

# Identification of the Dynamics of Human Arms after Stroke<sup>\*</sup>

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**Abstract:** The design of controllers to enable the application of Functional Electrical Stimulation as part of a rehabilitation programme for stroke patients requires an accurate model of electrically stimulated muscle. In this paper, nonlinear dynamics of the electrically stimulated muscle under isometric conditions is investigated, leading to the requirement to identify a Hammerstein model structure. Here we develop a two-stage identification method based on a preliminary construction of the linear part that is used as an initial estimate. Then the two-stage method is applied to identify the nonlinear part and optimize the linear part. The traditional ramp deconvolution method are implemented here and compared with the proposed method using a simulated muscle system that is based on experimental data from stroke patients. The results show that the proposed method outperforms the other one when implemented on the simulated muscle system with different noise levels.

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## 1. INTRODUCTION

Strokes affect between 174 and 216 people per 100,000 of the population in the UK each year (Mant et al., 2004) and one of the most devastating and common consequences of the stroke is loss of the use of the arm and hand (Gowland et al., 1992). Currently there is increasing interest in the application of control schemes as part of a rehabilitation programme for survivors of a stroke who are in a position to take part. Here the starting point is a research programme that has built a robotic workstation for use by stroke patients in order to regain voluntary control of their impaired arms (Freeman et al., 2008). This research is based on the application of Functional Electrical Stimulation (FES) to assist a patient's voluntary attempt to perform repeated tracking tasks using their impaired arms.

During treatment, FES is applied to the patient's triceps and Iterative Learning Control (ILC) is used to update the stimulation level in order to ensure that the assistance provided coincides as much as possible with the patient's voluntary intention. In particular, the patients hand is connected to the robot and they attempt to follow an illuminated track which represents reaching out for an object. Fig. 1 shows a stroke participant using the robotic workstation.

As they complete the task, the error between the required and achieved paths is measured and once they reach the end the robot returns their arm to the starting position, and in this resetting time an ILC algorithm is used to update the stimulation to be applied on the next attempt. This research has already had patient trials that demonstrate the basic feasibility of the approach but also highlighted the need for improved modeling of the



Fig. 1. The Robotic Workstation

patient's arm and hence the motivation for the results given here.

Identification of electrically stimulated muscle is a challenging problem. The task of modeling requires both determination of a particular model structure and estimation of the model parameters that relate the model to the electrically stimulated muscle system. Although the muscular contraction mechanism is generally well known, the model based on the structure and behavior of the underlying muscle physiology (Dorgan and OMalley, 1997) is still too complicated for control applications. In order to achieve the tractability of the muscle response to electrical stimulation, attention should be focused on the accuracy of the model rather than on its transparency. To achieve this, three modeling approaches are applicable where the first is the nonlinear black box modeling approach (Previdi, 2002). A discrete-time NARX model is used but selection of structure, in particular model family (polynomial ex-

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pansion or neural network) and the parameters for each structure need to be estimated. However, depending on the model family used, structure selection could be a difficult task. Therefore, in order to obtain an effective model, a long testing time could be needed which is highly undesirable, especially for stroke patients.

The second method is the light-grey box modeling approach where physical insights are included. For example, in (Shue et al., 1995) and (Chizeck et al., 1999), three factors (activation dynamics, torque-velocity and torque-angle) are incorporated, which can be treated as coupled or not and if coupled the value of the coupled parameters are assumed to be unknown. Actually, these parameters are approximately mutually coupled and hence they are dependent on each other. Consequently identification of these parameters in such a complex model may be practically infeasible in a reasonable experiment time period.

The third approach is a block-oriented modeling technique. One commonly used example here is the Hammerstein-Wiener model that describes nonlinear dynamic systems using one or two static nonlinear blocks in series with a linear block. In (Bernotas et al., 1986), a discrete-time Hammerstein model is used to describe the electrically stimulated muscle under isometric conditions (the muscle is held at a fixed length when stimulated). Later, this work was extended to non-isometric muscle (Durfee and Palmer, 1994), (Schauer et al., 2005) and (Farahat and Herr, 2005), but the key nonlinear muscle activation is still modeled by a Hammerstein structure with one or two more linear blocks in series (Schauer et al., 2005)(Farahat and Herr, 2005), or parallel (Durfee and Palmer, 1994). Using such a model to describe the muscle activation system has the possible advantages of a well defined structure and the possibility of applying a separate identification procedure for each block. Therefore, we use this modeling approach and the model structure introduced by Durfee and Palmer (Durfee and Palmer, 1994) shown in Fig. 2, is adopted.

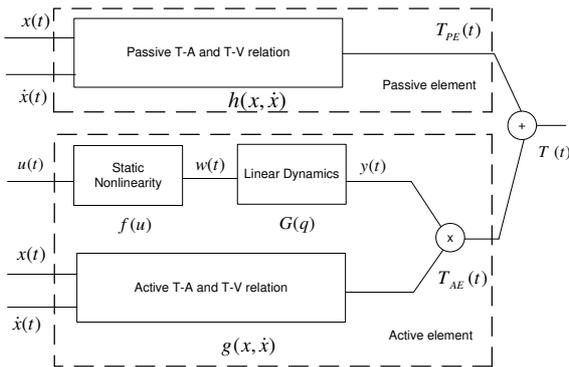


Fig. 2. Muscle Model Structure

The model input here is the stimulation pulse-width  $u(t)$  ( $\mu s$ ), the output  $T(t)$  is the torque generated at the elbow joint,  $x(t)$  is the joint angle and  $\dot{x}(t)$  is the joint angular velocity. The model structure consists of a passive element and an active element where the former is assumed to be time-invariant. The output of the passive element,  $T_{PE}(t)$ , is the torque generated by the elastic and viscous properties of the muscle without electrical stimulation. The active torque  $T_{AE}(t)$  is the torque generated by the electrical

stimulation of the triceps muscle and a multiplicative effect of joint angle and joint angular velocity on the resulting torque. The Passive Torque-Angle and Torque-Velocity relation  $h(x, \dot{x})$  and the Active Torque-Angle and Torque-Velocity relation  $g(x, \dot{x})$  represent the relationship between the joint state and the torque.

A Hammerstein structure is used to model nonlinear muscle activation in the model. It is composed of a static nonlinear block in series with linear dynamics. This is a commonly used structure due to its correspondence with biophysics. The static nonlinearity  $f(u)$  includes the nerve fiber recruitment properties. This is the so called Isometric Recruitment Curve (IRC), which is defined as the static gain relation between stimulus activation level and output torque when the muscle is held at a fixed length (Durfee and MacLean, 1989). The linear dynamic block  $G(q)$  corresponds to the dynamic response of electrically stimulated muscle.

Although models of the response of electrically stimulated muscle to FES have already been identified, see (Durfee and Palmer, 1994), it is necessary to perform a special identification procedure with stroke patients, who exhibit different properties of torque-generation parameters under isometric contraction (Mccrea et al., 2003). The peak torque and time-dependent parameters (time to develop and reduce torque) are impaired after stroke and are independent of each other. Consequently this paper considers nonlinear muscle activation, under isometric contraction for the case of stroke patients and, in particular, the identification and relative performance of the Hammerstein structure.

Next, we develop the two-stage optimized identification procedure based on a preliminary experiment which is the main novel feature of this work. Then the traditional ramp deconvolution method (Durfee and MacLean, 1989) is implemented hand compared with the two-stage method using a simulated muscle system that is based on experimental data from stroke patient.

## 2. THE IDENTIFICATION SCHEME

A nonlinear Hammerstein model in discrete-time is shown in Fig. 3. The stimulation input  $u(\cdot)$  is first scaled by the static nonlinear function  $f(\cdot)$  and then passed to a linear time-invariant system described by a transfer function  $G(q)$ . The system input, output and noise are denoted by  $u(\cdot)$ ,  $y(\cdot)$  and  $v(\cdot)$  respectively. The internal signal  $w(\cdot)$  is not measurable.

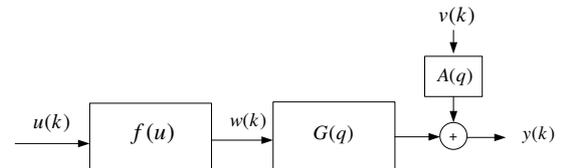


Fig. 3. Hammerstein structure

The linear system is assumed to be stable and represented by

$$G(q) = \frac{B(q)}{A(q)} \quad (1)$$

where

$$B(q) = b_0q^{-d} + b_1q^{-(d+1)} + \dots + b_nq^{-(n+d)} \quad (2)$$

$$A(q) = 1 + a_1q^{-1} + \dots + a_lq^{-l} \quad (3)$$

and  $B(q)$  and  $A(q)$  are assumed to be coprime.  $q^{-1}$  is the delay operator and  $n$ ,  $l$  and  $d$  are the number of zeros, poles and the time delay, respectively. These can be estimated in a number of ways where, for example,  $(l, n, d) = (2, 1, 1)$  has been used in a related problem by (Hunt et al., 1998).

When describing it as a linear difference equation:

$$y(k) + a_1y(k-1) + \dots + a_ly(k-l) = \bar{b}_0u(k-d) + \dots + \bar{b}_nu(k-(n+d)) + v(k) \quad (4)$$

$v(k)$ , the noise term, enters directly into the difference equation which is a quite simple noise model and at the same time is a prime choice in our application.

The nonlinear function is represented by a basis function expansion  $g_p(u)$ ,

$$f(u) = \sum_{p=1}^n \alpha_p g_p(u) \quad (5)$$

formed from scaled and translated versions of one ‘‘mother basis function’’  $\kappa(u)$ , as

$$g_p(u) = \kappa(\beta_p(u - \gamma_p)) \quad (6)$$

where  $\beta_p$  denotes the dilation parameters and  $\gamma_p$  denotes the translation parameters (Ljung, 1999, Pages 148-154). A common choice is the power series

$$g_p(u) = u^p \quad (7)$$

but if, for example, deadband and saturation are present, a high order polynomial is usually required and hence the possibility of oscillatory behavior. Another method is the spline function, a piecewise polynomial function which can overcome this problem. A straightforward choice here would be a piecewise linear function, but these type of functions are not smooth, which means that their derivatives are not continuous at their break points or knots. In order to avoid these drawbacks, we will use cubic splines to model the nonlinear block. A cubic spline is a piecewise cubic function defined by a series of knot points, where between each pair of knots the spline is defined by a third degree polynomial. The second derivatives of the spline polynomials are set equal to zero at the endpoints of the interval (Boor, 1978, Pages 49-57).

Let a set of knots  $u_1, u_2, \dots, u_m$  be defined which satisfy

$$u_{min} = u_1 < u_2 < u_3 < \dots < u_m = u_{max} \quad (8)$$

Then a cubic spline function is defined as

$$f(u) = \sum_{i=1}^{m-2} \beta_i |u - u_{i+1}|^3 + \beta_{m-1} + \beta_m u + \beta_{m+1} u^2 + \beta_{m+2} u^3 \quad (9)$$

where  $[\beta_1, \beta_2, \dots, \beta_{m+2}]$  are parameters to be estimated for the nonlinear block and the model for the whole process is

$$y(k) = G(q)f(u(k)) + \frac{1}{A(q)}v(k) \quad (10)$$

$$= \frac{B(q)}{A(q)}f(u(k)) + \frac{1}{A(q)}v(k) \quad (11)$$

Our task now is to estimate the parameter vector

$$\theta = [a_1, \dots, a_l, b_0, \dots, b_n, \beta_1, \beta_2, \dots, \beta_{m+2}]^T \quad (12)$$

that minimizes the cost function

$$V_N(\theta) = \frac{1}{2N} \sum_{k=1}^N (y(k) - \hat{y}(k, \theta))^2 \quad (13)$$

from collected input output data

$$[u(1), y(1), u(2), y(2), \dots, u(N), y(N)] \quad (14)$$

where  $N$  denotes the number of samples.

### 3. IDENTIFICATION PROCEDURE

The aim of the preliminary experiment in the two-stage method proposed here is to identify the linear part as an initial estimate. The reason is that the nonlinear muscle contraction mechanism of a stroke patient is impaired and varies, see (Mccrea et al., 2003), and may also vary between trials. In order to obtain accurate estimates for each patient at each treatment, the identification tests need to be performed before each treatment commences. In order to avoid patient fatigue, the identification test should be as short as possible. Therefore, before each treatment, just one test is applied with the patient and then the two-stage identification procedure is used to estimate the nonlinear part and optimize the linear part.

#### 3.1 Preliminary experiment: PRBS test

The Pseudo-Random Binary Signal (PRBS) changes between two levels and has white-noise-like properties. Although the PRBS signal is inappropriate for nonlinear system identification because it may not sufficiently excite the unknown nonlinearity, it is widely used in linear system identification of electrically stimulated muscle (Bernotas et al., 1986), (Shue et al., 1995), (Hunt et al., 1998) and (Munih et al., 2000). Therefore, the PRBS input is adopted in the preliminary study and Fig. 4 shows a typical muscle response to a PRBS input.

In this application, we assume the two levels are 0 and  $+c$  (where  $c > 0$ ) because the input pulse-width can not be negative. Also we know that  $f(0) = 0$  and hence we can choose  $c$  such that  $f(+c) = r$  (where  $r \neq 0$ ) and also the value of  $r$  is big enough compared to the possible noise level. Hence for all  $k$

$$w(k) = f(u(k)) = \alpha u(k) \quad (15)$$

where  $\alpha = r/c$ , and the output of the Hammerstein structure with PRBS inputs is

$$y(k) = G(q)f(u(k)) + \frac{v(k)}{A(q)} \quad (16)$$

$$= \frac{\alpha b_0 q^{-d} + \dots + \alpha b_n q^{-(n+d)}}{1 + a_1 q^{-1} + \dots + a_l q^{-l}} u(k) + \frac{1}{1 + a_1 q^{-1} + \dots + a_l q^{-l}} v(k) \quad (17)$$

$$= \frac{\bar{b}_0 q^{-d} + \dots + \bar{b}_n q^{-(n+d)}}{1 + a_1 q^{-1} + \dots + a_l q^{-l}} u(k) + \frac{1}{1 + a_1 q^{-1} + \dots + a_l q^{-l}} v(k) \quad (18)$$

or, with  $\bar{b}_j = \alpha b_j$   $j = 0, 1, \dots, n$ ,

$$y(k) = \bar{b}_0 u(k-d) + \dots + \bar{b}_n u(k-(n+d)) - a_1 y(k-1) - \dots - a_l y(k-l) + v(k) \quad (19)$$

Defining the parameter vector of the linear part as

$$\theta_l = [\bar{b}_0, \dots, \bar{b}_n, a_1, \dots, a_l]^T \quad (20)$$

where from (19) we have that  $k$  must be larger than both  $n+d$  and  $l$ . Without loss of generality, we assume  $l \geq n+d$  so that  $k$  starts from  $l+1$ . Also introduce

$$Y = [y(l+1) \ y(l+2) \ \dots \ y(N)]^T \quad (21)$$

$$V = [v(l+1) \ v(l+2) \ \dots \ v(N)]^T \quad (22)$$

$$\Phi = \begin{bmatrix} u(l+1-d) & \dots & u(N-d) \\ \vdots & & \vdots \\ u(l+1-n-d) & \dots & u(N-n-d) \\ -y(l) & \dots & -y(N-1) \\ \vdots & & \vdots \\ -y(1) & \dots & -y(N-l) \end{bmatrix}^T \quad (23)$$

then (19) can be rewritten as

$$Y = \Phi \theta_l + V \quad (24)$$

Hence the initial estimate,  $\hat{\theta}_l^0$ , can be obtained using the least squares method

$$\hat{\theta}_l^0 = [\hat{b}_0^0, \dots, \hat{b}_n^0, \hat{a}_1^0, \dots, \hat{a}_l^0]^T = (\Phi^T \Phi)^{-1} \Phi^T Y \quad (25)$$

Using (10), the gains of  $f(u)$  and  $G(q)$  are not unique. In order for these to be uniquely identifiable, we normalize the gains, e.g., set  $\hat{b}_0^0 = 1$ . If  $\hat{b}_0^0 = 0$ , set the first non-zero  $\hat{b}_j^0$  to be equal to one.

### 3.2 Two-stage Identification method: Triangular ramp test

In order to identify the nonlinear part, a triangular ramp test is applied to muscle, which is rich enough to excite the whole nonlinearity. Fig. 4 shows a typical muscle response to a triangular ramp input.

*Stage One* Identify the nonlinear part using the linear parameters  $\hat{\theta}_l^0$  estimated in the preliminary study. Without loss of generality, we assume  $\hat{b}_0^0 \neq 0$  and hence the intermediate signal  $w(k)$  can be estimated using

$$\hat{w}(k) = \hat{G}(q, \hat{\theta}_l^0)^{-1} y(k) \quad (26)$$

$$= \frac{1 + \hat{a}_1^0 q^{-1} + \dots + \hat{a}_l^0 q^{-l}}{q^{-d} + \frac{\hat{b}_1^0}{\hat{b}_0^0} q^{-(d+1)} + \dots + \frac{\hat{b}_n^0}{\hat{b}_0^0} q^{-(n+d)}} y(k) \quad (27)$$

The parameter vector of the nonlinear part

$$\theta_n = [\beta_1, \beta_2, \dots, \beta_{m+2}]^T$$

can be estimated by fitting a cubic spline function (9) in a least square sense

$$\hat{\theta}_n = \arg \min_{\hat{\theta}_n} \sum_{k=l+1}^N (w(k) - f(u(k), \hat{\theta}_n))^2 \quad (28)$$

*Stage Two* Optimize the linear parameters using the nonlinear parameters  $\hat{\theta}_n$  estimated in Stage One.

Once the nonlinear parameters are available, the intermediate signal  $w(k)$  can be easily recovered using

$$\hat{w}(k) = f(u(k), \hat{\theta}_n) \quad (29)$$

This is now a general linear system identification problem and hence we can optimize the linear parameters  $\hat{\theta}_l$  by

$$\hat{\theta}_l = \arg \min_{\hat{\theta}_l} \sum_{k=l+1}^N (y(k) - G(q, \hat{\theta}_l) w(k))^2 \quad (30)$$

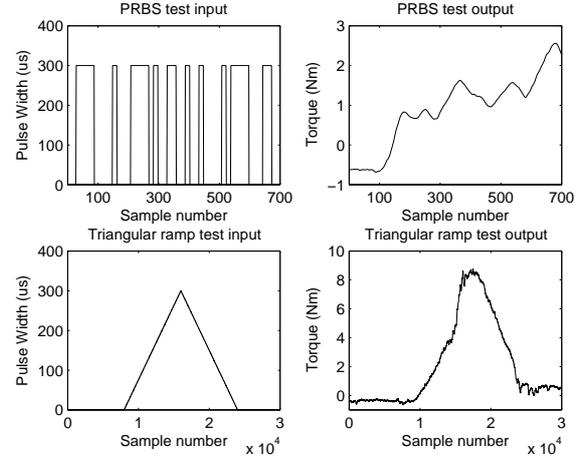


Fig. 4. Example of input and output in PRBS test and Triangular ramp test. The stimulus frequency is  $40Hz$ . The sample frequency is  $1600Hz$ . In the PRBS test, the input pulse width switches between 0 and  $300\mu s$ . In the Triangular ramp test, the pulse width rising from 0 to  $300\mu s$  and then back to 0.

This two-stage method detailed above can be repeatedly executed as an iterative algorithm:

- (1) Initial estimates of linear part,  $\hat{\theta}_l^0$ , from preliminary study: PRBS signal test.
- (2) update  $\hat{\theta}_n^j$  by solving (28).
- (3) update  $\hat{\theta}_l^j$  by solving (30).

Go back to 2). Stop when convergence occur.

Since the initial value of linear parameters from preliminary study is already a good estimate for the true system, we have reason to believe that just through one iteration, the optimized model can achieve acceptable accuracy with, crucially for this application area, short identification time. In the other hand, if the initial estimate is not a suitable one (due to many reasons: changes of patients' conditions, experimental setting and other environmental factors), we always can perform the identification stages iteratively to achieve convergence. Next we compare the performance against the other method that have been applied in the same general area.

## 4. RAMP DECONVOLUTION METHOD

In this method, the Isometric Recruitment Curve (IRC) is estimated by deconvolving the response of a muscle to a ramp input (Durfee and MacLean, 1989) and the Linear Activation Dynamics (LAD) is represented by a critically damped second-order system (Baratta and Solomonow, 1990). This method has already been used to identify models of the arm dynamics of stroke patients in the research programme on which this paper is based and it is implemented as follows. First, a triangular input is applied with 5 seconds up and 5 seconds down and the elbow torque is recorded. Second, the elbow torque is deconvolved using the Linear Activation Dynamics (LAD).

When plotted against the applied pulsewidth, this provides two isometric recruitment curves, corresponding to the increasing and decreasing ramps respectively. Then the expression

$$f(u) = a_1 \cdot \frac{e^{a_2 u} - 1}{e^{a_2 u} + a_3} \quad (31)$$

is selected to fit data in a least square sense. Third,  $h(\tau)$  is convolved with  $f(u)$  to produce the overall nonlinear dynamical model. The transfer function of Linear Activation Dynamics (LAD) is

$$h(s) = \frac{1}{T_\omega^2 s^2 + 2T_\omega s + 1} \quad (32)$$

and the natural frequency  $f_\omega = \frac{1}{T_\omega}$  of the system is chosen, based on evidence in the literature cited above, as  $0.85\pi$ .

## 5. SIMULATION RESULTS

In order to investigate the relative performance of the two identification methods, first we apply them on the data from a simulated muscle system and choose different noise levels. The simulated muscle system is defined as follows:

- (1) The nonlinearity  $f(\cdot)$ , shown in Fig. 5, is a sigmoid function:

$$f(u) = a_1 \cdot \frac{e^{a_2 u} - 1}{e^{a_2 u} + a_3} \quad (33)$$

where  $a_1 = 6.8994$ ,  $a_2 = 0.0410$  and  $a_3 = 2.3897 \times 10^3$

- (2) The linear dynamics  $G(q)$  is an underdamped second order system with  $T_\omega = 0.5284$ ,  $\zeta = 0.6369$  and unit gain.
- (3) The noise levels considered are:
  - $e(t) = 0$  — no noise.
  - $e(t)$  — normally distributed random process with zero mean and standard deviation  $\lambda$  ranging over the values 0.02, 0.04, 0.06, 0.08, 0.10.

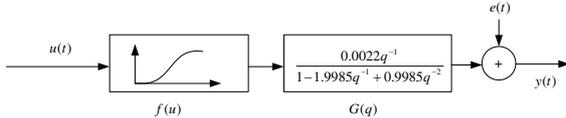


Fig. 5. Simulated Muscle System

This is the model corresponding to one of the stroke participants taking part in clinical trials, and has been used in the stimulation controller applied during their treatment. Analysis of the extensive experimental data collected throughout the intervention (consisting of 25 sessions of 1 hour duration) reveals it to be accurate. During tests the application of FES using this model in conjunction with a simple structure ILC algorithm (Freeman et al., 2008) generally led to a mean tracking error of less than 15mm, and in some cases less than 5mm. Comparison with the models corresponding to the remaining stroke participants taking part in the clinical trial also reveals that it is also representative of all the participants tested. The accuracy of the model is illustrated in Fig. 6 which shows the response of the simulated muscle system in comparison with experimental data from a stroke patient's muscle. The responses can be seen to be in close agreement.

The model will therefore be adopted as representative of the muscle of stroke patients, and will be used to compare the efficacy of the identification methods that have been

described. For each noise level, 100 independent trials have been performed and the mean and standard deviations of the best fit rates have been calculated, see Table 1. In the noise-free case, the two-stage method can be observed to almost reconstruct the simulated system. Although the variance is larger than Ramp Deconv, it is still comparably small enough not to significantly influence performance. It can be seen from Fig. 7 that under different noise levels, the two methods degrade but the method developed here is superior to the other one.

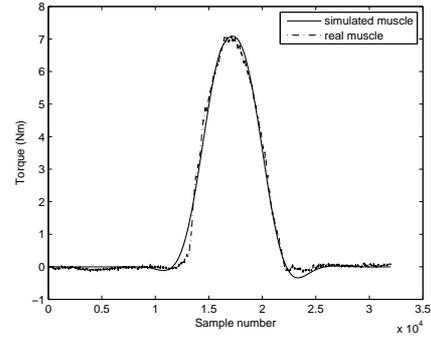


Fig. 6. Comparison of the responses of the simulated muscle system and the real muscle of a stroke patient to a triangular ramp test

Table 1. Best fit (%) for different noise levels

	Two-Stage	Ramp Deconv
noise-free	99.98	91.24
$\lambda = 0.02$	$99.07 \pm 0.07$	$91.19 \pm 0.00$
$\lambda = 0.04$	$98.14 \pm 0.02$	$91.07 \pm 0.00$
$\lambda = 0.06$	$97.27 \pm 0.03$	$90.86 \pm 0.00$
$\lambda = 0.08$	$96.31 \pm 0.15$	$90.58 \pm 0.00$
$\lambda = 0.10$	$95.33 \pm 0.22$	$90.24 \pm 0.00$

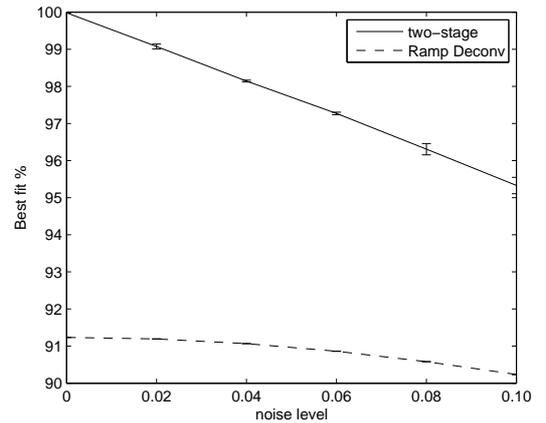


Fig. 7. Simulation results for different noise level

## 6. CONCLUSIONS AND FUTURE WORKS

In this paper a two-stage identification method was developed to identify the nonlinear dynamics of electrically stimulated muscle under isometric conditions. This method needs a preliminary experiment comprising a Pseudo-Random Binary Signal test and then a triangular ramp test before treatment. The proposed method then is

shown to outperform the Ramp Deconvolution method on a simulated muscle system with different noise levels.

Future work includes experiments on the real stroke patients. The workstation and its peripheral systems for the experiments are given in (Freeman et al., 2007). Experiments will enroll more stroke patients and tests and trials will be carried out on them. All these methods will be applied on the experiment data to determine the most promising method, balancing between efficacy and accuracy.

Future work also includes extension to non-isometric condition where Passive Torque-Angle and Torque-Velocity relation  $h(x, \dot{x})$  and Active Torque-Angle and Torque-Velocity relation  $g(x, \dot{x})$ , in Fig. 2, will be introduced. Separate experiments will be designed and an identification procedure will be implemented according to (Durfee and Palmer, 1994).

## REFERENCES

- Baratta, R. and Solomonow, M. (1990). The dynamic response model of nine different skeletal muscles. *IEEE Transactions on Biomedical Engineering*, 37(3), 243–251.
- Bernotas, L., Crago, P.E., and Chizeck, H.J. (1986). A discrete-time model of electrically stimulated muscle. *IEEE Transactions on Biomedical Engineering*, 33(9), 829–838.
- Boor, C.D. (1978). *A Practical Guide to Splines*, volume 27 of *Applied Mathematical Sciences*. New York: Springer-Verlag.
- Chizeck, H.J., Chang, S., and Stein, R.B. (1999). Identification of electrically stimulated quadriceps muscles in paraplegic subjects. *IEEE Transactions on Biomedical Engineering*, 46(1), 51–61.
- Dorgan, S.J. and OMalley, M.J. (1997). A nonlinear mathematical model of electrically stimulated skeletal muscle. *IEEE Transactions on Rehabilitation Engineering*, 5(2), 179–194.
- Durfee, W.K. and MacLean, K.E. (1989). Methods for estimating isometric recruitment curves of electrically stimulated muscle. *IEEE Transactions on Biomedical Engineering*, 36(7), 654–667.
- Durfee, W.K. and Palmer, K.I. (1994). Estimation of force-activation, force-length, and force-velocity properties in isolated, electrically stimulated muscle. *IEEE Transactions on Biomedical Engineering*, 41(3), 205–216.
- Farahat, W. and Herr, H. (2005). A method for identification of electrically stimulated muscle. In *Proceedings of the 2005 IEEE Engineering in Medicine and Biology 27th Annual Conference*, 1–4. Shanghai, China.
- Freeman, C.T., Hughes, A.M., Burridge, J.H., Chappell, P.H., Lewin, P.L., and Rogers, E. (2007). An experimental facility using functional electrical stimulation for stroke rehabilitation of the upper limb. In *Proceedings of 10th IEEE International Conference on Rehabilitation Robotics*, 393–400. Noordwijk aan Zee, The Netherlands.
- Freeman, C.T., Hughes, A.M., Burridge, J.H., Chappell, P.H., Lewin, P.L., and Rogers, E. (2008). A model of the upper extremity using surface FES for stroke rehabilitation. *ASME Journal of Biomechanical Engineering*, doi:10.1115/1.3005332.
- Gowland, C., deBruin, H., Basmajian, J.V., Plews, N., and Burcea, I. (1992). Agonist and antagonist activity during voluntary upper-limb movement in patients with stroke. *Physical Therapy*, 72(9), 624–633.
- Hunt, K.J., Munih, M., Donaldson, N.N., and Barr, F.M.D. (1998). Investigation of the hammerstein hypothesis in the modeling of electrically stimulated muscle. *IEEE Transactions on Biomedical Engineering*, 45(8), 998–1009.
- Ljung, L. (1999). *System Identification: Theory for the User*. Englewood Cliffs, NJ: Prentice Hall, 2nd edition.
- Mant, J., Wade, D., and Winner, S. (2004). Health care needs assessment: Stroke. In A. Stevens, J. Raftery, J. Mant, and S. Simpson (eds.), *Health Care Needs Assessment: the Epidemiologically Based Needs Assessment Reviews*. Oxford: Radcliffe Medical Press.
- Mccrea, P.H., Eng, J.J., and Hodgson, A.J. (2003). Time and magnitude of torque generation is impaired in both arms following stroke. *Muscle and Nerve*, 28, 46–53.
- Munih, M., Hunt, K.J., and Donaldson, N.N. (2000). Variation of recruitment nonlinearity and dynamic response of ankle plantarflexors. *Medical Engineering and Physics*, 22(2), 97–107.
- Previdi, F. (2002). Identification of black-box nonlinear models for lower limb movement control using functional electrical stimulation. *Control Engineering Practice*, 10(2), 91–99.
- Schauer, T., Negard, N.O., Previdi, F., Hunt, K.J., Fraser, M.H., Ferchland, E., and Raisch, J. (2005). Online identification and nonlinear control of the electrically stimulated quadriceps muscle. *Control Engineering Practice*, 13(9), 1207–1219.
- Shue, G., Crago, P.E., and Chizeck, H.J. (1995). Muscle-joint models incorporating activation dynamics, moment-angle, and moment-velocity properties. *IEEE Transactions on Biomedical Engineering*, 42(2), 213–223.