of $N$ symbols

- $M$-QAM constellation: $s_m(k) \in \mathcal{S} \triangleq \{s_{i,q} = u_i + ju_q, \ 1 \leq i, q \leq \sqrt{M}\}$

  with $\Re[s_{i,q}] = u_i = 2i - \sqrt{M} - 1$ and $\Im[s_{i,q}] = u_q = 2q - \sqrt{M} - 1$
Spatial Equalisation

- Bank of **spatial equalisers** for detecting transmitted symbols $s_m(k)$

$$y_m(k) = w_m^H x(k), \ 1 \leq m \leq n_T$$

- Given **initial training data** $X_K = [x(1) x(2) \cdots x(K)]$ and $S_K = [s(1) s(2) \cdots s(K)]$, **LSCE** of channel $H$

$$\hat{H} = [\hat{h}_1 \cdots \hat{h}_{n_T}] = X_K S_K^H (S_K S_K^H)^{-1}$$

with estimated noise variance $2\hat{\sigma}_n^2 = \frac{1}{K \cdot n_R} \|X_K - \hat{H} S_K\|^2$

- **Initial** spatial equalisers’ **weight vectors**

$$w_m(0) = \left( \hat{H} \hat{H}^H + \frac{2\hat{\sigma}_n^2}{\sigma_s^2} I_{n_R} \right)^{-1} \hat{h}_m, \ 1 \leq m \leq n_T$$

- For full rank $S_K S_K^H$, $K \geq n_T \Rightarrow$ **minimum training pilots** $K = n_T$
Concurrent Blind Adaptation

- **Concurrent** CMA and SDD equalisers: \( w_m = w_{m,c} + w_{m,d} \) with initial 
  \( w_{m,c}(0) = w_{m,d}(0) = 0.5w_m(0) \)

- **Constant modulus algorithm:**
  
  - Given spatial equaliser’s output \( y_m(k) = w_m^H(k)x(k) \) at sample \( k \)

  \[
  \begin{align*}
  \varepsilon_m(k) &= y_m(k) \left( \Delta - |y_m(k)|^2 \right), \\
  w_{m,c}(k+1) &= w_{m,c}(k) + \mu_{\text{CMA}} \varepsilon_m^*(k)x(k),
  \end{align*}
  \]

  - \( \Delta = E[|s_i(k)|^4] / E[|s_i(k)|^2] \) and \( \mu_{\text{CMA}} \) is step size

- **Soft decision directed equaliser:** maximise marginal PDF

  \[
  J_{\text{LMAP}}(w_m, y_m(k)) = \rho \log (\hat{p}(w_m, y_m(k)))
  \]

  of spatial equaliser’s output based on **stochastic gradient** optimisation
Soft Decision Directed Scheme

- Phasor plane is divided into $M/4$ regions

\[ S_{i,l} = \{ s_{p,q}, \quad p = 2i - 1, 2i, \quad q = 2l - 1, 2l \} \]

- If $y_m(k) \in S_{i,l}$, local approximation of marginal PDF of $y_m(k)$ is

\[ \hat{p}(w_m, y_m(k)) \approx \sum_{p=2i-1}^{2i} \sum_{q=2l-1}^{2l} \frac{1}{8\pi \rho} e^{-\frac{|y_m(k) - s_{p,q}|^2}{2\rho}} \]

- SDD weight updating:

\[ w_{m,d}(k + 1) = w_{m,d}(k) + \mu_{\text{SDD}} \frac{\partial J_{\text{LMAP}}(w_m(k), y_m(k))}{\partial w_{m,d}} \]
\( \mu_{\text{SDD}} \) is step size and \( \rho \) cluster width: when equalisation is done, 
\( y_m(k) \approx s_m(k) + e_m(k) \), where 
\( e_m(k) \) is Gaussian distributed with zero mean and variance \( 2\sigma_n^2 w_m^H w_m \)

\[ \rho \propto 2\sigma_n^2 w_m^H w_m \]

- **Soft DD nature**

\[
\frac{\partial J_{\text{LMAP}}(w_m, y_m(k))}{\partial w_{m,d}} = \frac{1}{Z_N} \sum_{p=2i-1}^{2i} \sum_{q=2l-1}^{2l} e^{-\frac{|y_m(k) - s_{p,q}|^2}{2\rho}} (s_{p,q} - y_m(k))^* x(k)
\]

with normalisation

\[
Z_N = \sum_{p=2i-1}^{2i} \sum_{q=2l-1}^{2l} e^{-\frac{|y_m(k) - s_{p,q}|^2}{2\rho}}
\]
Stationary MIMO Example

- **Stationary** $4 \times 4$ MIMO with 64 QAM, training pilots $K = 4$

- **Learning curve** of semi-blind CMA+SDD averaged over 10 runs and over all four spatial equalisers: average SNR $\approx 29$ dB, $\mu_{\text{CMA}} = 4 \times 10^{-7}$, $\mu_{\text{SDD}} = 2 \times 10^{-4}$

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<th>$-1.4 - 0.6j$</th>
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<td>$1.3 - 0.3j$</td>
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<td>$1.2 - 1.3j$</td>
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<td>$0.9 - 0.3j$</td>
<td>$-0.1 + 0.7j$</td>
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Average Symbol Error Rate

Sample

4 training symbols
CMA+SDD ($\rho=0.6$)
CMA+SDD ($\rho=0.2$)
perfect channel
Stationary MIMO Example (continue)

Average symbol error rates of spatial equalisation (a) training-based given different numbers of training symbols, and (b) semi-blind CMA+SDD, in comparison with minimum mean square error solution based on perfect channel knowledge.

(a)

(b)
Block Rayleigh Fading MIMO Example

- $5 \times 4$ MIMO with 16-QAM, simulated channel taps $h_{l,m}$, $1 \leq l \leq 5$ and $1 \leq m \leq 4$, were i.i.d. complex-valued Gaussian processes with zero mean and $E[|h_{l,m}|^2] = 1$
- Performance averaged over 100 channel realisations
- Number of pilot symbols $K = 5$, $\mu_{\text{CMA}} = 2 \times 10^{-6}$, $\mu_{\text{SDD}} = 5 \times 10^{-4}$ and $\rho = 0.5$
- Blind adaptive process typically converged within 300 samples
Conclusions

- A low-complexity high-performance semi-blind spatial equalisation scheme has been proposed for high-order QAM MIMO.
- Minimum number of pilot symbols, equal to the number of transmit antennas, are used for initial training.
- Constant modulus algorithm assisted soft decision directed scheme is applied for blind adaptation.
- The scheme converges fast and is capable of approaching the optimal MMSE solution based on perfect channel knowledge.
- Effectiveness of proposed semi-blind spatial equalisation scheme has been demonstrated using simulation.
THANK YOU.

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