

# Channel Coded Iterative Center-Shifting $K$ -Best Sphere Detection for Rank-Deficient Systems

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**Abstract**—Based on an EXtrinsic Information Transfer (EXIT) chart assisted receiver design, a low-complexity near-Maximum A Posteriori (MAP) detector is constructed for high-throughput MIMO systems. A high throughput is achieved by invoking high-order modulation schemes and/or multiple transmit antennas, while employing a novel sphere detector (SD) termed as a center-shifting SD scheme, which updates the SD's search center during its consecutive iterations with the aid of channel decoder. Two low-complexity iterative center-shifting SD aided receiver architectures are investigated, namely the direct-hard-decision center-shifting (DHDC) and the direct-soft-decision center-shifting (DSDC) schemes. Both of them are capable of attaining a considerable memory and complexity reduction over the conventional SD-aided iterative benchmark receiver. For example, the DSDC scheme reduces the candidate-list-generation-related and *extrinsic*-LLR-calculation related complexity by a factor of 3.5 and 16, respectively. As a further benefit, the associated memory requirements were also reduced by a factor of 16.

## I. INTRODUCTION

Sphere detection (SD) [1] techniques, have attracted wide interests in both the academic and industrial research communities, constituting a computationally efficient solution to the maximum likelihood (ML) detection problem in uncoded multiple input, multiple output (MIMO) systems [2, 3]. Various complexity reduction schemes optimizing the SD's search algorithm itself have been proposed in [2, 4, 5] in order to further reduce the complexity imposed by the SD. However, when employed in an iterative detection aided channel coded system, the soft-input-soft-output (SISO) SD termed as List SD (LSD) [2] has to generate soft information for every transmitted bit, which requires the observation of a high number of hypotheses about the transmitted MIMO symbol, resulting in a potentially excessive complexity. Explicitly, when aiming for achieving a near Maximum-A-Posteriori (MAP) performance, LSDs may still impose excessive computational complexities in high-throughput systems employing a large number of transmit antennas and/or high-order modulation schemes. This complexity-related predicament is further aggravated, when the number of transmit antennas exceeds that of the receive antennas, namely in the scenario of rank-deficient systems. Therefore, how to maintain a near-MAP performance with the aid of a small set of symbol hypothesis, i.e. small list size, is the key to the complexity reduction of soft-decision-aided SDs, which remains an open problem to be solved. Recently, the idea of choosing the hard decision ML symbol point as the LSD's search center was proposed by Boutros *et al.* [6], which has the advantage of maintaining a moderate the list size for the depth-first SD. *The novelty of this paper is outlined as follows:* 1) *We propose the center-shifting philosophy for the SD, which generalises the scheme of [6], leading to a potentially considerable reduction in the overall complexity imposed by the SD-aided iterative turbo receiver, as a benefit of its considerably reduced candidate list.* 2) *Based on the idea of the channel coded iterative center-shifting scheme, we proposed two low complexity center update schemes, resulting in two different iterative center-shifting SD aided receiver architectures, namely, the Direct-Hard-Decision Center-shifting (DHDC) and the Direct-Soft-Decision Center-shifting*

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(DSDC) aided receivers. Our Monte Carlo simulations and EXIT chart [7] analysis will demonstrate that the contrived center-shifting aided SD is capable of substantially reducing both the memory requirements and computational complexity imposed by the SD. In this paper, we demonstrate the benefits of the center-shifting in the context of the so-called  $K$ -Best SD when communicating in the worst-case scenario, namely in a high-throughput rank-deficient system.

The rest of this paper is organised as follows. Section II describes the system model of the MIMO systems considered. An overview of our novel center-shifting aided LSD, which is applicable to the channel coded system considered is given in Section III. The proposed center-shifting SD and the receiver architecture are presented in Sections IV, respectively. In Section V we provide our simulation results, while in Section VI we offer our conclusions.

## II. SYSTEM MODEL

Since Orthogonal Frequency-Division Multiplexing (OFDM) [8] has been developed to a promising candidate for next generation wideband digital communications, which is capable of coping with severe channel conditions, imposed by multipath-induced frequency-selective fading, we intend to investigate our proposed scheme in the scenario of a multiple antenna aided bandwidth-efficient Spatial-Division-Multiplexing (SDM) OFDM system. Consider the following generic MIMO system model employing  $M$  transmit and  $N$  receive antennas [8] per sub-carrier:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{w}, \quad (1)$$

where both  $\mathbf{y}$  and  $\mathbf{w}$  are  $(N \times 1)$ -element complex column vectors, which represent the received signal and the AWGN noise vectors, respectively, while the  $(M \times 1)$ -element complex-valued column vector  $\mathbf{s}$  denotes the transmitted signal vector. Moreover, the frequency-domain channel transfer factor (FDCHTF) matrix  $\mathbf{H}$  is a  $(N \times M)$ -element independent and identically distributed (i.i.d.) zero-mean unit variance complex Gaussian matrix, perfectly known to the receiver, with each column representing the unique spatial signature of the corresponding transmit antenna [8]. Furthermore, the AWGN noise,  $w_n$  encountered at the  $n$ th receive antenna element exhibits a zero-mean and a variance of  $\sigma_w^2$ .

## III. LIST $K$ -BEST SPHERE DETECTION

The well-known ML solution may be expressed as:

$$\hat{\mathbf{s}}_{ML} = \arg \min_{\mathbf{s} \in M_c^M} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|_2^2, \quad (2)$$

where  $M_c$  is the number of modulated symbol points in the constellation and again,  $M$  is the number of transmit antennas employed by the system. With the aid of the MMSE solution of  $\hat{\mathbf{x}}_c = (\mathbf{H}^H \mathbf{H} + \sigma_w^2 \mathbf{I})^{-1} \mathbf{H}^H \mathbf{y}$  and the Cholesky factorization [9], it may be readily shown that Eq.(2) can be expressed as [3]:

$$\hat{\mathbf{s}}_{ML} = \arg \min_{\mathbf{s} \in M_c^M} (\mathbf{s} - \hat{\mathbf{x}}_c)^H \mathbf{U}^H \mathbf{U} (\mathbf{s} - \hat{\mathbf{x}}_c), \quad (3)$$

$$= \arg \min_{\mathbf{s} \in M_c^M} \underbrace{\sum_{i=1}^M u_{ii}^2 [\tilde{s}_i - \hat{x}_i + \sum_{j=i+1}^M \frac{u_{ij}}{u_{ii}} (\tilde{s}_j - \hat{x}_j)]^2}_{\phi}, \quad (4)$$



1) *Direct-Hard-Decision Center-Shifting SD Assisted Receiver Architecture*: Following the above design principles and objectives, our first proposed center-calculation scheme is the so-called *Direct-Hard-Decision-Center-Shifting* (DHDC) scheme portrayed in Figure 3, which calculates the search center for the forthcoming detection iteration by imposing hard decisions on the interleaved *a posteriori* LLRs at the output of the channel decoder. Then it remodulates the resultant bit streams of all the SDM antennas, in order to generate the mapped symbol vector, which corresponds to the most recently obtained search center. Hence, as long as the search center  $\mathbf{x}_c$  is updated, the SD is required to regenerate the candidate list [2], which is used to calculate the *extrinsic* LLRs delivered to the outer decoder.

In this treatise we assume familiarity with the classic turbo detection principles [12]. In Figure 3 the interleaver and deinterleaver pair seen at the receiver side divides the receiver into two parts, namely, the MIMO detector (inner decoder) and the channel decoder (outer decoder). Note that in Figure 3,  $L_A$ ,  $L_E$  and  $L_D$  denote the *a priori*, the *extrinsic* and the *a posteriori* LLRs, while the subscripts '1' and '2' represent the bit LLRs associated with the inner detector and outer decoder, respectively.

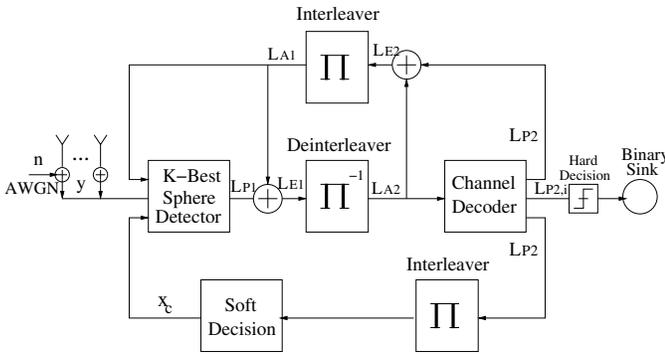


Fig. 4. Receiver architecture of the direct-soft-decision center-shifting aided iterative  $K$ -best SD scheme.

2) *Direct-Soft-Decision Center-Shifting SD Assisted Receiver Architecture*: Based on the idea of retaining the soft-bit-information contained in the *a posteriori* LLRs, we propose the iterative DSDC-aided  $K$ -Best SD receiver portrayed in Figure 4, where the soft-decision block substitutes the hard-decision and re-modulation functionality of the DHDC-aided iterative receiver shown in Figure 3. The scheme of Figure 4 provides a soft search center for the  $K$ -Best SD and based on the soft centers the SD is expected to generate a better candidate list for the following LLR calculation, which is then delivered to the outer channel decoder. Although the soft center calculation imposes a slightly higher computational complexity than its hard-decision based counterpart, the iterative DSDC-aided  $K$ -Best SD receiver is capable of attaining a higher performance gain over the conventional iterative receiver, as observed throughout our forthcoming EXIT chart analysis and in our BER results.

$m$	1	2	3	4
$b_{m,1} b_{m,2}$	00	01	10	11
$s_m$	$(1+i)/\sqrt{2}$	$(-1+i)/\sqrt{2}$	$(1-i)/\sqrt{2}$	$(-1-i)/\sqrt{2}$

TABLE I  
4-QAM SYMBOL ALPHABETS OVER THE COMPLEX NUMBERS

The *a posteriori* soft-bit-information delivered from the channel decoder to the SD is defined to be the logarithm of the bit-probability ratios of its two legitimate values, namely of  $+1$  and  $-1$ , given the received signal vector  $\mathbf{y}$ , which can be formulated as [12]:

$$L(b_k|\mathbf{y}) = \ln \frac{P[b_k = +1|\mathbf{y}]}{P[b_k = -1|\mathbf{y}]} \quad (6)$$

Therefore, bearing in mind that we have  $P[b_k = +1|\mathbf{y}] = 1 - P[b_k = -1|\mathbf{y}]$ , and taking the exponent of both sides in Eq.(6), it

is possible to derive the probability that  $b_k = +1$  or  $b_k = -1$  was transmitted in terms of their LLRs as follows:

$$e^{L(b_k|\mathbf{y})} = \frac{P[b_k = +1|\mathbf{y}]}{1 - P[b_k = +1|\mathbf{y}]} \quad (7)$$

From Eq.(7) we arrive at:

$$P[b_k = +1|\mathbf{y}] = \frac{1}{1 + e^{-L(b_k|\mathbf{y})}} \quad (8)$$

Similarly, we have:

$$P[b_k = -1|\mathbf{y}] = \frac{1}{1 + e^{+L(b_k|\mathbf{y})}} \quad (9)$$

In the following, we consider 4-QAM as an example to briefly discuss the soft-symbol calculation process with the aid of the LLR-to-probability conversion formula of Eq.(8) and Eq.(9). The symbol alphabet of the 4-QAM scheme is shown in Table I, which indicates that a 4-QAM symbol is constituted of two bits, the first of which determines the imaginary part of the symbol, while the second controls the real part. Specifically, given the probabilities of two successive bits, which constitute a 4-QAM symbol, from their two legitimate values of  $+1$  and  $-1$ , we can calculate the  $j$ th user's soft-symbol,  $s_j$ , as follows:

$$\begin{aligned} s_m &= [\Re(s_m); \Im(s_m)], \\ &= [P[b_{m,2} = -1|\mathbf{y}] \cdot (+1) + P[b_{m,2} = +1|\mathbf{y}] \cdot (-1); \\ &P[b_{m,1} = -1|\mathbf{y}] \cdot (+1) + P[b_{m,1} = +1|\mathbf{y}] \cdot (-1)]/\sqrt{2}, \end{aligned} \quad (10)$$

where we assumed that the two bits are independent of each other, which is not entirely true owing to their correlation imposed by the Gray mapping of bits to the 4-QAM symbols. The probabilities  $P[b_{m,k} = \pm 1|\mathbf{y}]$  can be calculated from Eq.(8) and Eq.(9) based on the *a posteriori* LLR values received from the outer channel decoder.

### C. Computational Complexity of the Iterative-Center-Shifting-Aided $K$ -Best SD

First of all, let us divide the complexity imposed by the  $K$ -best SD into two contributions, which are associated with the candidate-list generation (SD part) and the *extrinsic* LLR calculation (MAP part), respectively. Furthermore, we quantify the complexity of the list generation in terms of the number of PED evaluations corresponding to the term  $\phi$  of Eq.(4). Hence, the list-generation-related complexity can be approximated as:

$$C_{SD} \leq M \cdot \mathcal{M}_c \cdot K \quad (11)$$

number of PED evaluations. On the other hand, the complexity imposed by the *extrinsic* LLR calculation is quantified in terms of the number of objective function (OF) evaluations, which corresponds to the two terms in Eq.(5). The approximation in Eq.(5) becomes an equality, when  $\mathcal{L}$  represents the entire search space, constituted by  $\mathcal{N}_{cand} = \mathcal{M}_c^M = 2^{M \cdot BPS}$  number of OF evaluations, where  $BPS$  is the number of bits per symbol. Hence, the complexity of the resultant exact MAP detector can be calculated as the total number of OF evaluations given by:

$$C_{MAP} = M \cdot BPS \cdot 2^{(M \cdot BPS)} \quad (12)$$

Clearly, the complexity grows exponentially with the product of the number of transmit antennas  $M$  and the number of bits per symbol  $BPS$ . Let us consider an 8-transmit-antenna 4-QAM SDM system as an example, which imposes a potentially excessive complexity of  $C_{MAP} = 1,048,576$  OF evaluations.

Fortunately, the complexity may be significantly reduced by generating a list of candidates having a length of  $\mathcal{N}_{cand}$  with the aid of the  $K$ -best SD, where we have  $2^{M \cdot BPS} \geq \mathcal{N}_{cand} \geq 1$ , since the corresponding complexity can be expressed as:

$$C_{MAP} = M \cdot BPS \cdot \mathcal{N}_{cand} \quad (13)$$

Consequently, the complexity has become linearly proportional to the length of the list  $\mathcal{L}$ . We will demonstrate with the aid of our forthcoming simulation results that the value of  $N_{cand}$  can be set to a small fraction of  $2^{M \cdot BPS}$  with the aid of the proposed center-shifting scheme, especially when a high-throughput modulation scheme, such as 64-QAM and/or a large number of transmit antennas are employed by the system.

## V. SIMULATION RESULTS AND DISCUSSIONS

System	SDM/OFDM
No. of Sub-Carriers	1024
Modulation	4-QAM
No. of Transmit Antenna	8
No. of Receive Antenna	4
Block Length	20480
CIR Model	$P(\tau_k) = [0.5 \ 0.3 \ 0.2]$ , ( $k = 0, 1, 2$ )
CIR Tap Fading	OFDM symbol invariant
Channel Estimation	Perfect
Detector/MAP	$K$ -Best List-SD
List Length $N_{cand}$	$=K$
Channel Encoder	RSC(2,1,3)
	Generator Polynomials (6/13) Code Termination (Off)
Iteration Mode	Once no more iterative gain can be achieved by the conventional iterative receiver, the center-shifting function is switched on.

TABLE II  
SUMMARY OF SYSTEM PARAMETERS FOR THE  $K$ -BEST SD AIDED CODED SDM/SDMA OFDM SYSTEM

For the sake of investigating the performance of the center-shifting scheme in a worst-case scenario, a  $(8 \times 4)$ -element rank-deficient SDM/OFDM system is considered. The simulation parameters are provided in Table II. The EXtrinsic Information Transfer (EXIT) charts [7] of Figure 5 are used to analyze the iterative DHDC center-shifting assisted system. We evaluate the *extrinsic* mutual information (MI),  $I_E(MUD)$ , at the output of the SD, which is quantified on the vertical axis of Figure 5(a), after providing the SD with the two inputs required, as observed in Figures 3 and 4, which correspond to the *extrinsic* LLRs and the *a posteriori* LLRs gleaned from the channel decoder, respectively. The MI terms associated with above-mentioned two inputs denoted by  $I_E(CC)$  and  $I_D(CC)$ , are quantified on the two abscissa axes, namely on the x-axis and y-axis, respectively. Consequently, a 3D EXIT chart has to be employed to analyse the convergence characteristics of the iterative center-shifting assisted iterative receiver, where the SD is a double-input-single-output functional block.

In order to maintain an affordable  $K$ -best SD complexity,  $K$  and  $N_{cand}$  are set to a relatively small value of 128 despite considering a heavily rank-deficient system. Therefore, in the absence of our proposed center-shifting scheme the inner decoder's EXIT curve decayed upon increasing the *a priori* information owing to the flawed information exchange between the inner and outer decoders, as evidenced by Figure 5(a). This was caused by the employment of an insufficiently large candidate list size  $N_{cand}$ , which misinformed the channel decoder and the FEC decoder in turn 'deceived' the SD. For the sake of maintaining a low complexity, the center-shifting scheme is only activated, when the maximum iterative gain of the scheme using no center-shifting is achieved. This point is reached when the resultant detection trajectory reaches the intersection of the EXIT curves of the inner and the outer decoder in Figure 5(a). Hence, the stair-case-shaped decoding trajectory of Figure 5(a) follows exactly the same path as with the DHDC scheme disabled, until it reaches the point of intersection. Then, with the aid of the increasingly accurate search center provided by the DHDC center-shifting scheme, the decoding trajectory continues to evolve through the open EXIT tunnel of Figure 5(a) between the 3D-EXIT surface of the inner decoder and the EXIT curve of the outer decoder. In order to avoid the cumbersome 3D representation, let us now project the 3D EXIT

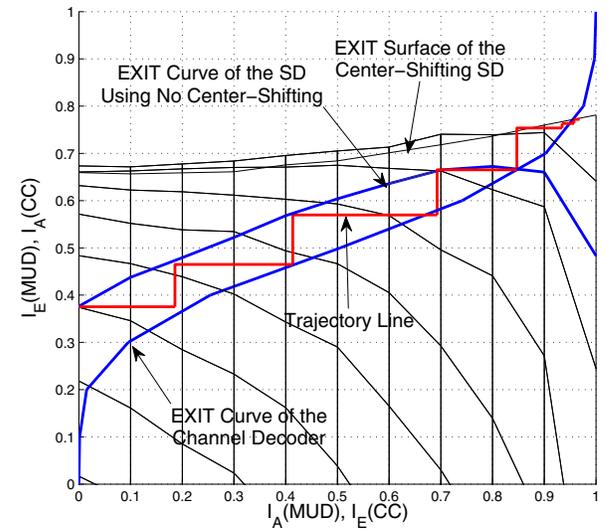
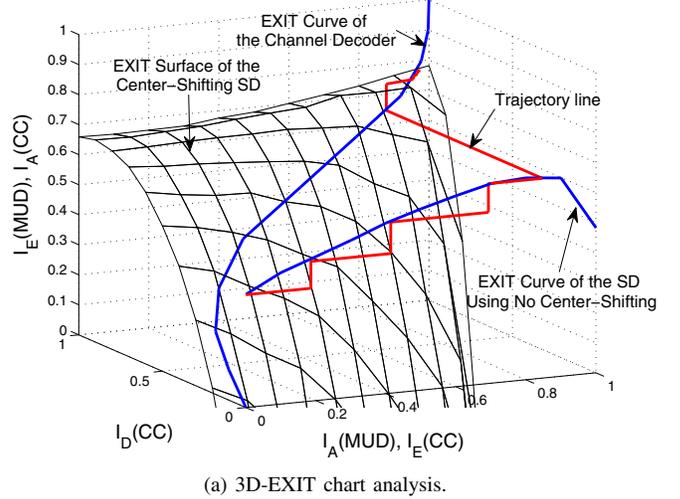


Fig. 5. EXIT chart analysis of the direct-hard-decision-center-shifting  $K$ -best SD aided iterative receiver in the scenario of  $(8 \times 4)$  rank-deficient 4-QAM SDM/OFDM system. (SNR=8 dB,  $K = N_{cand} = 128$ ). All other system parameters are listed in Table II.

curves of Figure 5(a) onto the 2D plane at  $I_D(CC) = 0$ , which is plotted in Figure 5(b). Hence, we can infer from Figure 5(b) that the DHDC-aided receiver is capable of achieving a near-MAP performance, despite using small values of  $K$  and  $N_{cand}$ .

Figure 6 depicts the BER curves of both the proposed center-shifting SD assisted receivers in comparison to those of the benchmark receiver dispensing with the center-shifting scheme. The system parameters used and the iteration mode control regime are summarized in Table II. As seen in Figure 6, a better BER performance can be achieved in both scenarios, where  $K = N_{cand} = 32$  and  $K = N_{cand} = 64$  were employed by the DSDC-aided iterative receiver than that by the DHDC-aided one. These further improvements attained by the DSDC scheme are indeed expected, because the action of subjecting the LLRs to hard decisions discards the valuable soft information, which indicates how accurate our estimate of the most recently obtained search center is. At a fixed list size of  $N_{cand} = 64$ , the DSDC center-shifting scheme produces a performance gain of about 2.5 dB over that of the conventional SD aided receiver. Remarkably, when having a list size of  $N_{cand} = K = 64$ , the DSDC-aided receiver slightly outperforms the conventional

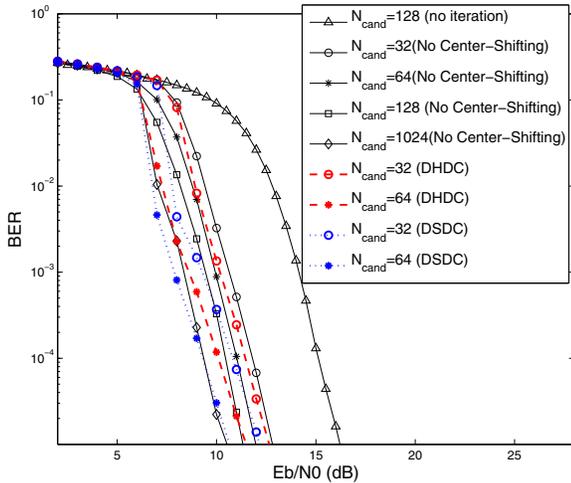


Fig. 6. BER performance improvements provided by the DSDC scheme in the context of an  $(8 \times 4)$ -element rank-deficient SDM/OFDM system. Compared to the DHDC-aided receiver, the DSDC-aided K-Best SD iterative receiver is capable of achieving a better BER performance at a slightly higher computational complexity imposed by the search center calculation process. All other system parameters are listed in Table II.

iterative receiver having a high complexity associated with a list size of  $K = N_{cand} = 1024$  but using no center-shifting. As a result, the DSDC scheme reduces the candidate-list-generation-related and *extrinsic*-LLR-calculation related complexity by a factor of 3.5 and 16, respectively. As a further benefit, the associated memory requirements were also reduced by a factor of 16. This remarkable complexity reduction is achieved, while simultaneously approaching the performance of the exact MAP detector, which may be implementationally infeasible, especially in such a high-throughput heavily rank-deficient system.

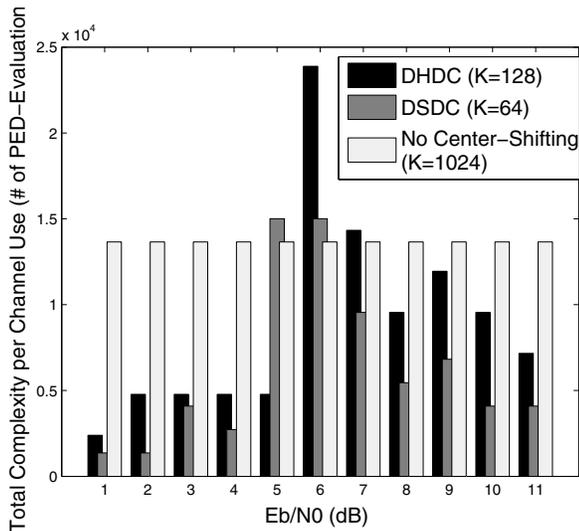


Fig. 7. Computational complexity comparison of the DHDC, the DSDC aided iterative center-shifting  $K$ -best SD ( $K = 32$ ) receivers as well as the receiver dispensing with center-shifting in the scenario of an  $(8 \times 4)$ -element rank-deficient 4-QAM SDM/OFDM system. All other system parameters are listed in Table II.

Furthermore, with reference to Figure 7, the candidate list generation complexity of both the DHDC and DSDC center-shifting-aided receivers is well below that of the receiver using no center-shifting for the SNR range spanning from 2 dB to 12 dB, except for SNRs in the vicinity of 6 dB, provided that our aim is to achieve the near-MAP BER performance quantified in Figure 6. By comparison, the near-MAP BER performance can only be attained

by having  $K = N_{cand} = 1024$  for the system operating without the center-shifting scheme or by setting  $K = N_{cand} = 64$  and 128 in the presence of the DSDC and DHDC schemes, respectively. Recall from Figure 6 that it is sufficient to generate the candidate list only once, namely when the SD is first invoked and then store it in memory for future *extrinsic* LLR calculations during the iterative detection process. Then, the total number of PED-evaluations per channel use carried out in the candidate-list-generation phase by the system dispensing with the center-shifting scheme remains as high as 13,652, regardless of the SNR and the number of iterations. On the other hand, in the presence of the center-shifting scheme, the candidate list has to be regenerated at each iteration when the search center is updated. Nonetheless, the total complexity imposed is substantially reduced as a benefit of the substantially reduced candidate list size  $N_{cand}$ . In the light of the simulation results of Figure 6, where the corresponding BER curves start to sharply decay around 6 dB, the EXIT curves of the inner and outer decoders seen in Figure 5(b) exhibit a narrow but still open tunnel. Therefore, in order to achieve the maximum iterative gain a high number of iterations have to be conducted, which explains the reason why a relatively high complexity is imposed by the center-shifting SD assisted receivers around 6 dB.

## VI. CONCLUSION

A novel center-shifting aided SD was proposed, which may be readily combined with any well-established linear or non-linear detector. Our proposed DHDC and DSDC aided solution may enable the iterative center-shifting SD to achieve a near-MAP performance, despite imposing a factor  $\frac{1}{3.5}$  reduced candidate-list-generation-related complexity and a factor  $\frac{1}{16}$  reduced complexity associated with the *extrinsic*-LLR-calculation in comparison to the receiver dispensing with the center-shifting scheme. As a further benefit, the memory requirements were also reduced by a factor of 16.

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