This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2008 proceedings.

A Systematic Luby Transform Coded V-BLAST System

T. D. Nguyen, M. El-Hajjar, L. L. Yang and [1]L. Hanzo
School of Electronics and Computer Science, University of Southampton, SO17 1BJ, UK.
Email: [1]lh@ecs.soton.ac.uk,
http://www-mobile.ecs.soton.ac.uk

***Abstract*— Systematic Luby Transform (SLT) codes have shown good performance for single antenna aided systems for transmission over AWGN and uncorrelated Rayleigh fading channels. For the sake of improving both the Bit Error Ratio (BER) performance and the diversity gain of Vertical Bell Laboratories Layered Space Time (V-BLAST) schemes, in this paper we propose a SLT coded V-BLAST system having four transmit and four receive antennas. As a benefit of iteratively exchanging the Log-Likelihood Ratios (LLRs) between the QPSK demapper and the SLT decoder of each antenna-specific stream of the V-BLAST system, the system exhibits an infinitesimally low BER for E_b/N_0 values in excess of 6.5 dB, when using an interleaver length of $L = 1,200$ bits. Additionally, the SLT coded system provides an E_b/N_0 gain of 5dB at a BER of 10^{-6} over its benchmark scheme employing iterative extrinsic information exchange between a Recursive Systematic Convolution (RSC) code and a unity-rate code, having an interleaver length of L= 1,200 bits.**

***Index Terms*– EXIT charts, set partitioning, syndrome, systematic Luby transform, V-BLAST, MIMO.**

I. INTRODUCTION

The fundamental limitations of reliable wireless transmissions are imposed by the time-varying nature of typical multipath fading channels, which may be efficiently circumvented by sophisticated transceiver design [1] employing multiple antennas at both the transmitter and the receiver. Recent information theoretic studies [2] [3] have revealed that employing a Multiple-Input Multiple-Output (MIMO) system significantly increases the capacity of the system. In [4], Wolniansky et al. proposed the popular multi-layer MIMO structure, known as the Vertical Bell Labs Layered Space-Time (V-BLAST) scheme. In V-BLAST systems, each transmit antenna simultaneously transmits an independent data stream within the same carrier frequency band. At the receiver side, provided that the number of receive antennas is higher than or equal to the number of transmit antennas, a low-complexity Successive Interference Cancellation (SIC) based detection algorithm may be applied for detecting the transmitted data [5]. The V-BLAST receiver is capable of providing a tremendous increase of a single user's effective bit-rate without the need for any increase in the transmitted power or the system's bandwidth. However, its impediment is that it was not designed for exploiting transmit diversity and the decision errors of a particular antenna's detector propagate to other bits of the multi-antenna symbol, when erroneously cancelling the effects of the sliced bits from the composite MIMO signal.

The financial support of the EPSRC, UK and of the European Union is gratefully acknowledged.

Systematic Luby Transform (SLT) codes were firstly proposed in [6] for exploiting that soft-bit decoding algorithms are capable of providing good BER performances, while communicating over both the traditional Binary Erasure Channel (BEC) modelling the Internet and over faded as well as noise contaminated channels. Recently, the authors of this paper further developed a new version of the systematic Luby transform codes [7] that are capable of achieving an exceptionally good performance in single-antenna aided systems, when communicating over a wide class of channels such as the BEC channel, the Additive White Gaussian Noise (AWGN) and uncorrelated Rayleigh fading channels. The SLT codes advocated outperform conventional quasi-regular LDPC codes in the wireless channel.

In this paper we study the performance of the recently developed SLT aided V-BLAST system communicating over a MIMO channel. The proposed scheme employs set partitioning based bit-to-QPSK mapping and at the receiver side iterative extrinsic information exchange is used between the SLT decoder and the QPSK demapper. The SLT coded scheme is compared to a Recursive Systematic Convolutional (RSC) coded and Unity Rate Coded (URC) scheme. We will demonstrate that the SLT coded system outperforms its benchmark scheme.

The organisation of our paper is as follows. In Section II-A we describe the basic concept of the V-BLAST architecture and in Section II-B we detail the SLT design, while in Section II-C the SLT coded V-BLAST system. In Section III we analyse the SLT coded V-BLAST system using EXIT charts. Finally, in Section IV we present our BER performance results followed by our conclusions in Section V.

II. SYSTEM ARCHITECTURE

A. V-BLAST

Again, V-BLAST provides a high throughput, at the cost of a modest diversity gain. Let $\mathbf{x}^T = [x_1 \quad x_2 \quad x_3 \quad x_4]$ denote the vector of QPSK symbols to be transmitted by the four antennas during a symbol interval. Then the corresponding received vector can be represented as

$$\mathbf{r}_t = \mathbf{H} \cdot \mathbf{x}_t + \mathbf{n}_t, \quad (1)$$

where $\mathbf{r}_t$ is the received signal vector, $\mathbf{H}$ is the $(n_r \times n_t)$-element Channel Impulse Response (CIR) matrix, where n_t is the number of transmit antennas, n_r is the number of receive antennas and h_{ij} represents the CIR coefficients between transmit antenna j and receive antenna i, while $\mathbf{n}_t$ denotes the noise vector at time instant t. V-BLAST detection may be carried out using for example SIC or the Zero Forcing (ZF) algorithm [4].

B. Systematic Luby Transform codes

The SLT codes of [8] were based on the Improved Robust Soliton Distribution (IRSD)[1] of [8] invoked for the sake of designing the degree distribution of the parity part of the generator matrix. This version of SLT codes might not outperform the best LDPC codes having the same code rate, but it was capable of outperforming the family of hard-decision-based Fountain codes with the advent of soft-bit decoding.

For convenience, in what follows, we provide a brief overview of our SLT code architecture. The generator matrix of SLT codes includes two parts, namely the information part and the parity part [6] seen in Figure 1, where K represents the number of SLT input packets and M defines the number of SLT parity packets. A binary 1 in the parity part indicates the presence of the input SLT packet in the parity SLT packet.

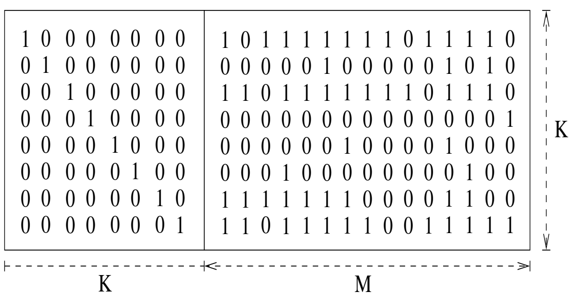

Fig. 1. SLT Generator Matrix.

The parity part of SLT codes is created by using the IRSD proposed in [8] and this distribution is described as follows:

$$D(d) = \begin{cases} 0 & \text{if } d = 1 \text{ or } d > \frac{K}{S}, \\ \frac{1+S+\nu}{Z \cdot K} & \text{if } d = 1, \\ \frac{1}{Z}\left(\frac{1}{d(d-1)} + \frac{S}{K} \cdot \frac{1}{d}\right) & \text{if } 2 \leq d < \frac{K}{S}, \\ \frac{S}{Z \cdot K} \cdot log\frac{S}{\delta} & \text{if } d = \frac{K}{S}, \end{cases} \tag{2}$$

where d is the degree of the encoded SLT packets belonging to the parity part of the SLT generator matrix, K is the number of source information packets, S is the number of packets having a degree of $d = 1$ and finally $Z = \sum_d \rho(d) + \tau(d) + \nu(d)$, where $\rho(d)$ and $\tau(d)$ were defined as part of the Robust Soliton Distribution (RSD) [9], while $\nu(d)$ is an extra factor of the IRSD as defined in [8].

In this contribution, we developed an improved version of SLT codes, which uses a novel Truncated Degree Distribution (TDD) and a new Conditional Random Integer Generator (CRIG) for generating the SLT parity packets and the SLT input packets, respectively. This version achieves a better BER performance than conventional quasi-regular LDPC codes in single antenna aided systems. The TDD $\Omega(d)$ of the SLT parity packets is given by [7]:

$$\Omega(d) = \begin{cases} \frac{1}{Z}\left[1 + \frac{S}{K} + \nu(d)\right] & \text{for } d = \gamma, \\ \frac{1}{Z}\left[\frac{\gamma}{d \cdot (\frac{\gamma}{d}+1)} + \frac{S}{K}\frac{\gamma}{d}\right] & \text{for } d = 2\gamma, \cdots, \frac{K \cdot \gamma}{S} - 1, \\ \frac{S}{Z \cdot K}\log(\frac{S}{\delta}) & \text{for } d = \frac{\gamma \cdot K}{S}, \\ 0 & \text{for } d > \frac{\gamma \cdot K}{S} \text{ and } d = 1, \end{cases} \tag{3}$$

where K is the number of input packets and S is the number of packets having a degree of γ. Furthermore, $S \equiv c \cdot log_e \cdot (K/\delta) \cdot \sqrt{K}$, $\nu(d)$ represents the extra fraction of SLT packets having a degree of γ required for the IRSD of [8] to ensure decodability and as in the context of Equation (2), $Z = \sum_d(\rho(d) + \tau(d)) + \nu(\gamma)$, where γ is an integer number higher than 1 required to ensure that the degree distribution becomes a truncated degree distribution and the smallest degree of the TDD equals to γ. When the highest degree of an SLT-encoded packet is $\frac{\gamma \cdot K}{S}$, this ensures that all input packets will be represented by the encoded packets of the parity part of the SLT code at least γ times. Hence, the degree distribution now becomes a truncated distribution, where the smallest degree difference between the encoded parity packets having different degrees is γ. The above-mentioned CRIG used for coining the specific degree of a particular SLT-encoded packet is based on the Bit Swapping Random Integer Generator (BSRIG) of [10] constrained by the condition of $d_m \leq \overline{D}$, where d_m is the degree of the SLT source packets and $\overline{D}$ is the average degree of the SLT source packets, which is calculated as [6].

$$\overline{D} = \frac{1-R}{R} \cdot (\overline{d} + 1), \tag{4}$$

where $\overline{d}$ is the average degree of the parity packets in Equation (3).

C. SLT coded V-BLAST system Overview

The SLT coded V-BLAST system's structure is portrayed in Figure 2, which employed four transmit and four receive antennas.

At the transmitter side, the source information bits are encoded by the SLT encoder, which are then mapped to the QPSK symbols of the modulator employing set partitioning based mapping [11]. The symbols at the output of the QPSK modulator are encoded by the VBLAST encoder and transmitted over a correlated narrow-band Rayleigh fading channel having a normalised Doppler frequency of 0.01.

At the receiver side of Figure 2, the received signal is decoded by the V-BLAST decoder, as described briefly in Section II-A. The output of the V-BLAST decoder is passed to the QPSK demapper, which also receives $a\ priori$ information from the SLT decoder. The $extrinsic$ LLR values at the output

[1]The degree distribution of a SLT code determines the specific number of source packets, which contribute to an SLT-encoded packet. A further important code parameter is the particular random number, which determines the specific degree of a given SLT-encoded packet.

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2008 proceedings.

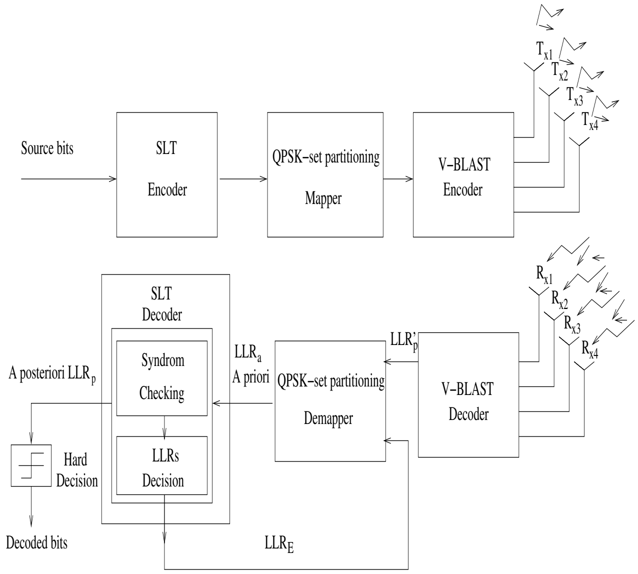

Fig. 2. The block diagram of the SLT coded V-BLAST system.

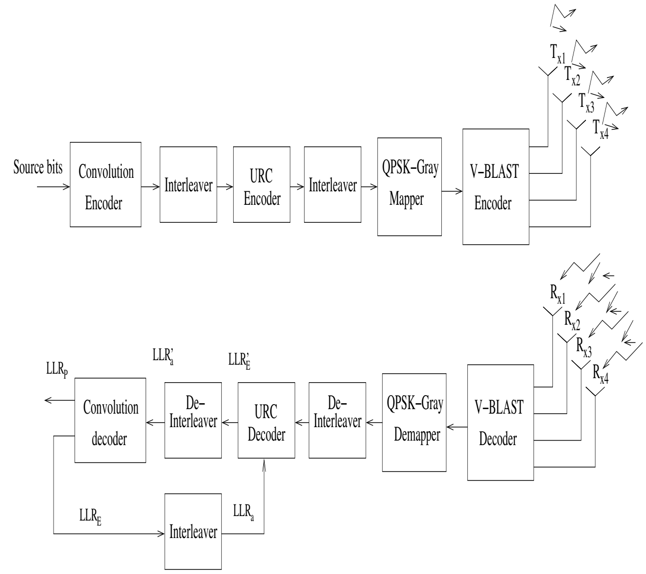

Fig. 3. The block diagram of the RSC-URC coded V-BLAST system.

of the QPSK demapper are passed to the SLT decoder as $a\ priori$ information. The SLT decoding process is assisted by the syndrome checking block of Figure 2. The output LLRs of the SLT decoder directly correspond to the $a\ posteriori$ LLRs, when the syndrome checking block detects the syndrome $S = C \times H^T = 0$, where C is a legitimate codeword of the SLT code and H^T is the transpose of the Parity Check Matrix (PCM) of the SLT code. However, when the syndrome becomes $S \neq 0$, then the output LLRs of the SLT decoder no longer constitute the $a\ posteriori$ LLRs. Instead, they constitute the $extrinsic$ LLR information, which is calculated by subtracting the $a\ priori$ LLRs from the $a\ posteriori$ LLR values.

As seen in Figure 2, the output LLRs of the SLT decoder are fed back to the QPSK demapper as $a\ priori$ LLRs. The extrinsic information aided iterative decoding process is then continued between the SLT decoder and the QPSK demapper, until all syndromes S at the SLT decoder become equal to zero or the number of iterations reaches the given maximum allowable value. During the last iteration, the $a\ posteriori$ LLR values generated at the output of the SLT decoder are passed to the hard decision block for the sake of recovering the original information bits.

The benchmark scheme considered in this paper consists of a Recursive Systematic Convolutional (RSC) code used as the outer code and a Unity Rate Code (URC) as the inner code, as shown in Figure 3. In the benchmark scheme, the information bits are firstly encoded by the RSC encoder and then precoded by the URC encoder, before they are Gray mapped by the QPSK modulator and transmitted using the V-BLAST scheme. The benchmark receiver carries out decoding iterations between the RSC decoder and the URC decoder.

III. EXIT CHART ANALYSIS

In the SLT coded V-BLAST system of Figure 2, the SLT code constitutes the outer code and the QPSK-set partitioning based mapper is the inner coder.

The iteration process is implemented by passing *extrinsic* LLRs between the SLT decoder and the QPSK-set partitioning based demapper at the receiver. In Figure 4 we plot the inverted EXIT curve of the SLT decoder as well as the EXIT curves of the QPSK demapper for various E_b/N_0 values. As seen in Figure 4, the interleaver length is $L = 1,200$ bits. The EXIT tunnel between the demapper and the SLT decoder is quite wide at $E_b/N_0 = 3dB$, when we use the set partitioning based mapper, but the bit-by-bit stair-case-shaped decoding trajectory does not accurately match the EXIT curves of the inner and outer codes, because for the short-duration 1200-bit interleaver length the LLRs are no longer Gaussian distributed, although this assumption is exploited by the EXIT chart[2]. Hence, the BER performance of the system employing an $L = 1,200$-bit interleaver does not match the predictions of the EXIT chart. By contrast, the decoding trajectory does match the EXIT curves for $E_b/N_0 > 6$ dB quite accurately. Observe that two EXIT curves are plotted for the SLT code in Figure 4, one of them is drawn using continuous lines and the other using dashed lines. The dashed curve represents the EXIT curve of the SLT decoder without using the syndrome checking block of Figure 2, while the other does employ the syndrome checking block. In other words, when using syndrome checking at the SLT decoder, the achievable BER performance improves and the size of the area S seen in

[2]Observe in Figure 4 that the stair-case-shaped decoding trajectory is unable to reach the point of perfect convergence to an infinitesimally low BER, namely the point of $(I_A, I_E) = (1, 1)$.

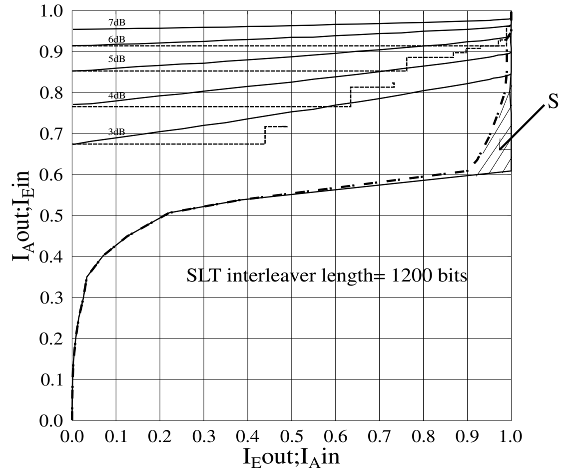

Fig. 4. EXIT charts and decoding trajectories of the SLT coded V-BLAST system for L= 1,200 bits.

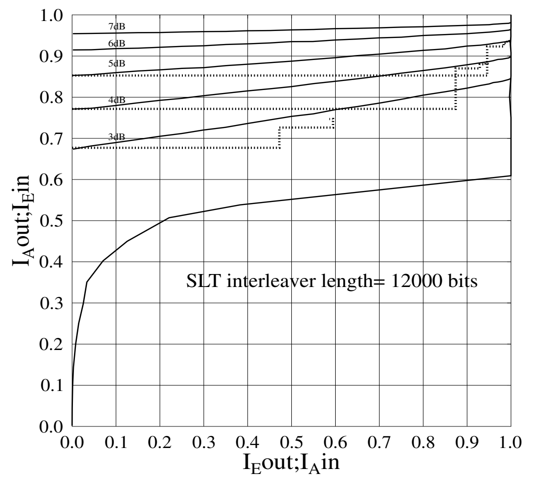

Fig. 5. EXIT charts and decoding trajectories of the SLT coded V-BLAST system for L= 12,000 bits.

Parameters in Equation (3)	δ= 0.5 c= 0.1
SLT code rates R	1/2
γ	2
The maximum number of inner iterations $I_{inner}max$ of SLT codes	30
Modulation	QPSK-set partition mapping
V-BLAST parameters	
Number of transmit antennas T_x	4
Number of receive antennas R_x	4
Interleaver lengths	1,200, 1,2000

TABLE I
SYSTEM PARAMETERS.

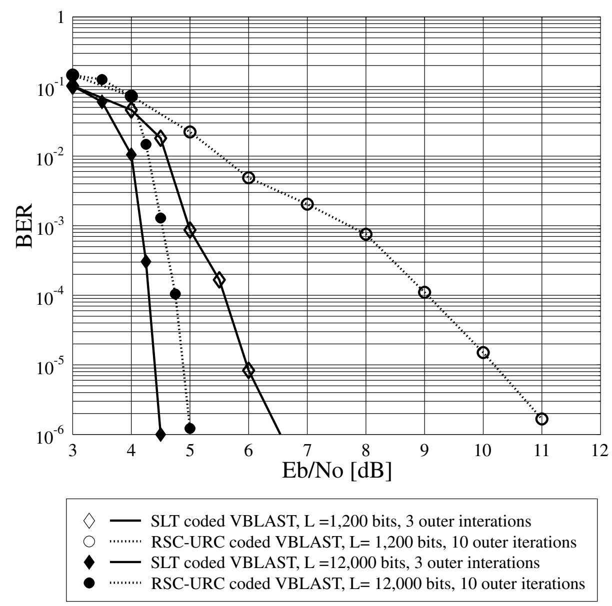

Fig. 6. BER performance of the proposed and the benchmark systems at an interleaver length of $L = 1,200$ and $L = 12,000$.

Figure 4 is characteristic of the achievable BER gain.

For the sake of improving the BER performance of the SLT coded V-BLAST scheme let us now increase the interleaver length to $L = 12,000$ bits. The system now becomes capable of achieving convergence at $E_b/N_0 = 5dB$ after $I = 2$ iterations, as seen in Figure 5. Increasing the interleaver length will improve its BER performance, but the complexity of the SLT coded V-BLAST system will also increase, as discussed in the next section.

IV. PERFORMANCE ANALYSIS

In this section we characterise the BER performance of the SLT coded V-BLAST scheme against that of the benchmark scheme of Figure 3. The benchmark scheme employs a 1/2-rate memory-2 RSC code having octally represented generator polynomials of $G_r = 7$ and $G = 5$, where G_r denotes the feedback generator polynomial and G denotes the feedforward generator polynomial. The URC code has a memory of 1. All simulation parameters are listed in Table I.

Figure 6 shows the BER performances of the SLT coded V-BLAST system and that of the RSC-URC coded V-BLAST system, when using an interleaver length of L= 1,200 bits and L= 12,000 bits. When employing an L=1,200-bit interleaver, the SLT coded V-BLAST system achieves BER $\leq 10^{-6}$ at E_b/N_0= 6.5 dB, while the RSC-URC coded V-BLAST arrangement requires an E_b/N_0 in excess of 11 dB to achieve the same BER. When we increase the interleaver length to $L = 12,000$ bits, as seen in Figure 6, the BER performance of the RSC-URC coded V-BLAST system improves more substantially than that of the SLT coded V-BLAST system, but still requires an E_b/N_0 value above 5dB for achieving BER $\leq 10^{-6}$. However, the SLT coded V-BLAST system still achieves BER$\leq 10^{-6}$ at E_b/N_0= 4.5 dB.

The complexity of the two systems mainly depends on that of the SLT decoder and on that of the RSC-URC decoders, when calculated based on the method proposed in [12]. We employed the Max-Log-MAP decoding algorithm for the RSC and URC decoders, while the message passing technique for the SLT decoder. Let us consider an interlever length of $L = 1,200$ bits. Recall furthermore that $R = 1/2$ is the code rate of the SLT and RSC codes. The number of outer iterations of the RSC-URC coded V-BLAST system was set to $I_{outer_{RSC-URC}}$= 10, while $m_1 = 2, m_2 = 1$ represents the memories of the RSC and URC codes, respectively. The message passing algorithm exchanged information between the message nodes and parity nodes. Hence, if $i = 8$ is the number of binary ones in a row of the PCM, then $j = i + 1 = 9$ is the number of binary ones in a column of the PCM of the SLT(1200,2400) code using the TDD distribution of Equation (3). Hence, the PCM of the SLT has $i = 8$ and $j = 9$. The maximum number of inner iterations used by the SLT code is $I_{inner}max = 30$ and the number of outer iterations is $I_{outer_{SLT}} = 3$. The complexity Λ_{SLT} of the SLT decoder is calculated in terms of the number of Add-Compare-Select (ACS) arithmetic operations as follows [12]:

$$\begin{aligned}\Lambda_{SLT} &= I_{outer_{SLT}} \times (I_{inner}max \times 4 \times i \times L \\ &\quad + I_{inner}max \times j \times L) \\ &= 3 \times (30 \times 4 \times 8 \times 1200 + 30 \times 9 \times 1200) \\ &= 4,428,000\ ACS. \end{aligned} \quad (5)$$

On the other hand, the complexity $\Lambda_{RSC-URC}$ of the benchmark scheme is calculated as follows [12]:

$$\begin{aligned}\Lambda_{RSC-URC} &= I_{outer_{RSC-URC}} \times L \times (10 \times 2^{m_1} \\ &\quad + 10^{m_2} + 2 \times 4 \times 2^{m_1} + 2 \times 2^{m_2} \\ &\quad + 2 \times 4 \times 2^{m_2} + 4^{m_1} - 2 + 4^{m_2} - 2) \\ &= 10 \times 1200 \times (10 \times 2^2 + 10 \times 2^1 \\ &\quad + 2 \times 4 \times 2^2 + 2 \times 4 \times 2^1 \\ &\quad + 4 \times 2^2 - 2 + 4 \times 2^1 - 2) \\ &= 1,536,000\ ACS. \end{aligned} \quad (6)$$

From Equations (5) and (6) we calculate the complexity ratio Λ_{ratio} between the SLT coded and the RSC-URC coded V-BLAST systems as follows:

$$\Lambda_{ratio} = \frac{\Lambda_{SLT}}{\Lambda_{RSC-URC}} = \frac{4,428,000}{1,536,000} = 2.88. \quad (7)$$

The number of inner iterations in the SLT decoder is related to the E_b/N_0 value and the syndrome checking process. In other words, as E_b/N_0 increases, the number of inner iterations required in order to arrive at a syndrome of $S = 0$ decreases. Hence, the complexity of the SLT aided system obeys $\Lambda_{SLT} \leq 2.88 \times \Lambda_{RSC-URC}$.

V. Conclusions

In this contribution we proposed a SLT coded V-BLAST scheme for the sake of improving the V-BLAST system's performance. EXIT charts were used to analyse the system's performance. The SLT coded system outperformed the benchmark scheme by about 5 dB for an interleaver length of L= 1,200 bits and by about 0.5 dB for L= 12,000 bits.

References

[1] L. Hanzo, S. X. Ng, T. Keller and W. Webb, *Quadrature Amplitude Modulation: From Basics to Adaptive Trellis-Coded, Turbo Equalised and Space-Time Coded OFDM, CDMA and MC-CDMA Systems, 2nd Edition*. Chichester, England: John Wiley and Sons Ltd and IEEE Press, 2004.

[2] G. Foschini and M. Gans, "On Limits of Wireless Communication in a Fading Environment when using Multiple Antennas," *Wireless Personal Comunications*, vol. 10, pp. 311–335, March 1998.

[3] E. Telatar, "Capacity of Multi-Antenna Gaussian Channels," *European Transactions on Telecommunications*, vol. 10, pp. 585–595, Nov./Dec. 1999.

[4] P.W. Wolniansky, G.J. Foschini, G.D. Golden and R.A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates over the Rich-scattering Wireless Channel," in *International Symposium on Signals, Systems, and Electronics*, (Pisa), pp. 295–300, September 1998.

[5] L. Hanzo, M. Münster, B.J. Choi and T. Keller, *OFDM and MC-CDMA for Broadband Multi-user Communications, WLANs and Broadcasting*. Chichester, UK: Wiley, 2003.

[6] T.D. Nguyen, L-L Yang and L. Hanzo, "Systematic Luby Transform Codes and Their Soft Decoding," *To appear in IEEE Workshop on Signal Processing Systems in Shanghai, China, October, 2007, at http://www.sips2007.org/AcceptedPapers.asp*.

[7] T. D. Nguyen, L. L. Yang, S. X. Ng and [1]L. Hanzo, "An optimal degree distribution design and a conditional random integer generator for the systematic Luby Transform coded wireless internet," *Submitted for WCNC 2008, http://eprints.ecs.soton.ac.uk/14587*.

[8] R. Y. S. Tee, T. D. Nguyen, L. L. Yang and L. Hanzo, "Serially Concatenated Luby Transform Coding and Bit-Interleaved Coded Modulation Using Iterative Decoding for the Wireless Internet," in *Vehicular Technology Conference*, vol. 5, (Melbourne, Austrialia), pp. 2494–2498, Spring 2006.

[9] M. Luby, "LT codes," *Proceeding of the 43rd Annual IEEE Symposium on Foundations of Computer Science*, pp. 271–282, 2002.

[10] R. Tee, T.D. Nguyen, L-L. Yang and L. Hanzo, "Luby Transform Coding Aided Bit-Interleaved Coded Modulation for the Wireless Internet," *To be presented in Proceeding of VTC Fall 2007, Baltimore, 30 Sept. -3 Oct*, 2007.

[11] G. Caire, G. Taricco and E. Biglieri, "Bit-Interleaved Coded Modulation," *IEEE Transactions on Information Theory*, vol. 44, pp. 927–946, May 1998.

[12] M. Sabbaghian, D. Falconer, "Comparison Between Convolutional and LDPC Code-based Turbo Frequency Domain Equalization," *IEEE International Conference on Communications*, vol. 12, pp. 5432–5437, June 2006.