Authors' response to the referees' comments

Manuscript title: Least squares contour alignment
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We thank the referees and the associate editor for their relevant and useful comments. In this document, we quote in **bold face** statements from the reports. Our replies follow in ordinary print.

Answer to Reviewer 1

1. Please try not to "abuse" the notation (first page, beginning).

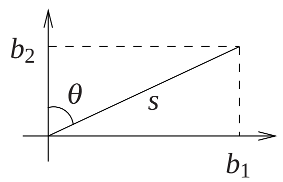
Corrected: The abuse of notation mentioned on page 1 refers to $\mathscr{C}_1$ and $\mathscr{C}_2$ denoting both sets of points (representing the contours) and matrices formed from these points (which matrices of course also represent the contours). It is natural to think of a contour as a set but algorithmically and numerically we work with the corresponding matrix. In our opinion, using the same letter for the set and the matrix is a minor abuse of notation (compared for example with the common expression "the function $f(x)$"). In order to be impeccable, however, in the revised version of the paper we reserve the notation $\mathscr{C}_1, \mathscr{C}_2$ for the contours (sets) and introduced the notation C_1, C_2 for the corresponding matrices. This resolves the problem with the clash of notation.

2. Is it necessary to introduce the Frobenius norm, when actually the Euclidean norm is used ?

Please note that the Frobenius norm is a *matrix* norm and the Euclidean norm, also called 2-norm, is a *vector* norm. Although the two notions are closely related, they are certainly not equivalent or interchangeable. As explained above, from a computational point of view, a contour is represented by a matrix (a single object) rather than the individual points (N objects). We can, in principle, work with the set of vectors and replace everywhere Frobenius norm by a sum of vector norms, i.e., write $\sqrt{\sum_{i=1}^N \|p\|_2^2}$ instead of $\|C_1\|_\mathrm{F}$, however, this makes all formulas look more complex than they actually are. Please note that (as the name suggests) the Frobenius norm is an old and widely used notion in the linear algebra.

3. A Figure depicting the vector $[b_1, b_2]$ with the θ and s as angle and magnitude would make the proof easier to follow.

Perhaps the figure that the reviewer has in mind is:



Since this figure is implicitly indicated by the first equation in (3) and due to the 4 page limit, we have not included the figure into the paper.

4. On page 4 you write: "In terms of the parameters a1, a2, b1, b2, (1) is reduced to a linear least squares problem", but this statement is not obvious and cannot be straightforwardly deduced from the previous part of the proof.

The statement follows from the definition of the Frobenius norm, the substitution of $\mathscr{A}_{a,\theta,s}(q^{(i)})$ with $\begin{bmatrix} q^{(i)} & q_{\mathrm{r}}^{(i)} \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ in the cost function of (1), and standard properties of the 2-norm. Here is the missing derivation:

$$\|\mathscr{C}_1 - \mathscr{A}_{a,\theta,s}(\mathscr{C}_2)\|_{\mathrm{F}} = \sqrt{\sum_{i=1}^{N} \left\| p^{(i)} - \begin{bmatrix} I_2 & p^{(i)} & p_{\mathrm{r}}^{(i)} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_1 \end{bmatrix} \right\|_2^2} \qquad \text{(by the definition of } \|\cdot\|_{\mathrm{F}} \text{ and by substitution)}$$

$$= \left\| \begin{bmatrix} p_1^{(1)} \\ p_2^{(1)} \\ \vdots \\ p_1^{(N)} \\ p_2^{(N)} \end{bmatrix} - \begin{bmatrix} 1 & 0 & q_1^{(1)} & q_{\mathrm{r}}^{(1)} \\ 0 & 1 & q_2^{(1)} & q_{\mathrm{r}}^{(1)} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & q_1^{(N)} & q_{\mathrm{r}}^{(N)} \\ 0 & 1 & q_2^{(N)} & q_{\mathrm{r}}^{(N)} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix} \right\|_2 \qquad \text{(by the property of } \|\cdot\|_2 \text{ that } \sum_{i=1}^{N} \|e_i\|_2^2 = \left\| \begin{bmatrix} e_1 \\ \vdots \\ e_N \end{bmatrix} \right\|_2^2 \text{).}$$

Since the above derivation is trivial and because of the 4 page limit we have not included it into the paper.

5. Please show 2, 3 more examples of contour alignments.

Three examples are shown at the end of this document. The file that has generated these examples is available in the software accompanying the paper

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http://users.ecs.soton.ac.uk/im/dist.tgz
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However, due to the restriction in the number of pages, we are unable to add any other examples in the paper. It is noted that the paper comes with a ready to use software implementation of the method, so that active readers can reproduce the additional examples or create their own examples.

6. The contour of a binary image can be found easily using many basic methods. Canny edge detector is useful for gray-scale images. Using it for binary images is weird.

Corrected as follows:

> First, the edges of the binary images are detected using an edge detection algorithm (e.g., the Prewitt algorithm) in order to obtain a binary edge map. Second, ...

7. Reference [5] should be removed, 2–3 more references should be added.

Reference [5] was removed and the following references [MA97, Mar98, PRR03, MSC$^+$00] were added.

Answer to Reviewer 2

Reject (Paper Is Not Of Sufficient Quality Or Novelty To Be Published In This Transactions)

We are sorry that the reviewer did not explain his/her decision. If it were the case that our results were not of sufficient quality or novelty, we could have benefited from the reviewer by knowing why this is the case. Since justification to the statement is not given, we insist that the reported results should be considered of sufficient quality and novelty.

How would you rate the technical novelty of the paper?: Not Novel

Can the reviewer please justify this statement by supplying references that contain the results in our paper? Unless this is done, we insist that the results should be considered original.

There are only 5 references (including a citation for the Canny edge detector). The most recent citation is from 2006. This is not an adequate set of references.

We would appreciate to know about additional or more recent references that are relevant for our results.

References

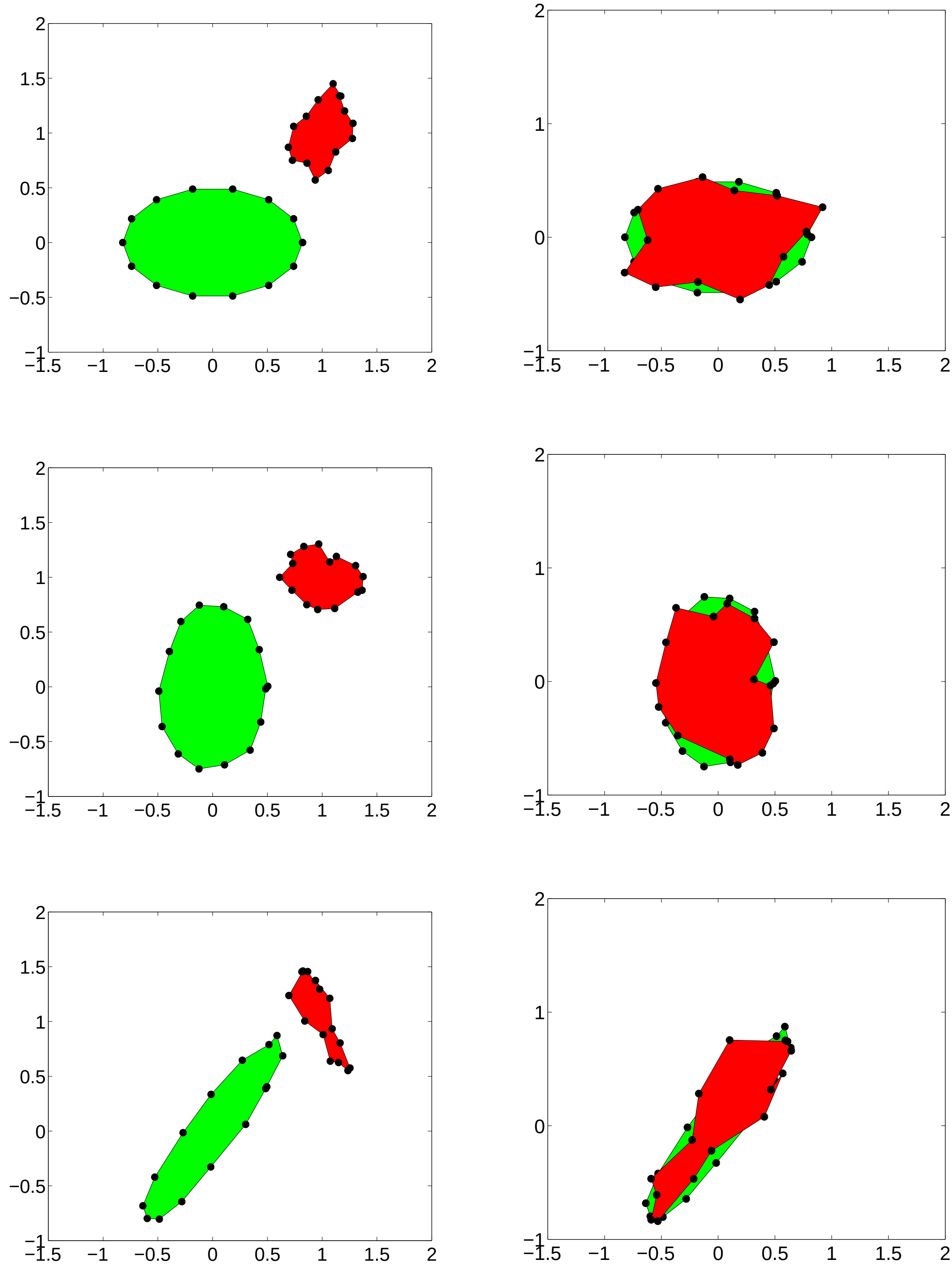
[MA97] J. Marques and A. Abrantes. Shape alignment—optimal initial point and pose estimation. *Pattern Recognition Letters*, 18:49–53, 1997.

[Mar98] J. Marques. A fuzzy algorithm for curve and surface alignment. *Pattern Recognition Letters*, 19:797–803, 1998.

[MSC+00] S. Mahmoodi, B. Sharif, E. Chester, J. Owen, and R. Lee. Skeletal growth estimation using radiographic image processing and analysis. *IEEE Trans. Information Technology in Biomedicine*, 4:292–297, 2000.

[PRR03] N. Paragios, M. Rousson, and V. Ramesh. Non-rigid registration using distance functions. *Computer Vision and Image Understanding*, 89:142–165, 2003.

Additional examples (by request of reviewer 1)



Left: original contours $\mathscr{C}_1$ (green) and $\mathscr{C}_2$ (red)
Right: least squares aligned contours $\mathscr{C}_1$ (green) and $\mathscr{A}_{a,\theta,s}(\mathscr{C}_2)$ (red)