Author's response to the reviewer's comments

Manuscript number TCON–2010–0088
Title: Closed-loop data-driven simulation
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May 27, 2010

We thank the reviewer for the relevant and useful comments. In this document, we quote in **bold face** statements from the reports. Our replies follow in ordinary print.

The major concern is whether vectors are column or row vectors?

Quoting from Section 1, "Notation":

> $w \in (\mathbb{R}^{\mathtt{w}})^T$ is the finite sequence
> $$w = \big(w(1), w(2), \dots, w(t), \dots, w(T)\big), \qquad \text{where} \quad w(t) \in \mathbb{R}^{\mathtt{w}},$$
> however, with some abuse of notation, we will view $w \in (\mathbb{R}^{\mathtt{w}})^T$ also as a $\mathtt{w}T$-dimensional vector.

We agree with the reviewer that "$\mathtt{w}T$-dimensional vector" does not specify whether w is a column vector or a row vector. The intended meaning is a *column* vector. We have corrected this omission in the revised version of the paper.

1. Notation: … the notation for w **reads** $w = \begin{bmatrix} u(1) & u(2) & \cdots & u(t) & \cdots \\ y(1) & y(2) & \cdots & y(t) & \cdots \end{bmatrix}$

As stated in Section 1, "Notation", w is a $\mathtt{w}T$-dimensional (column) vector, *i.e.*,

$$w = \begin{bmatrix} w(1) \\ w(2) \\ \vdots \\ w(t) \\ \vdots \end{bmatrix} \qquad \text{or in terms of } u \text{ and } y \qquad w = \begin{bmatrix} u(1) \\ y(1) \\ u(2) \\ y(2) \\ \vdots \\ u(t) \\ y(t) \\ \vdots \end{bmatrix}.$$

… the result of Theorem 1 [WRMM05] is denoted as: $\operatorname{row\,span}(\mathscr{H}_r(w)) = \mathscr{B}|_{[1,T]}$

This is a wrong interpretation of Theorem 1 in [WRMM05]. The result in [WRMM05] is that (under suitable conditions) $\operatorname{left\,ker}(\mathscr{H}_r(w)) = \mathscr{B}|_{[1,T]}$. Note, the left kernel and *not* the row space of $\mathscr{H}_r(w)$ gives the system's behavior.

With this notation (5) should be written as, $w_r = g\mathscr{H}_r(w_\mathrm{d})$**.**

As explained above the reviewer's interpretation of Theorem 1 in [WRMM05] is wrong. Moreover, in the suggested equation

$$\underbrace{\begin{bmatrix} u(1) & u(2) & \cdots & u(T-r+1) \\ y(1) & y(2) & \cdots & y(T-r+1) \end{bmatrix}}_{\mathtt{w}\times(T-r+1)} = \underbrace{\begin{bmatrix} g_1 & \cdots & g_r \end{bmatrix}}_{1\times\mathtt{w}r} \underbrace{\mathscr{H}_r(w_\mathrm{d})}_{\mathtt{w}r\times(T-r+1)}.$$

the left-hand-side is a $\mathtt{w} \times (T-r+1)$ matrix and the right-hand-side is a $1 \times (T-r+1)$ matrix. Since $\mathtt{w} \geq 2$, the dimensions do not match!

The correct expansion of the system of equations (5) is

$$\begin{bmatrix} w(1) \\ w(2) \\ \vdots \\ w(r) \end{bmatrix} = \begin{bmatrix} w(1) & w(2) & \cdots & w(T-r+1) \\ w(2) & w(3) & \cdots & w(T-r+2) \\ \vdots & \vdots & & \vdots \\ w(r) & w(r+1) & \cdots & w(T) \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{T-r+1} \end{bmatrix}$$

or in terms of u and y

$$\begin{bmatrix} u(1) \\ y(1) \\ u(2) \\ y(2) \\ \vdots \\ u(r) \\ y(r) \end{bmatrix} = \begin{bmatrix} u(1) & u(2) & \cdots & u(T-r+1) \\ y(1) & y(2) & \cdots & y(T-r+1) \\ u(2) & u(3) & \cdots & u(T-r+2) \\ y(2) & y(3) & \cdots & y(T-r+2) \\ \vdots & \vdots & & \vdots \\ w(r) & w(r+1) & \cdots & w(T) \\ y(r) & y(r+1) & \cdots & y(T) \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_{T-r+1} \end{bmatrix},$$

which is equation (5) (as typed in the paper), with the definition of w given in Section 1, "Notation".

The switching of notation is very confusing!

We hope the reviewer now agrees that there is *no* switch of notation. The confusion is perhaps caused by our unfortunate omission of stating in Section 1, "Notation" that w is a *column* vector.

2. It is indicated that (5) only depends on w_d, this is not true. It expresses w using the vector g. This vector g can be considered as some kind of (impulse) response model representation of the system. In that sense, the notion of data driven needs to be more precisely defined. It does not mean model free (or without identifying a representation of the system)!

Actually g parameterizes the system trajectories and is a *free* variable (any g is admissible). Therefore, g can not be a model representation of the system (which depends on the given system and is therefore constraint, if not unique). E.g., the impulse response of a discrete-time system is a *fixed* sequence, so it is certainly not a free variable.

An interpretation of g is as "input and initial conditions" of the system because g serves to uniquely specify the output of the system as do the input and initial conditions. However, the mapping from g to the system's input and initial conditions is not injective.

The statement that (5) only depends on w_d has been deleted and the following explanation was added:

> The vector g is related to the input and initial conditions of the system that generate the trajectory w_r. However, the mapping from g to the system's input and initial conditions is not injective. (Note that a solution g of (6) need not be unique.)

The (dis-)advantage of the solution set of (6) by G needs to be discussed. How does this compare to the case that a model of the plant is given?

The advantage of (6) is that it does not involve a model of the plant and this allow us to solve data-driven simulation and control problems. In a model based approach, obviously one needs a model of the plant.

The uniqueness is lost in the current setting and the reasons for that need to be discussed.

We are not sure to what "uniqueness" the reviewer refers. The solution G of (6) need not be unique but this does not matter as long as a solution of the simulation problem is concerned. As proven in the paper, the solution of the simulation problem has a unique solution under our assumptions. It is implicit from Theorem 1 that the solution of the data-driven simulation problem (computed by Algorithm 1) does not depend of the chosen particular solution G of (6).

Contribution of the paper. . . . The definitions are not precise . . .

The detailed comments of the reviewer, listed under "1. Notation" in the report show that the confusion comes from the missing word "column" in section 1 "Notation" of the paper. This has been corrected.

. . . the paper provides a minor extension to existing work.

We disagree. In our opinion, the extension of the open-loop simulation algorithm to the closed-loop setting is not trivial. Indeed, the solutions of the open-loop simulation problem (published before) and the solution of the closed-loop data-driven simulation problem (proposed in this paper) are rather different. In particular, the result of the open-loop simulation problem is a *single* trajectory, while the result of the cosed-loop simulation problem is a *set of trajectories* — closed loop system's behavior.

There is a major problem with the relevance of the work.

On application of the results presented in the paper is mentioned in the introduction:

> Our motivation for studying the closed-loop data-driven simulation problem comes from unfalsified control [ST97]. Unfalsified control is an switching adaptive control method that selects in real-time a controller satisfying the performance specification from a set of candidate controllers. The main step in unfalsified control is testing the performance of a candidate controller without applying it on the plant. The performance of the candidate controller is evaluated *directly* from data collected of the plant (possibly operating in closed-loop with another controller). Data-driven simulation allows us to evaluate the controller performance by computing the closed-loop behavior of the plant with the given controller. The standard performance test in the unfalsified control setting makes no assumptions about the plant (therefore it is applicable for a general nonlinear time-varying system), however, it computes a single trajectory of the closed-loop system, so that the performance test can be conservative. In contrast, closed-loop data-driven control uses an LTI assumption about the plant but computes the full behaviors of the closed-loop system, so that it is non-conservative in the LTI case.

In the revised version of the paper we have added a section (see Section 3 and Appendix A), which further clarifies the importance of our results for data-driven control and in particular the unfalsified control concept.

As such the paper is fundamentally not different from the exposure given in [WRMM05].

As explained above, we disagree with the statement that the presented results are minor extension of previous results derived for open-loop data driven simulation.

The rank condition of the input-output hankel matrix established in Theorem 1 of this reference, is the key for the current paper. As such the current paper is to be considered as a corollary to the former paper [WRMM05].

Theorem 1 of [WRMM05] is indeed a key ingredient in solving the closed-loop data-driven simulation problem. However, as explained above, we disagree that our new results are trivial application of Theorem 1 of [WRMM05].

But what is the relevance of the current contribution?

This question was answered above and the paper is revised accordingly (see the new Section 3 and Appendix A).

For establishing the value of this work in the introduction, the author only refers to his own work.

To the best of our knowledge the introduction gives a complete and honest account of previous work on the considered problem. We will be glad to consider and include any additional references suggested by the reviewer. In the new

Section 3 and Appendix A we have added references from the adaptive control literature but they are related only to one possible application of our results and are thus only indirectly related.

But please relate the contribution to existing main streams of research

As explained in the abstract and the introduction, our results derive and extend recent results on system identification in the behavioural setting [MWRM05a, MWRM05b, MR07, MR08]. Whether this literature represents a main streams of research is difficult for us to judge. As stated above, we will be glad to consider related publications that the reviewer knows of.

or describe and motivate in what sense you are addressing original and innovative research!

We believe that the problem is original. Moreover as discussed in the introduction and in more derails in Section 3, apart from being an interesting scientific problem, it has applications for data-driven control.

References

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