H-ARQ-Aided Systematic Luby Transform Codes


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Abstract – Hybrid Automatic-Repeat-Request (H-ARQ) Aided Systematic Luby Transform (SLT) Coded Modulation is proposed, where SLT codes are used both for correcting erroneous bits and for detecting as well as retransmitting erroneous Internet Protocol (IP) based packets. Erroneous IP packet detection is implemented using syndrome checking with the aid of the SLT codes’ Parity Check Matrix (PCM). Optimizing the mapping of SLT-encoded bits to modulated symbols and then using iterative decoding for exchanging extrinsic information between the SLT decoder and the demapper substantially improves the achievable Bit Error Ratio (BER) performance of the scheme. Quantitatively, at $E_b/N_0$ in excess of 3.8 dB, this scheme is capable of achieving a BER $\leq 10^{-5}$ and up to 1.5 times higher throughput in comparison to less sophisticated benchmarker schemes such as SLT codes, dispensing with ARQ-assistance or joint SLT coded modulation and H-ARQ-SLT codes, when communicating over AWGN channels, using 16-QAM and a half-rate SLT code.

Index Terms—EXIT charts, Systematic Luby Transform Codes, Hybrid Automatic-Repeat-reQuest, Syndrome Checking, Parity Check Matrix, Coded Modulation, Quadrature Amplitude Modulation.
I. INTRODUCTION

Hybrid-Automatic Repeat reQuest (H-ARQ) conveniently amalgamates packet retransmission techniques with error-correction and error-detection algorithms in order to improve the performance of wireless communication systems over hostile channels [1], [2]. There are two basic types of H-ARQ, namely H-ARQ type I and H-ARQ type II [3], [4]. The transmitter of the H-ARQ type I scheme retransmits both the information part and the parity part of corrupted packets, when the transmitter receives a repeat request from the receiver. This process is typically implemented within the Media Access Control (MAC) layer. Naturally, retransmitting both the information and parity part of the packets reduces the achievable effective throughput. Hence, the H-ARQ type I [3] scheme is often replaced by the H-ARQ type II [4] arrangement. In the H-ARQ type II scheme, the information part and the parity part are sent together during the first transmission attempt. However, during the second transmission attempt additional parity information is transmitted. The receiver then uses the parity received during all transmissions to correct the information received.

For the sake of achieving a low Bit Error Ratio (BER), H-ARQ coded modulation schemes were employed in [5], [6] which invoked various coded modulation techniques [7], [8] and [9]. Iterative decoding was used for exchanging extrinsic information between the Forward Error Correction (FEC) decoder and the demapper of the demodulator.

Systematic Luby Transform (SLT) codes were proposed in [10], [11], which achieved significant BER performance improvements for transmission over both noisy and fading channels. Against this background, the novel contribution of this paper is an amalgamated H-ARQ SLT coded modulation scheme, which outperforms the classic H-ARQ aided coded modulation schemes of [12], [13] using Cyclic Redundancy Checking (CRC) to detect erroneously received packets.

We used a modified version of the well-known H-ARQ Type II [4]. In the classic H-ARQ Type II scheme
the transmitter implements the retransmission process by initially puncturing some of the parity bits and then incrementally transmitting additional portions of the punctured parity, whenever it receives a request for further parity from the receiver. The receiver then uses both the previously and the currently received parity portions in order to repeat the decoding process with a higher chance of decoding success. If the receiver is still unable to generate error-free decoded information, this process is repeated until the maximum number of retransmissions is exhausted. By contrast, our simplified H-ARQ Type II scheme was implemented as follows. The receiver invokes the syndrome checking technique of [14] for detecting the legitimate codewords at the output of the SLT decoder. When an illegitimate codeword is deemed to be present at the output of the SLT decoder, the receiver requests the transmitter to retransmit the entire parity part of the illegitimate SLT codeword. The SLT decoder retains the previous LLRs of the information bits generated by the message passing between the information part and the previous parity part of the parity matrix H. The new parity part of the parity matrix H continues its message passing algorithm with the aid of the updated LLRs, until a legitimate SLT codeword is arrived at or the maximum number of iterations is exhausted.

The novelty and rationale of our H-ARQ aided SLT coded scheme is summarised as follows:

1) In contrast to conventional LT codes, the SLT code advocated does not impose a high delay nor does it inflict an avalanche-like inter-packet error-propagation in the presence of channel errors.

2) The SLT code of [11] has a fixed packet-level overhead, regardless of the channel-quality encountered, which may be excessive or insufficient. This results in either a reduced effective throughput or a failure to recover the original source file. By contrast, the proposed H-ARQ-aided SLT scheme always uses just sufficient redundancy for error-free SLT packet recovery and hence maximizes the effective throughput.

3) The H-ARQ aided SLT coded modulation scheme invokes its own inherent syndrome checking for detecting corrupted packets and hence for activating the H-ARQ scheme. The parity check matrix \( H \) of the SLT code contains many elements in its rows and columns, hence checking the integrity of
the packets by the syndrome checking of the iterative SLT decoding process [14] instead of using an additional overhead for Cyclic Redundancy Checking (CRC) is more reliable and does not impose any extra redundancy. Quantitatively, we will demonstrate that our novel scheme can achieve BER \( \leq 10^{-5} \) at an \( E_b/N_0 \) value, which is 3 dB lower than the SLT coded scheme having the same code rate \( r = 1/2 \), but dispensing with H-ARQ assistance. Moreover, our novel scheme’s throughput is about 0.4 bit/symbol higher than that of the corresponding SLT coded scheme, when using 16-QAM modulation.

4) Semi-analytical EXIT-chart analysis will be used for finding the most appropriate bit-to-symbol mapping scheme without having to resort to time-consuming Monte-Carlo simulations, when designing either for achieving the lowest possible complexity or the lowest \( E_b/N_0 \) requirement.

In a next shell, we propose H-ARQ aided SLT coded modulation using syndrome based H-ARQ activations, which achieves BER performance and an improved throughput in comparison to the state-of-the-art.

The paper is organized as follows. Section II presents the proposed H-ARQ aided SLT coded modulation scheme, detailing its architecture. Section III analyses the H-ARQ-SLT coded modulation scheme using different mappers with the aid of EXIT charts. Section IV details the associated system parameters and characterizes the achievable BER performance of the H-ARQ SLT coded modulation scheme. Section V presents our conclusions and future research ideas.

II. H-ARQ-SLT CODED MODULATION SCHEME

As seen in Fig. 1, the source data is packetized by the Internet Protocol (IP) packetizer at the transmitter [15], before being passed to the SLT encoder block. A frame of \( k \) IP packets is buffered and then passed to the SLT encoder. Due to the fact that the number of bits in the IP header part is lower than in the data part, the IP header bits may be protected by strong SLT codes without substantially increasing the overheads. An example of the relation between the SLT packets and IP packets is shown in Fig. 2. Here, \( k \) bits of each SLT packet are arranged in the horizontal dimension and \( K' \) bits of each IP packet are assigned
in the vertical dimension. Each IP packet contains an \( L \)-bit header part and an \( S \)-bit data part, while \( K \) source SLT packets are formed from \( k \) IP packets. An SLT codeword is constituted by \( K \) SLT source bits representing an IP source packet plus \( M \) number of SLT parity bits. The SLT codewords at the output of the SLT encoder are interleaved, where the interleaver has a length of \( N \) bits and then passed to the mapper for forwarding in terms of 4-bit symbols to the 16-Quadrature Amplitude Modulation (16-QAM) modulator, where \( N \) is the SLT codeword’s total length. We term this serially concatenated system as the Hybrid-ARQ aided Systematic Luby Transform coded modulation (H-ARQ-SLT) scheme. The mapper of the 16-QAM modulator employs different types of mapping schemes, namely Gray mapping and set-partitioning based mapping [16], [17]. The mappers are designed to attain the largest minimum Euclidean distance between phaser points having a Hamming distance of one and the smallest average number of nearest neighbor signal points [18]. Viewing the constellation from a different perspective, the average number of bits \( N_{\text{min}} \) that differs between two closest phaser points in the constellation is calculated as follows [18]

\[
N_{\text{min}} = \frac{1}{L} \cdot N_0 \cdot \sum_{s \in S} \sum_{s' \in S_s} d_H(s, s'),
\]

(1)

where \( s \) denotes the specific phaser considered in a two-dimensional signal set \( S \), \( s' \) represents a phaser in the neighboring signal subset \( S_s \) of the phaser \( s \), \( L = 2^m \) denotes the number of phaser points in the constellation, with \( m \) being the number of bits in a symbol of the constellation and finally, \( d_H(s, s') \) denotes the Hamming distance between \( s \) and \( s' \). Still referring to (1), \( N_0 \) physically represents the average number of nearest neighbor phaser points in the constellation and it is calculated as follows:

\[
N_0 = \frac{1}{L} \sum_{s \in S} N_s,
\]

(2)

where \( N_s \) is the number of nearest neighbor phaser points adjacent to \( s \), i.e. the number of phasers in the subset and \( N_{\text{min}} = 1 \), when using the Gray mapper [16].

Again, the modulation scheme is 16-QAM, hence each modulated symbol is represented by four bits. The transmitter is also equipped with an ACKnowledgement (ACK) receiver, while the receiver has an ACK
transmitter in order to feed back the ACK signal to the transmitter. The ACK transmitter employed at the receiver side is controlled by the SLT decoder and the syndrome checking block of Fig. 1, as proposed in [14] in order to identify the error-free legitimate SLT codewords. When a legitimate codeword is detected at the output of the SLT decoder, the SLT decoder generates a signal $S_c$ for the ACK transmitter of the receiver, which is then conveyed to the system’s transmitter. By contrast, if the output of the SLT decoder is an illegitimate codeword, the SLT decoder activates the ACK transmitter of the receiver to send a Negative-ACK (NACK) signal to the system’s transmitter to request the retransmission of this codeword.

For each QAM symbol $x = \Gamma(b)$, where $b$ is the m-bit sequence $b= (b_0, b_1, \cdots, b_{m-1})$, associated with the a priori LLRs spanning from $\text{LLR}(b_0)$ to $\text{LLR}(b_{m-1})$ we have [19]:

$$\text{LLR}(b_j) = \log[P(b_j = 1)]/\log[P(b_j = 0)] \quad j = 0, \cdots, m-1. \quad (3)$$

The received signal $y_k$ is represented as $y_k = a_k x_k + n_k$ [20], where $n_k$ is the complex-valued zero-mean Gaussian noise process having a variance of $\sigma^2 = N_0/2$ per dimension. The notation $a_k$ represents the complex-valued time-variant fading gain and $x_k$ denotes the transmitted QAM symbol, when detection benefitting from coherent perfect channel knowledge is used. Then the conditional Probability Density Function (PDF) of the received signal $y$ may be calculated as follows [20]:

$$P(y|x, a) = \left(\frac{1}{\pi N_0}\right)\exp(-\frac{E_s}{N_0}|y - ax|^2). \quad (4)$$

Taking the logarithm of both two sides of (4), we arrive at the logarithmic-probability of [20]:

$$\log P(y|x, a) = \log \left(\frac{1}{\pi N_0}\right) - \frac{E_s}{N_0}|y|^2 - |a|^2 \frac{E_s}{N_0}|x|^2 + 2|a| \frac{E_s}{N_0}(y_I x_I + y_Q x_Q), \quad (5)$$

where the inphase and quadrature-phase transmitted signal $x$ and received signal $y$ can be represented as $x = x_I + j x_Q$ and $y = y_I + j y_Q$, respectively, and $E_s/N_0$ is the modulated symbol energy-to-noise ratio. If all bits $b_j$ in the bit-sequence $b$ are independent, we have:

$$\log P(x) = \log P(b) = \log P(b_0) P(b_1) \cdots P(b_{m-1}), \quad (6)$$
although in case of Gray-coding this is clearly not the case. From (4) and (6), we have the soft output of the demapper, expressed in terms of the LLRs as [20]:

\[ \text{LLR}(\hat{b}_j) = \log \frac{P(b_j = 1 | y)}{P(b_j = 0 | y)} = \log \frac{\sum_{x:b_j=1} \exp [\log P(y|x, a) + \log P(x)]}{\sum_{x:b_j=0} \exp [\log P(y|x, a) + \log P(x)]} \]  

(7)

\[ = \log \frac{\sum_{x:b_j=1} \exp [\gamma(y, x)]}{\sum_{x:b_j=0} \exp [\gamma(y, x)]}, \]  

(8)

where the receiver’s estimate of \( x \) is denoted as \( \hat{x} \) and we have \( \hat{b} = (\hat{b}_0, \cdots, \hat{b}_{m-1}) \), while the notation \( \gamma(y, x) \) represents [20]:

\[ \gamma(y, x) = -|a|^2 \frac{E_s}{N_0} |x|^2 + 2|a| \frac{E_s}{N_0} (y_I x_I + y_Q x_Q) + \sum_{j=0}^{m-1} b_j \text{LLR}(b_j). \]  

(9)

The extrinsic information output is calculated as follows [20]:

\[ \text{LLR}_E(\hat{b}_j) = \text{LLR}(\hat{b}_j) - \text{LLR}(b_j). \]  

(10)

The capacity of the SLT coded modulation scheme is denoted as \( C \), and when transmitting over the AWGN channel and using 16-QAM, we have [9]:

\[ C = \frac{1}{16} \sum_{x \in 16} I(x; y) = \frac{1}{16} \sum_{x \in 16} \int_{-\infty}^{\infty} P(y|x) \log_2 \frac{P(y|x)}{P(y)} dy, \]  

(12)

where \( I(x; y) \) denotes the mutual information between the received signal \( y \) and the transmitted signal \( x \), while \( P(y|x) \) is given in (5) with \( a \) is a constant.

In the block diagram of the H-ARQ-SLT coded modulation scheme seen in Fig. 1, the demapper constitutes an inner decoder, while the SLT decoder acts as an outer decoder. Following deinterleaving the output extrinsic information LLRs gleaned from the demapper are fed to the input of the SLT decoder. These LLRs are processed by the inner SLT decoder. The extrinsix LLRs at the output of the SLT decoder are fed back to the demapper after interleaving, as seen in Fig. 1. If the syndrome checking block of the SLT decoder detects an illegitimate decoded codeword, it will send the control signal \( S_c \) to activate the ACK transmitter. An ACK will then be sent to the transmitter, requesting the transmission of extra parity. The
receiver will continue the iterative decoding process. This retransmission process will then be repeated, until a legitimate SLT codeword is decoded or upon reaching the maximum number of retransmissions.

III. EXIT Chart Analysis

In this section we use EXtrinsic Information Transfer (EXIT) charts [21], [22] to analyse the iterative decoding convergence of the H-ARQ-SLT coded modulation scheme (Scheme-1) in Table. I. The EXIT chart of the H-ARQ-SLT coded modulation scheme includes three curves in Fig. 3 and Fig. 4, namely the inner decoder’s curve, the outer decoder’s curve and the Monte-Carlo simulation based trajectory. The inner decoder’s curve constitutes that of the demapper, the outer decoder’s curve constitutes that of the SLT decoder and the decoding trajectory represents the extrinsic information exchange between the demapper and the SLT decoder.

The inner decoder’s curve seen in Fig. 3 and Fig. 4 visualises the apriori input mutual information $I_{A_{map}}$ and output extrinsic information $I_{E_{map}}$ relationship of the demapper. By contrast, the outer decoder’s EXIT curve seen in Fig. 3 and Fig. 4 represents the relationship between the input mutual information $I_{A_{SLT}}$ and the output mutual information $I_{E_{LST}}$ of the SLT decoder of Fig. 1. If there is no retransmission between the transmitter and receiver of the H-ARQ-SLT coded modulation scheme, then the a priori LLRs of the SLT decoder are exactly the same as during the previous iteration, because the extrinsic LLRs of the demapper remain unchanged. Hence, the decoding trajectory of the H-ARQ-SLT coded modulation scheme represents the relationship between the extrinsic LLR output of the demapper and the extrinsic LLR output of the SLT decoder after $l$ iterations. The output mutual information obeys the following function [11] [22]:

$$I_{E_{SLT}} = f (I_{A_{SLT}}, I_{A_{m}}),$$

where $I_{A_{m}} = I(X; R)$ is the average a priori information at the input of the SLT decoder’s message node and $R$ is the LLR message gleaned from the check nodes of the SLT decoder.

When there is a retransmission between the transmitter and the receiver of the H-ARQ-SLT coded
modulation scheme, the decoding trajectory of the system is a function of both the input mutual information of the demapper as well as of the output mutual information of the SLT decoder and of the mutual information $I_{A_{\text{retrans}}}$ input to the SLT decoder. The retransmission-aided extra input mutual information $I_{A_{\text{retrans}}}$ of the SLT decoder is related to the number of retransmissions.

Again, Fig. 3 shows the corresponding EXIT curves and the decoding trajectory of the H-ARQ-SLT coded modulation scheme at $E_b/N_0 = 4$ dB, when using 16-QAM and the classic Gray mapper. As seen in Fig. 3, the dashed-dotted line represents the inner EXIT curve of the original Gray mapper at $E_b/N_0=4$ dB, while the bold-continuous line marks the EXIT curve of the Gray mapper in the H-ARQ-SLT coded modulation scheme. Initially, the H-ARQ-SLT receiver attempts to iteratively decode the received codewords by exchanging extrinsic information between the SLT decoder and the demapper. However, the trajectory reaches the point of intersection between the two EXIT curves after a single iteration, which is marked by a circle, as seen in Fig 3. Hence, we cannot achieve an infinitesimally low BER. In the presence of SLT errors the H-ARQ-SLT receiver requires the H-ARQ-SLT transmitter to retransmit the parity part of the corrupted codewords. At this time, the SLT decoder receives an extra LLR information from the retransmitted parity part, hence the EXIT curve of the inner mapper is correspondingly moved upward, as indicated by the bold line seen in Fig. 3. The $a \text{ priori}$ LLRs input to the SLT decoder are gleaned from the extrinsic LLRs of the previous decoding iteration at the output of the SLT decoder. The receiver decodes again the received codewords and hence the decoding trajectory now reaches the point $(0.64, 1)$, as seen in Fig. 3. The EXIT curve of the outer SLT code having a code rate of 0.5 [14] has a specific form, as seen in Fig. 3 and after two iterations the H-ARQ aided SLT decoder becomes capable of achieving $\text{BER} \leq 10^{-5}$ at $E_b/N_0= 4$ dB, as seen in Fig. 5.

Let us now embark on improving the achievable system performance by creating a more beneficial EXIT-curve shape for the inner demapper with the aid of set-partitioning. The EXIT chart of the H-ARQ-SLT coded modulation scheme using the set-partition demapper is portrayed in Fig. 4. The original EXIT curve of the inner set-partitioning mapper is plotted by the dashed-dotted line, while the EXIT curve after one
retransmission is represented by the bold-thick line seen Fig. 4. If the receiver using the set-partitioning based demapper without the aid of retransmission invokes only four decoding iterations between the SLT decoder and the demapper, the decoded output codewords cannot achieve an infinitesimally low BER, because the trajectory’s evolution is curtailed at the point marked by the asterisk seen in Fig. 4. For the sake of achieving $\text{BER} \leq 10^{-5}$, the receiver has to allow for up to 8 iterations.

We can see by comparing Fig. 3 and Fig. 4 that at $E_b/N_0 = 4$ dB the H-ARQ-SLT coded modulation scheme using the Gray mapper requires a lower number of iterations for achieving $\text{BER} \leq 10^{-5}$ in comparison to the H-ARQ-SLT coded modulation scheme using the set-partitioning mapper. By contrast, the intercept point of the two EXIT curves is closer to the (1,1) point of perfect convergence, where an infinitesimally low BER can be achieved, if the associated higher number of decoding iterations and the ARQ-aided retransmission of the set-partitioning aided scheme are deemed affordable.

\section*{IV. Simulation Results}

Our simulation parameters are provided in Table I. The different proposed schemes are assigned as Scheme-1, Scheme-2 and Scheme-3 seen in Table I. Fig. 5 represents the BER performance of the H-ARQ-SLT coded modulation scheme using the parameters of Table I. The left vertical axis represents the BER values, while the right vertical axis represents normalized throughput values and the horizontal axis the $E_b/N_0$ values. The normalized throughput values are calculated by normalizing the effective throughput by the throughput\(^1\) of the H-ARQ-SLT coded modulation scheme, recorded at high $E_b/N_0$ values, i.e. when the data transmission is error-free. The SLT code rate of the scheme is 0.5, hence the maximum throughput of the 16-QAM H-ARQ-SLT coded modulation scheme is 2 bits/symbol, which represents our throughput normalization factor. As seen in Fig. 5 and, the continuous line marked by the hollow diamond represents the BER versus $E_b/N_0$ performance of the H-ARQ-SLT coded modulation scheme, which attains $\text{BER} < 10^{-5}$ for $E_b/N_0$ values in excess of 3.8 dB. When we briefly compare the performance of the three schemes in Fig. 5 in terms of their achievable throughput, a near-unity

\(^1\)The effective throughput is defined as the ratio of the number of received correct bits per the total number of transmitted bits.
throughput is achieved by the amalgamated H-ARQ-aided SLT coded modulation scheme for $E_b/N_0$ values in excess of about 4 dB. The other two schemes exhibit both a higher BER and a lower normalized throughput, indicating the superiority of the proposed scheme. Finally, the curves denoted by the circled-plus-sign represent a H-ARQ-aided but uncoded 16-QAM benchmarker, which hence has a 4 bits/symbol effective throughput. In the high-SNR region the throughput of this H-ARQ-aided but uncoded 16-QAM benchmarker is twice as high as that of the 0.5-rate SLT-coded system. By contrast, in the low-SNR region the normalized throughput of the uncoded H-ARQ-aided scheme becomes $\sim 0.33$, because in this region the receiver is unable to attain an infinitesimally low BER at the output, hence it requires the maximum affordable number of retransmissions. Scheme-2 has an approximately 3 dB higher $E_b/N_0$ requirement than our proposed Scheme-1, as indicated by the continuous line marked by the triangle in Fig. 5. The ARQ scheme operating without the assistance of the SLT code requires an $E_b/N_0$ value in excess of 18 dB to attain $BER < 10^{-5}$, although this BER value is not reached within the $E_b/N_0$ range shown in Fig. 5, hence indicating $BER \approx 0.5$. Finally, in Fig. 6 we compare the set-partitioned and Gray-coded H-ARQ-aided SLT-16-QAM schemes. Observe that the Gray-coded scheme performs better both in terms of its BER and normalized throughput, when using two iterations between the demapper and the SLT decoder.

V. CONCLUSIONS AND FUTURE WORKS

The proposed H-ARQ-SLT aided scheme is capable of achieving an $E_b/N_0$ gain ranging from 0.5 dB upto 3.5 dB and a higher throughput in comparison to the benchmark schemes. The H-ARQ-SLT aided scheme advocated is also capable of attaining an infinitesimally low BER at a low $E_b/N_0$ value, while maintaining a high normalized throughput. By exploiting the syndrome checking capability of the SLT decoder we can reduce the complexity of the scheme, while simultaneously eliminating the redundant bits used for checking the parity of the IP packets in classic CRC schemes. Furthermore, it was shown that the H-ARQ-SLT coded modulation scheme using the Gray mapper performs better in terms of its BER, while imposing a lower complexity and achieving a higher throughput in comparison to the set-partitioning mapper, which was proposed in [14].
REFERENCES


Fig. 1. The block diagram of H-ARQ-SLT coded modulation scheme.
Fig. 2. An Example of the relation between IP packets and SLT packets.
<table>
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<tr>
<th>SLT code rates $r$</th>
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<tr>
<td>Parity packet degree distribution</td>
<td>Truncated Degree Distribution proposed in [11]</td>
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<tr>
<td>The maximum number of inner iterations</td>
<td>$I_{max}$ of SLT codes 12 and 20</td>
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<tr>
<td>The total number of outer iterations</td>
<td>2 and 4</td>
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<tr>
<td>H-ARQ scheme</td>
<td>Scheme-3</td>
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**TABLE I**

**SYSTEM PARAMETERS.**
Fig. 3. EXIT chart and the decoding trajectory of the H-ARQ-SLT coded modulation scheme using the classic Gray mapper [16].
Fig. 4. EXIT chart and the decoding trajectory of the H-ARQ-SLT coded modulation scheme using the set-partitioning mapper (Scheme-1) [17].
Fig. 5. BER performances and normalized throughputs of Scheme-1, Scheme-2 and Scheme-3 seen in Table I, having a maximum of $I=20$ SLT decoder iterations, when transmitting data over an AWGN channel using 16-QAM.
The BER performance of Scheme-1 using SP demapper, outer iter $I = 2$

The BER performance of Scheme-1 using Gray demapper, outer iter $I = 2$

Normalized-throughput of Scheme-1 using SP demapper, outer iter $I = 2$

Normalized-throughput of Scheme-1 using SP demapper, outer iter $I = 2$

Fig. 6. BER performances and normalized throughputs of the H-ARQ-SLT system having a maximum of $I = 12$ SLT decoder iterations, when transmitting data over an AWGN channel using 16-QAM and employing either SP or Gray-coding.