Network Representation of Conducting Regions in 3-D Finite-Element Description of Electrical Machines

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The paper introduces a network description of conducting regions in electrical machines. Resistance models are considered, where loop equations are equivalent to an edge element formulation using the electric vector potential $\boldsymbol{T}$, as well as conductance models, for which the nodal equations refer to a nodal element description by means of the scalar potential V. Network models for multiply connected regions are derived for both Ω–T–T_0 and A–T–T_0 formulations. A network representation of the edge value of potential T_0 is suggested. Convergence of the iterations of the T–T_0 method may be accelerated by supplementing equations for the edge values of T_0.

***Index Terms*—Coupling circuits, eddy currents, electrical engineering education, electromagnetic fields, finite-element methods (FEMs), magnetic circuits.**

I. INTRODUCTION

DESIGN and analysis of electrical machines increasingly exploit numerical field simulations. By far the most popular is the finite-element method (FEM), although equivalent magnetic and electric circuits continue to be useful as they provide good physical insight and aid understanding of complicated electromagnetic phenomena. It was shown previously [1]–[3] that FEM formulations may be considered as analogous to loop or nodal descriptions of equivalent electric or magnetic circuits (networks). Thus models established using FEM approach may be treated as network models. The number of branches in such models equals the number of edges or facets of the discretising mesh. This paper builds on previous publications and extends the treatment by focussing on network description of regions with conduction currents. The aim is to facilitate the connection between field equations due to such currents and the equations of the supplying circuitry. Coupling between magnetic and electric networks is also considered for models of electric windings.

II. NETWORK REPRESENTATION OF FE MODELS

It was shown in [1] that finite-element formulations using potentials may be seen as equivalent to network models of either edge elements (EN), with branches coinciding with element edges, or facet elements (FN), where the branches connecting the nodes are associated with the facets, while the nodes are positioned in the middle of the volumes. Fig. 1 depicts the edge and facet models of a tetrahedron. The nodal equations of EN are equivalent to the nodal element description (NEM) of the scalar potential formulation, while the loop equations of FN correspond to the edge element formulation (EEM) using vector potential. The edge values of the vector potentials $\boldsymbol{A}$ and $\boldsymbol{T}$ represent the loop fluxes and currents in loops around the edges, respectively [1].

In regions with conduction currents, a conductance network (CN) may be created from an electric edge model, whereas a resistance network (RN) stems from an electric facet model. The CN conductances may be established from the interpolating functions of the edge element, while resistances of the RN from those of the facet element. In the networks arising from the FE method, coupling between the branches may occur, i.e., mutual conducatnces and resistances may be present, which distinguishes such networks from classical circuits. The voltage across a conductance of the ith branch may force a current in the jth branch of the CN; similarly, a current in the ith branch of the RN may create a voltage in the jth branch.

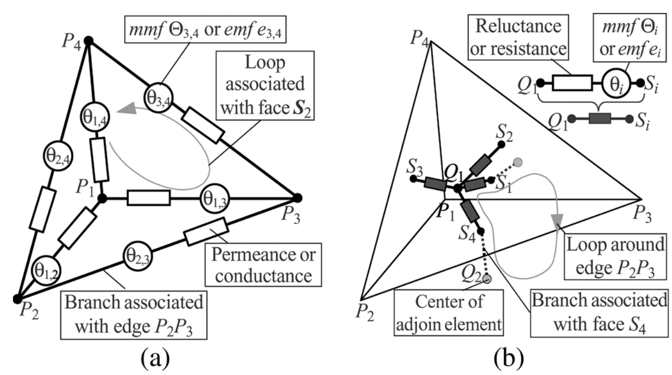

Fig. 1. (a) Edge and (b) facet model of a tetrahedron.

III. ELECTROMOTIVE AND MAGNETOMOTIVE FORCES

The task of describing conductors in the FE domain necessitates defining the mmfs set up by currents in conducting regions and emfs due to changing magnetic fluxes. The edge networks (EN) are analysed using the nodal method, thus branch sources need to be introduced. On the other hand, a loop method is applied to evaluate the facet networks (FN), hence either branch or loop sources may be used.

In the models under consideration, the branch mmfs and emfs are established from loop currents and fluxes. In the case of EN, currents and fluxes "around" the edges are relevant, whereas for the FN case currents and fluxes of the loops associated with facets need to be used (see Fig. 1). Loop mmfs in EN represent facet values of $\boldsymbol{J}$, whereas loop emfs arise from time derivatives of facet values of $\boldsymbol{B}$. The loop sources of FN, on the other hand, may be defined using branch currents and fluxes corresponding to element edges. Table I collects expressions that describe branch and loop currents, fluxes, mmfs, and emfs for electric and magnetic networks.

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TABLE I
CURRENTS, FLUXES, mmfS AND emfS ASSOCIATED WITH AN ELEMENT

Branch currents i_b, fluxes ϕ_b, *mmf*s θ_b and *emf*s e_b		Loop currents i_m, fluxes ϕ_m, *mmf*s θ_m and *emf*s e_m	
Branch $N_{i,j}$ associated with edge P_iP_j (EN)	Branch q-th associated with face S_q (FN)	Loop $N_{i,j}$ around edge P_iP_j (FN)	Loop q-th associated with face S_q (EN)
$i_{bN_{i,j}}=\iiint_{V_e}\boldsymbol{w}_{eN_{i,j}}\boldsymbol{J}\,\mathrm{d}v$	$i_{bq}=\iint_{S_q}\boldsymbol{J}\,\mathrm{d}\boldsymbol{s}$	$i_{mN_{i,j}}=\int_{P_i}^{P_j}\boldsymbol{T}\,\mathrm{d}\boldsymbol{l}$	$i_{mq}=\iiint_{V_e}\boldsymbol{w}_{fq}\boldsymbol{T}\,\mathrm{d}v$
$\phi_{bN_{i,j}}=\iiint_{V_e}\boldsymbol{w}_{eN_{i,j}}\boldsymbol{B}\,\mathrm{d}v$	$\phi_{bq}=\iint_{S_q}\boldsymbol{B}\,\mathrm{d}\boldsymbol{s}$	$\phi_{mN_{i,j}}=\int_{P_i}^{P_j}\boldsymbol{A}\,\mathrm{d}\boldsymbol{l}$	$\phi_{mq}=\iiint_{V_e}\boldsymbol{w}_{fq}\boldsymbol{A}\,\mathrm{d}v$
$\theta_{bN_{i,j}}=i_{mN_{i,j}}$	$\theta_{bq}=i_{mq}$	$\theta_{mN_{i,j}}=i_{bN_{i,j}}$	$\theta_{mq}=i_{bq}$
$e_{bN_{i,j}}=-\mathrm{p}\phi_{mN_{i,j}}$	$e_{bq}=-\mathrm{p}\phi_{mq}$	$e_{mN_{i,j}}=-\mathrm{p}\phi_{bN_{i,j}}$	$e_{mq}=-\mathrm{p}\phi_{bq}$
Note: $\boldsymbol{w}_{eN_{i,j}}$ is the interpolation function of an edge element for the edge P_iP_j, V_e is the element volume, $\boldsymbol{w}_{fq}$ is the interpolation function of the facet element for face S_q and p=d/dt			

IV. SIMPLY CONNECTED CONDUCTING REGIONS

In field analysis of simply connected conducting regions, e.g., solid parts of a core with no "holes," it is possible to use the $\boldsymbol{A}$–V combination of potentials, as well as Ω–$\boldsymbol{T}$ or $\boldsymbol{A}$–$\boldsymbol{T}$. The FEM formulation using $\boldsymbol{A}$–V is equivalent to equations of the magnetic FN and electric EN [3]. The less popular application of Ω–$\boldsymbol{T}$ and $\boldsymbol{A}$–$\boldsymbol{T}$ formulations in the FEM description leads to analogies with representative resistance networks coupled with magnetic EN or magnetic FN, respectively. In the Ω–$\boldsymbol{T}$ model we define branch mmfs $\theta_{bN_{i,j}}$ in branches associated with edges and loop emfs $e_{mN_{i,j}}$ in loops around the edges. The aforesaid emfs may be established directly from the branch fluxes of the magnetic EN, whereas mmfs from the loop currents of the electric FN. When forming the sources no further transformation of network quantities is necessary, while deriving the sources for the network model using the $\boldsymbol{A}$–$\boldsymbol{T}$ approach (i.e., electric and magnetic facet network) requires such additional steps to be taken.

From Table I it follows that the branch sources in FN are described as

$$\theta_{bq} = i_{mq} = \iiint_{V_e} \boldsymbol{w}_{fq}\boldsymbol{T}dv \tag{1a}$$

$$e_{bq} = -p\phi_{mq} = -p\iiint_{V_e} \boldsymbol{w}_{fq}\boldsymbol{A}dv. \tag{1b}$$

The vectors $\boldsymbol{T}$ and $\boldsymbol{A}$ may be obtained from the loop currents and fluxes in FN as

$$\boldsymbol{T} = \sum_{N_{i,j}} \boldsymbol{w}_{eN_{i,j}} i_{mN_{i,j}} \tag{2a}$$

$$\boldsymbol{A} = \sum_{N_{i,j}} \boldsymbol{w}_{eN_{i,j}} \phi_{mN_{i,j}}. \tag{2b}$$

As a result, after relevant substitutions, expressions for branch sources in FN are found in terms of branch currents and fluxes

$$\theta_{bq} = \sum_{N_{i,j}} K_{N_{i,j}}^{(q)} i_{mN_{i,j}} \tag{3a}$$

$$e_{bq} = -p\sum_{N_{i,j}} K_{N_{i,j}}^{(q)} \phi_{mN_{i,j}} \tag{3b}$$

where

$$K_{N_{i,j}}^{(q)} = \iiint_{V_e} \boldsymbol{w}_{fq}\boldsymbol{w}_{eN_{i,j}} dv. \tag{4}$$

The facet networks (FNs) are analysed using a loop method, thus the knowledge of loop mmfs and emfs will suffice. These may be derived using branch sources or directly from branch currents and fluxes in FN. Using the relationships in the third and first columns of Table I, and bearing in mind that $\boldsymbol{J} = \sum_q \boldsymbol{w}_{fq} i_{bq}$, $\boldsymbol{B} = \sum_q \boldsymbol{w}_{fq}\phi_{bq}$, yields

$$\theta_{mN_{i,j}} = i_{bN_{i,j}} = \iiint_{V_e} \boldsymbol{w}_{eN_{i,j}}\boldsymbol{J}dv = \sum_q K_{N_{i,j}}^{(q)} i_{bq} \tag{5a}$$

$$\begin{aligned} e_{mN_{i,j}} &= -p\phi_{bN_{i,j}} = -p\iiint_{V_e} \boldsymbol{w}_{eN_{i,j}}\boldsymbol{B}dv \\ &= -p\sum_q K_{N_{i,j}}^{(q)}\phi_{bq}. \end{aligned} \tag{5b}$$

Both methods of finding sources in FN are comparable when it comes to their computational complexity; however, the algorithm based on (3) is more reliable in terms of the convergence of the iterative solution. Even if the branch source estimates are not very accurate, a condition is satisfied that guarantees good convergence of the ICCG procedure of solving FN's equations for ungauged formulations [7].

V. MULTIPLY CONNECTED CONDUCTORS—WINDINGS

In analysis of windings of electrical machines two cases are considered: 1) thin (filament) conductors of cross-section less than the facet area of the elements; 2) solid conductors, such as in a cage rotor of an induction motor, whose cross-section is larger than the element facet area. In a special case the two areas may actually coincide (Fig. 2) and it is convenient to use this case to explain the differences between the resistance model ($\boldsymbol{T}$ potential) and the conductance model (V potential). The FEM equations for the classical $\boldsymbol{T}$ formulation refer to loops around the element edges. From Fig. 2(a) it is transparent that all loops around the edge are "open" and the classical $\boldsymbol{T}$ solution gives incorrect zero result [4]. It is thus necessary to introduce an additional equation describing the loop current i_c flowing around the "hole"—Fig. 2(a). This current is a circuit representation of the edge value of $\boldsymbol{T}_0$ introduced in [5] and [6]. The equation for i_c uniquely describes the current flow in Fig. 2(a). A more accurate model of a single winding turn may be obtained by using a conductance network. However, a large number of nodal equations will result, even if skin effect is neglected and a condition is imposed that nodes on the surface of the conductor cross section are shorted, e.g., nodes P_i, P_j, P_k. It may therefore be concluded that for systems with thin conductors the use of potential $\boldsymbol{T}_0$ is to be recommended.

A mixed approach, linking the above description with equations based on a scalar potential Ω (equations of magnetic EN) and a vector potential $\boldsymbol{A}$ (equations of magnetic FN) is now considered, whereby classical formulations involving potential $\boldsymbol{T}$ may be added, i.e., equations for loops containing eddy currents

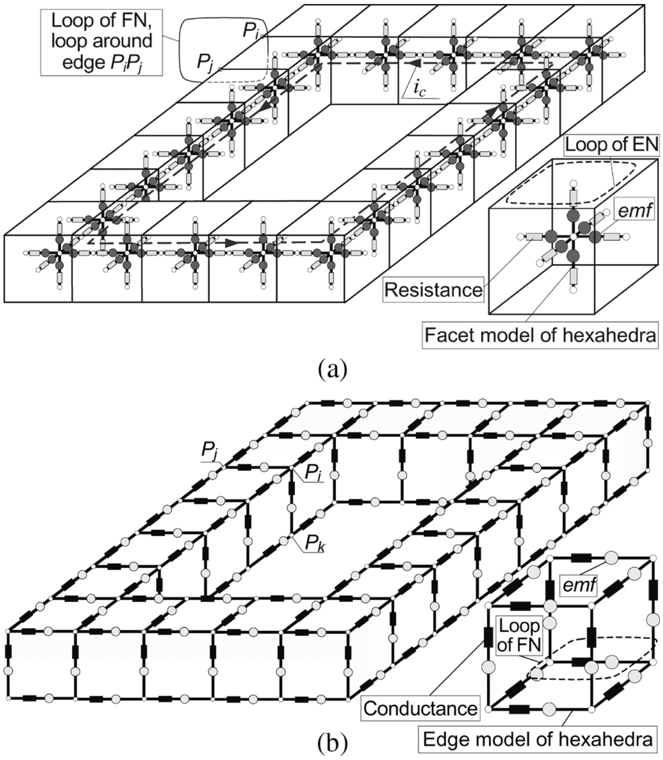

Fig. 2. Models of a turn split into hexahedrons. (a) Resistance and (b) conductance.

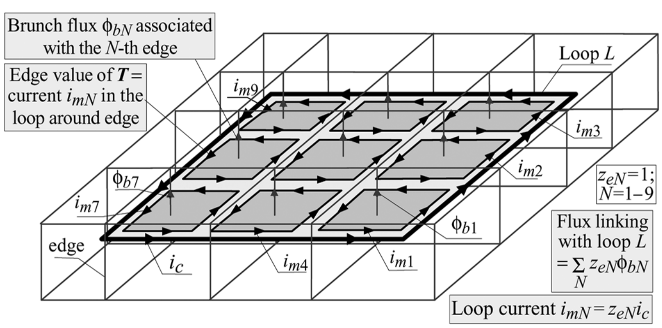

Fig. 3. Loop with current in space of an edge magnetic network.

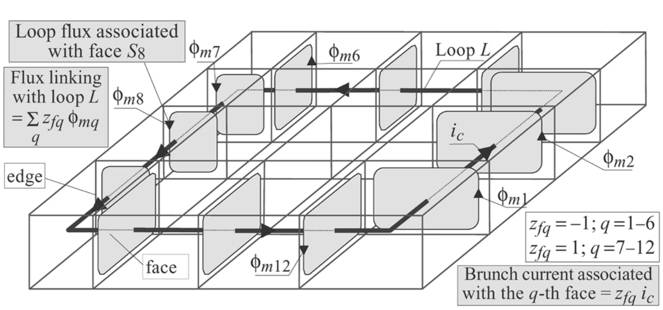

Fig. 4. Loop with current in space of a facet magnetic network.

(the methods Ω–$\boldsymbol{T}$–$\boldsymbol{T}_0$ and $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$). It has been assumed that the "loop" L shown if Figs. 3 and 4 is part of a winding consisting of filament conductors or represents a loop around the hole of a solid conductor, e.g., a loop with current i_c in Fig. 2.

It follows that to define mmfs in EN it is necessary to form loops around edges. The L loop must therefore be replaced by these loops, as in Fig. 3 (loops with current i_{mk}). A matrix $\boldsymbol{z}_e$ is being formed transposing a current in L into a vector of currents i_{mk} representing branch mmfs in EN. Multiplying the transposed matrix $\boldsymbol{z}_e$ by the vector of fluxes associated with edges yields the flux linkage with L as shown in Fig. 3. Since currents i_{mN} are edge values of potential $\boldsymbol{T}_0$, by using the interpolation functions of the edge element the values of $\boldsymbol{T}_0$ may be established—see (2a). Next, via the relationship (3a), we can determine the currents i_{mq}, which represent branch mmfs θ_{mq} in the magnetic FN. Combining matrix $\boldsymbol{z}_e$ with (3) and (4) yields the following equations for the $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$ method for a system with eddy currents $\boldsymbol{i}_m$ and currents $\boldsymbol{i}_c$ in loops L:

$$\begin{bmatrix} \boldsymbol{k}_e^T\boldsymbol{R}_\mu\boldsymbol{k}_e & -\boldsymbol{k}_e^T\boldsymbol{K} & -\boldsymbol{k}_e^T\boldsymbol{K}\boldsymbol{z}_e \\ p\boldsymbol{K}^T\boldsymbol{k}_e & \boldsymbol{k}_e^T\boldsymbol{R}\boldsymbol{k}_e & \boldsymbol{k}_e^T\boldsymbol{R}\boldsymbol{k}_e\boldsymbol{z}_e \\ p\boldsymbol{z}_e^T\boldsymbol{K}^T\boldsymbol{k}_e & \boldsymbol{z}_e^T\boldsymbol{k}_e^T\boldsymbol{R}\boldsymbol{k}_e & \boldsymbol{z}_e^T\boldsymbol{k}_e^T\boldsymbol{R}\boldsymbol{k}_e\boldsymbol{z}_e \end{bmatrix} \begin{bmatrix} \phi_m \\ \boldsymbol{i}_m \\ \boldsymbol{i}_c \end{bmatrix} = \begin{bmatrix} \theta_m \\ 0 \\ \boldsymbol{e}_{cu} \end{bmatrix}. \tag{6}$$

Here $\boldsymbol{R}_\mu$ and $\boldsymbol{R}$ are the matrices of branch reluctances and resistances for FNs, $\boldsymbol{k}_e$ is a transposed loop matrix for the FNs, $\boldsymbol{\phi}_m$ is the vector of loop fluxes, and $\boldsymbol{K}$ consists of entries defined by (4). When setting up (6) it has been recognized that additional loop mmfs, $\boldsymbol{\theta}_m$, and emfs, $\boldsymbol{e}_{cu}$, may exist in loops L due to external sources.

When solving the loop equations of magnetic FN it is sufficient to define loop mmfs $\theta_{mNi,j}$, i.e., currents $i_{bNi,j}$, in branches associated with edges. The currents $i_{bNi,j}$ may be found from (5a) using currents i_{bq} in branches associated with facets. The matrix of currents i_{bq} may be written as a product of the current i_c in L and the matrix $\boldsymbol{z}_f$ of cuts of L with the facets (Fig. 4). The product of the transposed matrix $\boldsymbol{z}_f$ and the vector of fluxes θ_{mq} through loops associated with facets yields the flux linking with L (Fig. 4). The fluxes ϕ_{mq} are found from loop fluxes in FN by applying a similar procedure as used when forming (3b). If a formulation involving $\boldsymbol{z}_f$ is needed, equations using $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$ may be derived in a similar way to (6) in terms of the previously defined matrix $\boldsymbol{z}_e$ by noting that in an equation which follows from (6) the following holds:

$$\boldsymbol{k}_e^T\boldsymbol{K} = \boldsymbol{K}^T\boldsymbol{k}_e \qquad \boldsymbol{k}_e\boldsymbol{z}_e = \boldsymbol{z}_f. \tag{7}$$

Description of the loop L using $\boldsymbol{z}_f$ is simpler but less general. Moreover, it is not possible to use it for the Ω–$\boldsymbol{T}$–$\boldsymbol{T}_0$ method, the equations for which may be written as

$$\begin{bmatrix} \boldsymbol{k}_n^T\Lambda\boldsymbol{k}_n & \boldsymbol{k}_n^T\Lambda & \boldsymbol{k}_n^T\Lambda\boldsymbol{z}_e \\ p\Lambda\boldsymbol{k}_n & \boldsymbol{k}_e^T\boldsymbol{R}\boldsymbol{k}_e + p\Lambda & \left(\boldsymbol{k}_e^T\boldsymbol{R}\boldsymbol{k}_e + p\Lambda\right)\boldsymbol{z}_e \\ p\boldsymbol{z}_e^T\Lambda\boldsymbol{k}_n & \boldsymbol{z}_e^T\boldsymbol{k}_e^T\boldsymbol{R}\boldsymbol{k}_e & \boldsymbol{z}_e^T\left(\boldsymbol{k}_e^T\boldsymbol{R}\boldsymbol{k}_e + p\Lambda\right)\boldsymbol{z}_e \end{bmatrix} \begin{bmatrix} \Omega \\ \boldsymbol{i}_m \\ \boldsymbol{i}_c \end{bmatrix} = \begin{bmatrix} \boldsymbol{k}_n^T\Lambda\theta_b \\ 0 \\ \boldsymbol{e}_{cu} \end{bmatrix} \tag{8}$$

where $\boldsymbol{k}_n$ is a transposed nodal incidence matrix for EN, $\boldsymbol{\Lambda}$ is the matrix of branch permeances for EN [1], $\boldsymbol{\Omega}$ is the vector of nodal potentials Ω and $\boldsymbol{\theta}_b$ is the vector of additional branch mmfs, e.g., in the permanent magnet region. The above matrix description, applicable to a single turn, may be easily extended to multiturn coils. The values in the matrix $\boldsymbol{z}_f$, will then be equal to the number of conductors crossing the facets, while for the matrix $\boldsymbol{z}_e$ to the number of turns around the edges.

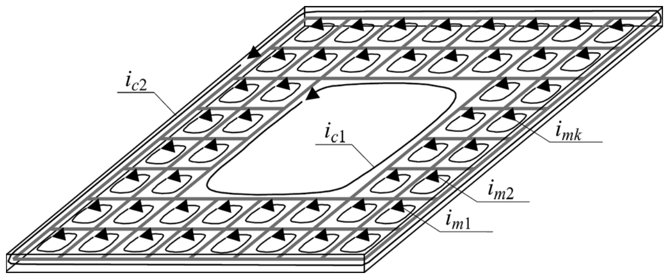

Fig. 5. Resistance model (FN) of a conducting plate with a hole.

VI. Shape and Location of Additional Loops

We consider now how to select the shape of the loops L in regions containing a solid conductor. These need to be formed in such a way so that the additional loops with currents $\boldsymbol{i}_c$ complement the set of the main loops. For example, when analysing the cage winding of an induction motor with N bars, we must add $N+1$ loops, i.e., N loops associated with the currents in the bars and one loop related to the end ring. The choice of the loops with currents $\boldsymbol{i}_c$ will be guided by the method of solution used to calculate currents $\boldsymbol{i}_m$ and $\boldsymbol{i}_c$ in the windings. Most contemporary solvers use iterative methods, in which case the number of loops with currents $\boldsymbol{i}_m$ and $\boldsymbol{i}_c$ may actually be higher than the number of main loops. It is perfectly acceptable, for example, to assume that all possible loops surrounding the holes are to be defined as loops with currents $\boldsymbol{i}_c$. The number of such loops may be quite high. In the system depicted in Fig. 5, containing one "hole" and 48 loops with eddy currents, it is possible to form over a thousand such loops with currents $\boldsymbol{i}_c$. The two most characteristic loops have been marked: 1) an external loop with the current i_{c2}; and 2) an internal loop with the current i_{c1}. The matrix of coefficients describing the currents $\boldsymbol{i}_m$ and $\boldsymbol{i}_c$ will be the most sparse if a loop with the current i_{c1} is chosen. However, numerical tests have revealed that this does not result in the most economical calculations. In fact the iterations for finding the current distributions in Fig. 5 were significantly accelerated when the system was effectively "overspecified" by adding to the loop with i_{c1} the other loop with i_{c2}.

Table II demonstrates the relative error in the iterative calculation of current distribution for the case of Fig. 5 under the excitation by a uniform time varying magnetic flux passing through the plate. Two popular iterative methods were employed: 1) successive over relaxation (SOR) and 2) incomplete Cholesky conjugate gradient (ICCG). Three cases were considered: one loop with current i_{c1}, one loop with current i_{c2}, and two loops containing both currents i_{c1} and i_{c2}. The measure of the error at iteration k was taken as

$$\delta = \sum_{i=1}^{N} |i_{k,i} - i_{a,i}| \Bigg/ \sum_{i=1}^{N} |i_{a,i}| \tag{9}$$

where $i_{k,i}$ is the calculated current in the ith branch for the kth iteration step, $i_{a,i}$ is the exact value of current in the ith branch, and N is the total number of branches ($N = 116$ in the example of Fig. 5). It appears that thanks to the increase in the number of loops with current $\boldsymbol{i}_c$, the rate of convergence has been improved significantly. In the case of the ICCG method, with the two additional loops, the calculation error after only 40 iterations is already at the level of rounding errors.

TABLE II
Errors in Current Distributions After k Iterations

	Errors for SOR			Errors for ICCG		
Iteration number k	1 loop with i_{c1}	1 loop with i_{c2}	2 loops with i_{c1}, i_{c2}	1 loop with i_{c1}	1 loop with i_{c2}	2 loops with i_{c1}, i_{c2}
10	1.53E-01	2.80E-01	3.14E-03	8.24E-03	1.21E-01	1.76E-04
20	1.94E-02	1.07E-01	2.60E-05	4.20E-05	1.22E-02	1.26E-08
30	2.48E-03	4.04E-02	4.83E-08	2.14E-07	1.23E-03	8.93E-13
40	3.17E-04	1.54E-02	4.21E-10	1.09E-09	1.24E-04	2.97E-16
50	4.04E-05	5.83E-03	7.35E-13	5.58E-12	1.25E-05	2.97E-16
100	1.37E-09	4.60E-05	7.68E-15	2.97E-16	1.29E-10	2.97E-16

VII. Conclusion

A network description of FEM equations has been derived for systems containing conducting regions. An interpretation of the edge value of potential $\boldsymbol{T}_0$ has been put forward where this potential is related to a current in a loop. Ways of describing a loop with a current in the FE domain have been proposed using both potentials Ω and $\boldsymbol{A}$. The method is applicable to multiply connected regions, including cage rotors and windings connected to external circuits and sources. It also adds to understanding of such systems.

It has been shown that when solving iteratively equations of the $\boldsymbol{T}$–$\boldsymbol{T}_0$ formulation, i.e., equations describing current distributions in systems containing holes, it is beneficial to introduce superfluous loops (more than required). This results in more economical computation and faster convergence.

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