Space Time Block Coded Multi-user CDMA Systems over Rayleigh Fading Channel

M. Shahriar Mamun, Jamil Hussain, Md. Hasibul Haque, Rishad Ahmed Shafik
Department of Electrical and Electronic Engineering,
Islamic University of Technology (IUT), Boardbazar, Gazipur-1704, Bangladesh.
Email: {*ron_iut, jamil_iut, tanim_022420, rishad09*}*@yahoo.com*

***Abstract*—This paper investigates the performance and capacity of space time block coded (STBC) multi-user CDMA system over Rayleigh fading channel condition using multiple transmit antennas. Using simulation and analytical approach, we show that STBC CDMA system has increased performance in cellular networks. We also compare the performance of this system with the typical CDMA system and show that STBC and multiple transmit antennas for multi-user CDMA system provide performance gain without any need of extra processing or bandwidth.**

***Index Terms*—Code Division Multiple Access (CDMA), CDMA capacity, Space Time Block Code (STBC), Rayleigh Fading, Transmit Diversity.**

I. Introduction

At present, wireless communication is experiencing an exponential growth rate. Providing adequate quality of service (QoS) under restricted bandwidth is now one of the greatest concerns [1], [5]. The next generation wireless systems are expected to meet the ever increasing demands, such as, high voice quality and bit rate, coverage, bandwidth and power efficiency, less effect of channel impairments, ability to be deployed in diverse environments, and so on. The remote units need to be small and lightweight to provide better service and work efficiently in any sort of environments [1]. At the same time the cost effectiveness should also be taken into account. To meet up all these ambitious objectives, Code Division Multiple Access (CDMA) can be considered as a remarkable technology [5].

CDMA is a form of direct sequence spread spectrum communications that supports simultaneous digital transmission of several users' signals in multiple access environments. It provides a unique property of supporting multiplicity of users in the same radio channel with graceful degradation of performance due to multi-user interference. Hence any reduction in interference leads to explicit increase in capacity. The most notable advantage of CDMA is its ability to combat Rayleigh fading [5]. In CDMA, the bandwidth is spread by means of a code, generally known as pseudo-random noise (PN) code, which is independent of the data and the receiver synchronizes to the code to retrieve the data. Several detection techniques have been developed to manipulate the transmitted data. The use of such independent code and synchronous reception allows multiple users to access the same frequency band at the same time [5]. This leads to the fact that CDMA system has a very high spectral density and it can accommodate more users per MHz of bandwidth than any other technology.

Fading is a common phenomenon which deteriorates the original signal while transmitted through wireless channel. This signal experiences both small-scale and large-scale fading. Small-scale fading is also known as Rayleigh Fading because if the multiple reflective paths are large in number and there is no line of sight signal component, the envelope of the received signal can statistically be described by a Rayleigh pdf [6].

In most scattering environments, antenna diversity is a practical, effective and, hence, a widely applied technique for reducing the effect of Rayleigh fading [1]. The number of antennas at the transmitter or the receiver decides the type of the system that will finally be implemented. Space-time processing will either be receive diversity or transmit diversity. In receive diversity, the channel can be estimated and there can be multiple antennas at the receiver. The major problem with using the receive diversity approach is the cost, size, and power of the remote units. The use of multiple antennas and radio frequency (RF) chains (or selection and switching circuits) makes the remote units larger and more expensive. As a result, diversity techniques have almost exclusively been applied to base stations to improve their reception quality [3]. In this paper, we show two diversity techniques using STBC for multi-user CDMA systems over Rayleigh Fading channel condition.

The paper is organised as follows. Section II presents the capacity of CDMA system, while in Section III, multi-user CDMA system using STBC has been discussed including two transmit diversity schemes (Alamouti 2:1, Alamouti 2:2) and corresponding capacities of the schemes have been evaluated mathematically. Section IV conducts the simulated results and finally the paper is concluded by a summary of comparison of the schemes.

II. Capacity of CDMA System

The capacity of a CDMA system is chracterised by the maximum number of simultaneous users that can be supported by the system, while the service quality requirements of each user, such as the data rate, bit error rate (BER), and outage probability, are being satisfied [7]. The performance and capacity of a CDMA system are limited by near-far effect and multiple access interference (MAI) caused by signals from other users. The actual capacity of a CDMA cell depends on several different factors, such as receiver demodulation, power-control accuracy, and actual interference power introduced by other users in the same cell and in neighboring cells as well [7], [8]. In case of CDMA systems, transmit power constraints and the system's self-generated interference ultimately restrict CDMA capacity. Normally, the capacity of a CDMA system depends on the reverse link (MS to BS) capacity. The maximum number of users that can be supported by the forward link (BS to MS) of a CDMA system is different from the maximum number that can be supported by the reverse link [8].

A. Single Cell CDMA Capacity

For a single cell CDMA system, each user occupies the entire allocated spectrum and the neighboring cell interference has no effect on the capacity [7], [8]. For a single cell system, it is assumed that reverse link signals are received at the same power level. For N users, the received signal consists of desired signal of power, S and interfering signal of power, $(N-1)S$. From this point of view, Signal to Noise Ratio (SNR) can be defined as

$$SNR = \frac{S}{(N-1)S} = \frac{1}{N-1} \quad (1)$$

For a reliable system operation, it is desired to express performance by bit energy-to-noise density ratio. Bit energy can be obtained by dividing the signal power by information bit rate, R; where noise density can be calculated by dividing the noise (or interference) by the bandwidth, W. This results

$$E_b/N_0 = \frac{S/R}{(N-1)S/W} = \frac{W/R}{N-1} \quad (2)$$

Equation (2) ignores background noise, η, which occurs due to spurious interference as well as thermal noise contained in the total bandwidth, W. Including this additive term in the denominator of (2) results in a required

$$E_b/N_0 = \frac{W/R}{(N-1)+(\eta/S)}. \quad (3)$$

From (3), capacity can be evaluated as

$$N = 1 + \frac{W/R}{E_b/N_0} - \frac{\eta}{S}. \quad (4)$$

From (4), it is obvious that background noise plays quite a vital role for the capacity of the system. In forward link, coherent modulation is done by inserting pilot symbol which is being tracked and multiple transmitted signals are combined synchronously, its performance is much superior to that of the reverse link in a single cell system [8].

B. Multiple Cell CDMA Capacity

For a multiple cell system, users in neighboring cells generate additional interference, which is quite a different scenario compared to that of a single cell system, where the other users in the same cell generate the interference to the desired user.

In Section II-A, it has been shown that, for N users per cell, the total interference can never be greater than $(N-1)S$, but on an average it can be reduced by the voice activity factor, α. While considering multiple cell system, when the interfering user is in other cell at a distance r_m from its cell site and r_0 from the cell site of the desired user, then the interference produced by the interfering user, when active, to the desired user's cell site is

$$\begin{aligned}\frac{I(r_0,r_m)}{S} &= \left(\frac{10^{\xi_0/10}}{r_0^4}\right)\left(\frac{r_m^4}{10^{\xi_m/10}}\right) \\ &= \left(\frac{r_m}{r_0}\right)^4 10^{(\xi_0-\xi_m)/10} \leq 1, \end{aligned} \quad (5)$$

where the first term is due to the attenuation caused by distance and blockage to the given cell site, while the second term is the effect of power control to compensate for the corresponding attenuation to the cell site of the out-of-cell interferer [8]. The Gaussian random variables, ξ_m and ξ_0 are independent so that the difference has zero mean and variance $2\alpha^2$.

Assuming a uniform density of subscribers and normalizing the hexagonal cell radius to unity, and since the average number of subscribers per cell is $N = 3N_s$, the density of users per unit area is

$$\rho = \frac{2N}{3\sqrt{3}} = \frac{2N_s}{\sqrt{3}}. \quad (6)$$

The other cell interference in the CDMA system for the voice service can be modeled as a Gaussian random variable [7], where the mean and variance effectively influence the characterization of the capacity of the system. Now the total other-cell user interference-to-signal ratio is

$$\begin{aligned} I/S = \iint \psi \left(\frac{r_m}{r_0}\right)^4 & 10^{(\xi_0-\xi_m)/10} \\ & .\O(\xi_0-\xi_m, r_0/r_m) \;\; \rho \, dA, \end{aligned} \quad (7)$$

where ψ is the voice activity variable, a binomial random variable which equals to 1 with probability α, and to 0 with probability $(1-\alpha)$ and m is the cell-site index, for which

$$r_m^4 10^{-\xi_m} = \min_{k\neq 0} r_k^4 10^{-\xi_k}, \quad (8)$$

and

$$\O(\xi_0-\xi_m, r_0/r_m) = \begin{cases} 1, & if (r_m/r_0)^4 10^{(\xi_0-\xi_m)/10} \leq 1 \\ & or(\xi_0-\xi_m) \leq 40 log_{10}(r_0/r_m) \\ 0, & otherwise. \end{cases} \quad (9)$$

To compute the moment statistics of random variable I, if smallest distance is considered instead of smallest attenuation, the calculation becomes simpler and result does not change that much. Hence (7), with (9), holds as an upper bound if in place of (8) we use that value of m, for which

$$r_m = \min_{k\neq 0} r_k \quad (10)$$

Now, by complex calculation of reverse link outer cell interference, it can be shown that the mean or first moment, of the random variable I/S is upper bounded (using 10 rather than 8 for m) by the expression

$$E(I/S) = \alpha \iint \frac{r_m^4}{r_0} f\left(\frac{r_m}{r_0}\right) \rho \, dA$$

where

$$f\left(\frac{r_m}{r_0}\right) = \exp[(\sigma \ln 10/10)^2] \left\{ 1 - Q\left[\frac{40}{\sqrt{2\sigma^2}} \right.\right. \\ \left.\left. . \log_{10}\left(\frac{r_0}{r_m}\right) - \sqrt{2\sigma^2}\,\frac{\ln 10}{10} \right] \right\} \quad (11)$$

and

$$Q(x) = \int_x^\infty e^{(-y^2/2)} dy/\sqrt{2\pi}.$$

where

$$m = y^2/2.$$

To compute this integral over the two-dimensional area numerically, values of r_0, distance to the desired cell site and r_m, distance to the closest cell site, along with the function of (11) are required to be evaluated. The result for $\sigma = 8dB$ is

$$E(I/S) \leq 0.247 N_s \quad (12)$$

From (12), the number of user per cell, (N_s) can be calculated.

III. Space-Time Multi-user CDMA Systems

Space Time Block Code (STBC) is an effective transmit diversity technique used to transmit symbols from multiple antennas, which ensures that transmission from various antennas is orthogonal [2], [5]. Wireless transmission with high data rate, as well as diversity and coding gain, is quite achievable using STBC, which combats fading in wireless communications [2]. STBC is a highly efficient approach to signaling within wireless communication that takes the advantage of the spatial dimension by transmitting a number of data streams using multiple co-located antennas [3]. The main feature of STBC is the provision of full diversity with very simple, yet effective encoding and decoding mechanism.

STBC operates on a block of data-stream which is to be transmitted and produces a matrix whose rows and columns represent antennas and time, respectively. The usual and simplest representation of STBC is shown below

$$\text{time-slots} \downarrow \overset{\text{transmit antennas}\ \longrightarrow}{\begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n_T} \\ s_{21} & s_{22} & \cdots & s_{2n_T} \\ \vdots & \vdots & & \vdots \\ s_{T1} & s_{T2} & \cdots & s_{Tn_T} \end{bmatrix}}$$

Here, s_{ij} is the modulated symbol to be transmitted from antenna j in time-slot i. There should be T time-slots and nT transmit antennas as well as nR receive antennas. This block is usually considered to be of length T. We consider two diversity schemes for our analyses:

1) *Scheme-I*: two transmit antennas, one receive antenna
2) *Scheme-II*: two transmit antennas, two receive antennas

A. Scheme-I: Two transmit antennas, one receive antenna

For the scenario where the combination of two transmitters and one receiver is employed, a diversity scheme, proposed by S. M. Alamouti, has been adopted. This particularly simple and prevalent scheme, with two transmit antennas and one receive antenna, uses simple coding which is the only STBC that can achieve its full diversity gain without needing to sacrifice its data rate. As per Alamouti's scheme, the transmitter sends out data in groups of 2 (two) bits. The scheme may be analyzed by the following three functions [1]:

- The encoding and transmission sequence of information symbols at the transmitter;
- The combining scheme at the receiver;
- the decision rule for maximum likelihood detection.

1) The Encoding and Transmission Sequence: At a given symbol period, two signals, transmitted from two antennas: antenna zero and antenna one, are denoted by s_0 and s_1 simultaneously. During the next symbol period, signal $(-s_1^*)$ is transmitted from antenna zero, and signal s_0^* is transmitted from antenna one, where $*$ stands for complex conjugate operation. The encoding is done in space and time (and hence space-time coding). The assumption made for this scheme is that the channel state remains fairly constant over the transmission of 2 (two) consecutive symbols [1], [4].

Assuming that fading is constant across two consecutive symbols, the channel at time may be modeled as

$$h_0(t) = h_0(t+T) = h_0 = \alpha_0 e^{j\theta_0} \\ h_1(t) = h_1(t+T) = h_1 = \alpha_1 e^{j\theta_1} \quad (13)$$

where T is the symbol duration.

The received signals, r_0 and r_1 at time T and $t+T$ respectively can be expressed as

$$r_0 = r(t) = h_0 s_0 + h_1 s_1 + n_0 \\ r_1 = r(t+T) = -h_0 s_1^* + h_1 s_0^* + n_1 \quad (14)$$

where n_0, n_1 are complex random variable representing receiver noise and interference.

2) The Combining Scheme: The combiner builds the following two combined signals that are sent to the maximum likelihood detector

$$\tilde{s}_0 = h_0^* r_0 + h_1 r_1^* \\ \tilde{s}_1 = h_1^* r_0 - h_0 r_1^* \quad (15)$$

3) The Maximum Likelihood Decision Rule: Finally in the maximum likelihood detector, signals are chosen either s_0 or s_1 according to corresponding decision rule. Thus, the scheme reduces complexity and simplifies transmission greatly.

B. Capacity of Scheme-I

Capacity is generally associated with a particular communication channel rather than a transmission scheme. However, it is also possible to consider the maximum achievable rate of some STBCs, by treating the channel as a standard Additive

White Gaussian Noise (AWGN) channel and re-assigning the received SNR according to the properties of the code. This quantity is referred as $C_E = log_2(1+\rho)$ as the effective capacity, where ρ is the post-detection SNR achieved by the code. By considering the 2:1 Alamouti STBC just discussed in Section III-A, channel matrix is $\bar{h}^T = [h_1 \quad h_2]$ and the SNR seen at the receiver is given by $\frac{\rho}{2}(|h_1|^2 + |h_2|^2)$.

The instantaneous capacity supported by the channel is then given by

$$\begin{aligned} C_{2:1|\bar{h}} &= log_2 \det\left[I_1 + \frac{\rho}{2}\,\bar{h}^T\,\bar{h}^*\right] \\ &= log_2\left[1 + \frac{\rho}{2}\big(|h_1|^2 + |h_2|^2\big)\right] \\ &= \frac{\rho}{2}\,C_{E,Scheme-I} \end{aligned}$$

Therefore it is obvious that the Alamouti code is able to achieve the maximum capacity offered by the 2:1 MISO (Multiple Input Single Output) channel. Note that this statement is not meant to imply that the Alamouti STBC does in fact achieve full channel capacity, only that it is not restricted by its structure to some fraction of the available capacity.

C. Scheme-II: Two transmit antennas, two Receive Antennas

When a higher order of diversity is needed and multiple receive antennas at the remote units are feasible, it is possible to provide a diversity order of $2M$ with two transmit and M receive antennas [1]. In this section, a special case considering two transmitters and two receivers has been illustrated briefly in almost similar fashion as already done in Section III-A. The generalization to M receive antennas is trivial.

1) The Encoding and Transmission Sequence: The encoding and transmission sequence for this configuration is identical to the case discussed in Section III-A1. The channel at time t can be modeled by complex multiplicative distortions, $h_0(t)$, $h_1(t)$, $h_2(t)$, $h_3(t)$ between transmit antenna zero and receive antenna zero, transmit antenna one and receive antenna zero, transmit antenna zero and receive antenna one, transmit antenna one and receive antenna one, respectively.

Assuming that fading is constant across two consecutive symbols, it can be written

$$\begin{aligned} h_0(t) &= h_0(t+T) = h_0 = \alpha_0 e^{j\theta_0} \\ h_1(t) &= h_1(t+T) = h_1 = \alpha_1 e^{j\theta_1} \\ h_2(t) &= h_2(t+T) = h_2 = \alpha_2 e^{j\theta_2} \\ h_3(t) &= h_3(t+T) = h_3 = \alpha_3 e^{j\theta_3} \end{aligned} \qquad (16)$$

where T is the symbol duration.

The received signals can then be expressed as

$$\begin{aligned} r_0 &= h_0 s_0 + h_1 s_1 + n_0 \\ r_1 &= -h_0 s_1^* + h_1 s_0^* + n_1 \\ r_2 &= h_2 s_0 + h_3 s_1 + n_2 \\ r_3 &= -h_2 s_1^* + h_3 s_0^* + n_3 \end{aligned} \qquad (17)$$

The complex random variables, n_0, n_1, n_2 and n_3 represent receiver thermal noise and interference.

2) The Combining Scheme: The combiner builds the following two combined signals that are sent to the maximum likelihood detector

$$\begin{aligned} \tilde{s}_0 &= h_0^* r_0 + h_1 r_1^* + h_2^* r_2 + h_3 r_3^* \\ \tilde{s}_1 &= h_1^* r_0 - h_0 r_1^* + h_3^* r_2 - h_2 r_3^* \end{aligned} \qquad (18)$$

3) The Maximum Likelihood Decision Rule: The maximum likelihood detector chooses either s_0 or s_1 according to corresponding decision rule, as was done in Section III-A3.

D. Capacity of Scheme-II

Considering the scheme discussed in Section III-C, the channel matrix and the received SNR are given by

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

and

$$\rho_{Scheme-II} = \frac{\rho}{2}(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2)$$

where ρ is the instantaneous received SNR and $\rho_{scheme-II}$ is the instantaneous received SNR for scheme-II, H is channel matrix.

The instantaneous capacity of the channel is given by

$$\begin{aligned} C_{2:2|H} &= log_2 \det\left[I_2 + \frac{\rho}{2}\,HH^+\right] \\ &= log_2\Bigg[1 + \frac{\rho}{2}\big(|h_{11}|^2 + |h_{12}|^2 + |h_{21}|^2 + |h_{22}|^2\big) \\ &\qquad + \left(\frac{\rho}{2}\right)^2 \det(HH^+)\Bigg] \end{aligned}$$

Which is certainly more than that of Scheme-I. Strict inequality follows by defining $\det(HH^+) \cong \prod_{i=1}^{rankH} \sigma_i^2$, where σ_i is the eigenvalue of the H matrix. Thus we can see that although the structured use of channel resources made by Alamouti's STBC leads to efficient detection algorithms, there is an implicit sacrifice of effective capacity. This class of codes is only optimal in terms of diversity advantage and rate for the 2:1 (Tx-2, Rx-1) MISO channel.

IV. Simulations and Results

Fig. 1 shows BER performance for both uncoded BPSK and coded CDMA system using Alamouti's STBC technique in Rayleigh fading condition. It is assumed that the receiver has perfect knowledge of the channel condition. It is obvious that the more the number of users, the more the degradation in performance. It is clear that Alamouti's STBC technique using two transmitting antennas and two receiving antennas for CDMA system yields around 1–2dB improvement in performance than using two transmitting antenna and one receiving antenna.

Fig. 2 presents comparison of performance among typical CDMA system, BPSK system using Alamouti's STBC technique and CDMA system implementing Alamouti's STBC technique. When Alamouti's STBC technique is used for

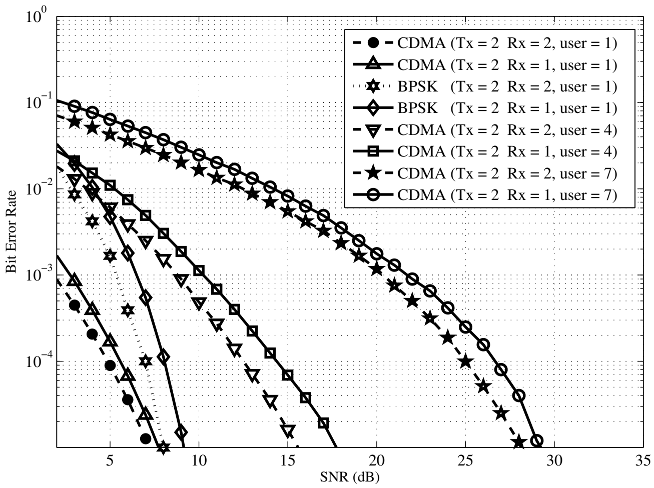

Fig. 1. Comparison of different schemes using Alamouti's STBC technique

CDMA system, the performance drastically improves by around 5dB. That means, it will require less power to transmit for same BER for CDMA system using Alamouti's STBC technique. It can be explained alternatively that for transmitting signal at a same power will give better BER for CDMA system with Alamouti's STBC technique than the typical CDMA system.

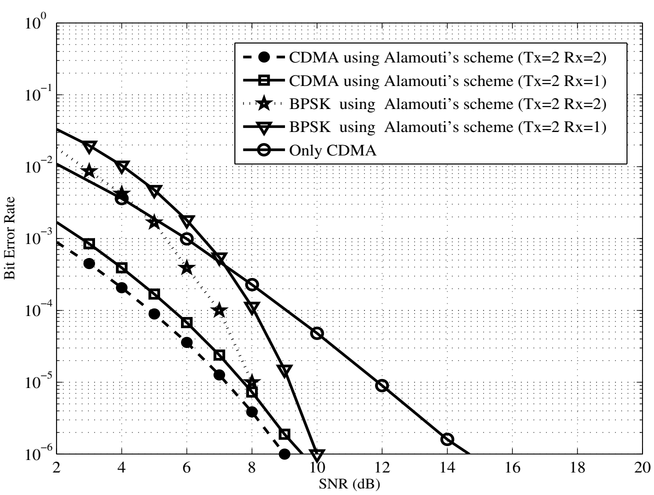

Fig. 2. Stylized BER versus SNR Performance of CDMA with and without Alamouti's scheme

It is evident that E_b/N_0 is decreased for CDMA system when Alamouti's scheme is used. Capacity of any system is inversely proportional to E_b/N_0, which indicates that the capacity increases for using Alamouti's scheme. It can be explained in other way that implementation of Alamouti's scheme in CDMA system achieves reduction in transmitting power which allows the number of users to be increased.

V. Conclusion

In this paper, multi-user STBC CDMA system has been implemented and analysed. Using and analytical and simulation approach, we have shown that using STBC in CDMA system is advantageous over traditional CDMA system including better BER performance, lower complexity and higher user capacity. In CDMA, where users simultaneously share the same spectrum, each user is an interferer to each of the other users in the same cell as well as nearby cells. Hence, the capacity per cell is inversely proportional to SNR. When Alamouti's STBC technique using multiple antennas is applied to CDMA system, SNR gets decreased which results in capacity increment. We hope that the findings of this paper would help clearly identifying advantages of such spatial and temporal multiplexing for CDMA systems.

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