Multiple antenna assisted hard versus soft decoding-and-forwarding for network coding aided relaying systems

Kyungchun Lee[1] and Lajos Hanzo[2]

[1]Samsung Electronics, P.O. Box 105, Suwon, Korea, e-mail: kl7v@esc.soton.ac.uk
[2]School of ECS, Univ. of Southampton, SO17 1BJ, UK, e-mail: lh@ecs.soton.ac.uk

Abstract— **In this paper, we propose two types of new decoding algorithms for a network coding aided relaying (NCR) system, which adopts multiple antennas at both the transmitter and receiver. We consider the realistic scenario of encountering decoding errors at the relay station (RS), which results in erroneous forwarded data. Under this assumption, we derive decoding algorithms for both the base station (BS) and the mobile station (MS) in order to reduce the deleterious effects of imperfect decoding at the RS. We first propose a decoding algorithm for a hard decision based forwarding (HDF) system. Then, for the sake of achieving further performance improvements, we also employ soft decision forwarding (SDF) and propose a novel decoding error model, which divides the decoding error pattern into two components: hard and soft errors. Given this error model, we then modify the HDF decoder for employment in SDF systems. Our simulation results show that the proposed algorithms provide substantial performance improvements in terms of the attainable packet error rate as a benefit of our more accurate decoding error model.**

I. Introduction

Multiple-input multiple-output (MIMO) diversity techniques are capable of achieving substantial diversity gains, provided that their elements experience independent or moderately correlated fading. However, in practical communication systems, multiple antennas are typically allocated insufficiently far apart, especially at the mobile station (MS) but often even at the base station (BS), which results in correlated spatial channels. Hence the spatial diversity promise of independently faded signals is often eroded. In order to cope with this problem, MIMO systems can be combined with adaptive modulation [1] as well as cooperative and relay station (RS) aided transmission schemes [2]–[4]. In relaying systems a MS receives both the two-hop downlink (DL) signal via the RS as well as the directly detected signal of the BS. Since these two signals typically arrive via completely different paths, the correlation between the fading of these channels is typically low. Furthermore, the RS is capable of extending the cell area and/or improving the quality of the reception at the cell-edge, which results in requiring a reduced number of BSs for maintaining seamless coverage.

Naturally, the relaying of the DL signal requires additional resources. For example, the traditional time multiplexing based relaying scheme [4] requires four time slots (TSs) for duplex communications, which is twice higher than that of the direct link between a MS and a BS. Therefore, relaying schemes requiring lower resources have been investigated in [5], [6]. The network coding aided relaying (NCR) scheme jointly encodes the signals received from two source nodes (such as a MS and a BS) into a single coded stream $\mathbf{x}_A$ [5], [6]. Once the RS received the data to be conveyed to the BS and MS in the uplink (UL) and DL in its two receiver TSs, it forwards their jointly encoded data to both the MS and the BS in the same time slot, which is seen as a DL slot for the MS and a UL slot for the BS, rather than independently transmitting their respective data in a different TS. Accordingly, the NCR system requires only three TSs, which leads to a 33% throughput enhancement compared to the traditional relaying scheme. Naturally, this TS reduction is achieved at the cost of potential error-propagation, which is reminiscent of the that experienced in differentially encoded systems.

The NCR is based on the decode-and-forward (DF) relaying strategy [3]. In DF relaying, the relay node fully decodes the received signal and forwards its re-encoded version in order to avoid the noise amplification of amplify-and-forward (AF) schemes. However, upon slicing the transmitted signal, the DF relay discards the soft information, which would be helpful at the destination node. To take the advantage of DF relaying, while also retaining the soft information at the same time, soft decision aided forwarding (SDF) techniques have been proposed in [7], [8]. In SDF relaying, the relay retransmits soft-valued estimates of the decoded signals, rather than their hard-decision based sliced versions in order to retain the soft information.

In this paper, we propose novel decoding algorithms for the destination nodes of NCR systems employing multiple antennas. In Section III-A, we derive an optimal decoding algorithm for hard decision forwarding (HDF) aided MIMO relaying systems, which transmits hard-decision values from the RS. Then, in Section III-B we extend this algorithm to the SDF aided MIMO relaying. When deriving the proposed algorithms, we employ an accurate error model for the sake of attaining a valuable performance improvement.

This paper is organized as follows. Section II describes the NCR system model, while Section III derives the proposed decoding algorithms. In Section IV our simulation results are provided for performance comparisons. Finally, we present our conclusions in Section V.

II. System Model

We assume that the BS, the MS, and the RS have the same number of transmit/receive antennas for notational convenience. Furthermore, we assume that the number of channel uses is fixed in both the UL and DL [1]. We consider vertically encoded MIMO systems [9], where the BS encodes the DL information bit stream $\mathbf{b}_B$ into a codeword $\mathbf{c}'_B$, which is then interleaved to obtain $\bar{\mathbf{c}}_B = \{\bar{\mathbf{c}}_B^1,\ \bar{\mathbf{c}}_B^2, \cdots, \bar{\mathbf{c}}_B^l, \cdots, \bar{\mathbf{c}}_B^T\}$, where T is the number of channel uses and $\bar{\mathbf{c}}_B^l = \{\bar{c}_{B,1},\ \bar{c}_{B,2}, \cdots, \bar{c}_{B,N_tM_c}\}$ denotes the $N_t \cdot M_c$ coded bits to be transmitted during the lth channel use. Here, N_t denotes the number of transmit antennas and M_c is the number of coded bits associated with a single modulated symbol. Then, $\bar{\mathbf{c}}_B^l$ is mapped to the N_t-element MIMO symbol $\mathbf{x}_B \in \mathcal{C}^{N_t}$, which is transmitted across N_t transmit antennas.

During the lth channel use of the first TS, the signals encountered at the RS and the MS are formulated as $\mathbf{y}_{BR}$ and $\mathbf{y}_{BM}$ [2], respectively, where $\mathbf{y}_{BR}$ and $\mathbf{y}_{BM}$ denote $(N_r \times 1)$-dimensional complex-valued

[1]In realistic environments where the packet sizes are different in the UL and DL, we can employ zero-padding or repetition coding to make them equal-length when joint encoding of the UL and DL signals is performed at the RS [5].

[2]For notational convenience, we omit the channel use index throughout the paper, when it does not cause any confusion.

978-1-4244-2324-8/08/$25.00 © 2008 IEEE.
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE "GLOBECOM" 2008 proceedings.

received signal vectors at the RS and the MS, while N_r indicates the number of receive antennas.

Similarly, the MS encodes the vector of UL information bits $\mathbf{b}_M$ into a codeword $\mathbf{c}'_M$ and interleaves it to obtain $\bar{\mathbf{c}}_M = \{\bar{\mathbf{c}}^1_M, \bar{\mathbf{c}}^2_M, \cdots, \bar{\mathbf{c}}^l_M, \cdots, \bar{\mathbf{c}}^T_M\}$. Finally, $\bar{\mathbf{c}}^l_M$ is converted to a symbol vector $\mathbf{x}_M$ before commencing transmission. During the lth channel use of the second TS, the signals received at the RS and the BS become $\mathbf{y}_{MR}$ and $\mathbf{y}_{MB}$, which denote $(N_r \times 1)$-element complex-valued received signal vectors.

The RS's estimates $\mathbf{c}'_B$ and $\mathbf{c}'_M$ are generated by the iterative detection/decoding (IDD) aided MIMO systems [9]. If $\mathbf{c}'_B$ and $\mathbf{c}'_M$ are perfectly decoded, then the RS combines $\mathbf{c}'_B$ and $\mathbf{c}'_M$ using the element-wise XOR operation into a composite packet[3] and sends the resultant message to the interleaver $\mathbf{\Pi}_R(\cdot)$ in order to obtain $\mathbf{c}_A = \{\mathbf{c}^1_A, \mathbf{c}^2_A, \cdots, \mathbf{c}^l_A, \cdots, \mathbf{c}^T_A\}$, where $\mathbf{c}^l_A = \{c_{A,1}, c_{A,2}, \cdots, c_{A,m}, \cdots, c_{A,N_tM_c}\}$ represents a bit stream of length $(N_t \cdot M_c)$ to be forwarded to the destination nodes during the lth channel use of the third TS.

The MS receives the combined signal of $\mathbf{c}_A = \mathbf{c}_B \oplus \mathbf{c}_M$, where $\mathbf{c}_B$ and $\mathbf{c}_M$ are defined as $\mathbf{c}_B = \mathbf{\Pi}_R(\mathbf{c}'_B)$ and $\mathbf{c}_M = \mathbf{\Pi}_R(\mathbf{c}'_M)$, respectively, while $\oplus$ denotes the element-wise XOR operation. To decode $\mathbf{c}_B$, we perform the IDD algorithm of Fig. 1. When applying the IDD, we first generate the log-likelihood ratio (LLR) $\mathbf{L}^{e,1}_{RM}(\mathbf{c}_A)$ for the coded bit vector $\mathbf{c}_A$ using the MIMO detector. Before inserting $\mathbf{L}^{e,1}_{RM}(\mathbf{c}_A)$ into the channel decoder, we remove the effect of $\mathbf{c}_M$, which is already known at the MS. Based on the XOR operation, we change the signs of those elements of $\mathbf{L}^{e,1}_{RM}(\mathbf{c}_A)$, whose corresponding bit values in $\mathbf{c}_M$ are 1, formulated as:

$$L^{e,1}_{RM}(c_{B,k}) = \begin{cases} L^{e,1}_{RM}(c_{A,k}), & \text{if the } k\text{th element of } \mathbf{c}_M \text{ is } 0 \\ -L^{e,1}_{RM}(c_{A,k}), & \text{if the } k\text{th element of } \mathbf{c}_M \text{ is } 1 \end{cases}, \quad (1)$$

where $L^{e,1}_{RM}(c_{A,k})$ is the kth element of $\mathbf{L}^{e,1}_{RM}(\mathbf{c}_A)$, and $L^{e,1}_{RM}(c_{B,k})$ is the kth element of the resultant LLR $\mathbf{L}^{e,1}_{RM}(\mathbf{c}_B)$ for $\mathbf{c}_B$, which is used to obtain the input of the channel decoder.

When decoding $\mathbf{c}_B$, we also exploit the directly detected signal $\mathbf{y}_{BM}$ as shown in Fig. 1 in order to improve the attainable performance. The MIMO detector extracts the LLR $\mathbf{L}^{e,1}_{BM}$ representing $\mathbf{c}_B$ from $\mathbf{y}_{BM}$, which is then interleaved and combined with $\mathbf{L}^{a,2}_{RM}$, namely the interleaved version of the LLRs obtained from the relayed signal $\mathbf{y}_{RM}$ of Fig. 1. We note that the a-priori information is subtracted, before the a-posteriori LLRs are combined and forwarded to the channel decoder.

III. Decoding schemes for network coding aided relaying

A. MIMO decoder for hard decision relaying

In HDF relaying, the relay obtains the estimates of $\mathbf{c}_B$ or $\mathbf{c}_M$ by slicing the elements of the LLR vector, even if decoding errors occur. The estimated $\mathbf{c}_B$ and $\mathbf{c}_M$ values are combined to generate $\hat{\mathbf{c}}_A$, which is the estimate of $\mathbf{c}_A$, even if $\mathbf{c}_B$ or $\mathbf{c}_M$ contains errors. Considering this scenario, we derive the optimal decoding scheme for MIMO relaying systems.

We assume that the bit error rate of the composite packet $\hat{\mathbf{c}}_A$ is $q(< 1/2)$, i.e. $q = d(\hat{\mathbf{c}}_A, \mathbf{c}_A)/(TN_tM_c)$, where $d(\cdot,\cdot)$ denotes the Hamming distance between two vectors. We also define $\hat{c}_{A,m}$ and $\hat{\mathbf{c}}^l_A$ as the mth estimated coded bit and the estimated $(N_tM_c \times 1)$-element bit stream vector corresponding to $c_{A,m}$ and $\mathbf{c}^l_A$, respectively. Here, we express $\hat{c}_{A,m}$ and $\hat{\mathbf{c}}^l_A$ as $\hat{c}_{A,m} = c_{A,m} \oplus \Delta c_{A,m}$ and

[3]The XOR operation essentially creates the difference of the MS's and BS's transmitted signals, hence the RS's transmission may be viewed as sending to both destinations, namely the MS and the BS, a sequence, which has a binary one in the transmitted frame, where the sequence destined for their reception is different from their transmitted sequence.

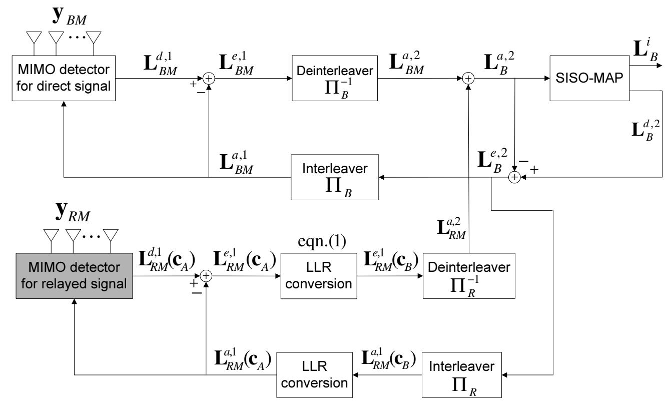

Fig. 1. Block diagram of the proposed decoding scheme at the MS

$\hat{\mathbf{c}}^l_A = \mathbf{c}^l_A \oplus \Delta\mathbf{c}^l_A$, where $\Delta c_{A,m} \in \{0, 1\}$ and $\Delta\mathbf{c}^l_A$ represent the corresponding decoding errors in $c_{A,m}$ and $\mathbf{c}^l_A$, respectively. We define $\hat{\mathbf{x}}^h_A$ as the modulated signal vector of $\hat{\mathbf{c}}^l_A$. The modulated signal $\hat{\mathbf{x}}^h_A$ is transmitted and the corresponding received signals at the MS and the BS are formulated as

$$\mathbf{y}^h_{RM} = \mathbf{H}_{RM}\hat{\mathbf{x}}^h_A + \mathbf{v}_{RM}, \quad (2)$$
$$\mathbf{y}^h_{RB} = \mathbf{H}_{RB}\hat{\mathbf{x}}^h_A + \mathbf{v}_{RB}, \quad (3)$$

where $\mathbf{H}_{RM}$ and $\mathbf{H}_{RB}$ denote the $(N_r \times N_t)$-element complex-valued channel matrices, while $\mathbf{v}_{RM}$ and $\mathbf{v}_{RB}$ are the $(N_r \times 1)$-element Gaussian distributed noise vectors of $\mathbf{v}_{RM}, \mathbf{v}_{RB} \sim \mathcal{CN}(\mathbf{0}_{N_r}, \sigma^2_v\mathbf{I}_{N_r})$.

Hereafter, we will derive the optimal MIMO decoder for the DL signal of the HDF system. The BS's decoder of the UL signal can be readily obtained from the decoder of the DL at the MS as a benefit of the NCR system's symmetric structure.

For the signal directly transmitted from the BS, the conventional MIMO detector of [9] is applied without any modification, because it does not contain the RS's decision error hosted by $\Delta\mathbf{c}^l_A$. Therefore, we only have to modify the MIMO detection block of the RS seen in Fig. 1 by considering the decoding error vector $\Delta\mathbf{c}^l_A$. The LLR of $c_{A,m}$ is formulated as

$$L^{d,1}_{RM}(c_{A,m}) = \log \frac{\sum_{\{\mathbf{c}^l_A, \Delta\mathbf{c}^l_A\}: c_{A,m}=1} e^{-\|\mathbf{y}^h_{RM} - \mathbf{H}_{RM}\hat{\mathbf{x}}_A\|^2/\sigma^2} p(\mathbf{c}^l_A, \Delta\mathbf{c}^l_A)}{\sum_{\{\mathbf{c}^l_A, \Delta\mathbf{c}^l_A\}: c_{A,m}=0} e^{-\|\mathbf{y}^h_{RM} - \mathbf{H}_{RM}\hat{\mathbf{x}}_A\|^2/\sigma^2} p(\mathbf{c}^l_A, \Delta\mathbf{c}^l_A)}, \quad (4)$$

where $\hat{\mathbf{x}}_A$ is a trial of $\hat{\mathbf{x}}^h_A$ corresponding to $\{\mathbf{c}^l_A, \Delta\mathbf{c}^l_A\}$ and $p(\mathbf{c}^l_A, \Delta\mathbf{c}^l_A)$ is the joint probability of $\mathbf{c}^l_A$ and $\Delta\mathbf{c}^l_A$. Furthermore, $\|\cdot\|$ indicates the Frobenius norm of a matrix. Assuming that the elements of $\mathbf{c}^l_A$ and $\Delta\mathbf{c}^l_A$ are mutually independent thanks to the random interleaver at the transmitter, respectively, $p(\mathbf{c}^l_A, \Delta\mathbf{c}^l_A)$ can be written as

$$p(\mathbf{c}^l_A, \Delta\mathbf{c}^l_A) = \prod_{k=1}^{N_tM_c} p(\Delta c_{A,k})p(c_{A,k}). \quad (5)$$

Since the error rate of $\mathbf{c}_A$ is q, $p(\Delta c_{A,k})$ becomes

$$p(\Delta c_{A,k}) = \begin{cases} 1-q, & \text{if } \Delta c_{A,k} = 0 \\ q, & \text{if } \Delta c_{A,k} = 1 \end{cases}. \quad (6)$$

$$\sum_{\{\mathbf{c}_A^l,\Delta\mathbf{c}_A^l\}:c_{A,m}=1} d(\hat{\mathbf{x}}_A)p(\mathbf{c}_A^l,\Delta\mathbf{c}_A^l) = \sum_{\{\mathbf{c}_A^l,\Delta\mathbf{c}_A^l\}:c_{A,m}=1,c_{A,1}=0,\Delta c_{A,1}=0} d(\hat{\mathbf{x}}_A)p(\Delta c_{A,1}=0)p(c_{A,1}=0)\prod_{k=2}^{N_tM_c} p(\Delta c_{A,k})p(c_{A,k})$$
$$+\sum_{\{\mathbf{c}_A^l,\Delta\mathbf{c}_A^l\}:c_{A,m}=1,c_{A,1}=0,\Delta c_{A,1}=1} d(\hat{\mathbf{x}}_A)p(\Delta c_{A,1}=1)p(c_{A,1}=0)\prod_{k=2}^{N_tM_c} p(\Delta c_{A,k})p(c_{A,k})$$
$$+\sum_{\{\mathbf{c}_A^l,\Delta\mathbf{c}_A^l\}:c_{A,m}=1,c_{A,1}=1,\Delta c_{A,1}=0} d(\hat{\mathbf{x}}_A)p(\Delta c_{A,1}=0)p(c_{A,1}=1)\prod_{k=2}^{N_tM_c} p(\Delta c_{A,k})p(c_{A,k})$$
$$+\sum_{\{\mathbf{c}_A^l,\Delta\mathbf{c}_A^l\}:c_{A,m}=1,c_{A,1}=1,\Delta c_{A,1}=1} d(\hat{\mathbf{x}}_A)p(\Delta c_{A,1}=1)p(c_{A,1}=1)\prod_{k=2}^{N_tM_c} p(\Delta c_{A,k})p(c_{A,k}). \quad (8)$$

$$L_{RM}^{d,1}(c_{A,m})=\log\frac{p(c_{A,m}=1)}{p(c_{A,m}=0)}+\log\frac{(1-q)\sum_{\hat{\mathbf{c}}_A^l:\hat{c}_{A,m}=1} d(\hat{\mathbf{x}}_A)\prod_{k=1,k\neq m}^{N_tM_c} p(\hat{c}_{A,k})+q\sum_{\hat{\mathbf{c}}_A^l:\hat{c}_{A,m}=0} d(\hat{\mathbf{x}}_A)\prod_{k=1,k\neq m}^{N_tM_c} p(\hat{c}_{A,k})}{(1-q)\sum_{\hat{\mathbf{c}}_A^l:\hat{c}_{A,m}=0} d(\hat{\mathbf{x}}_A)\prod_{k=1,k\neq m}^{N_tM_c} p(\hat{c}_{A,k})+q\sum_{\hat{\mathbf{c}}_A^l:\hat{c}_{A,m}=1} d(\hat{\mathbf{x}}_A)\prod_{k=1,k\neq m}^{N_tM_c} p(\hat{c}_{A,k})}. \quad (13)$$

Then, the likelihood function for $c_{A,m}=1$ in (4) is expressed as

$$\sum_{\{\mathbf{c}_A^l,\Delta\mathbf{c}_A^l\}:c_{A,m}=1} e^{-\|\mathbf{y}_{RM}^h-\mathbf{H}_{RM}\hat{\mathbf{x}}_A\|^2/\sigma^2} p(\mathbf{c}_A^l,\Delta\mathbf{c}_A^l)$$
$$=\sum_{\{\mathbf{c}_A^l,\Delta\mathbf{c}_A^l\}:c_{A,m}=1} d(\hat{\mathbf{x}}_A)\prod_{k=1}^{N_tM_c} p(\Delta c_{A,k})p(c_{A,k}), \quad (7)$$

where $d(\hat{\mathbf{x}}_A)=e^{-\|\mathbf{y}_{RM}^h-\mathbf{H}_{RM}\hat{\mathbf{x}}_A\|^2/\sigma^2}$.

We can divide (7) into four terms with respect to $c_{A,1}$ and $\Delta c_{A,1}$ in order to obtain (8). We note that for both $\{c_{A,1}=0,\Delta c_{A,1}=0\}$ and $\{\Delta c_{A,1}=1,c_{A,1}=1\}$, $\hat{c}_{A,1}$ is constant (i.e., $\hat{c}_{A,1}=0$). Therefore, for $\{c_{A,1}=0,\Delta c_{A,1}=0\}$ and $\{\Delta c_{A,1}=1,c_{A,1}=1\}$, $\mathbf{x}_A$ has the same value and we can combine the first and last terms in (8). In a similar manner, we combine the second and third terms and use (6) to obtain

$$\sum_{\{\mathbf{c}_A^l,\Delta\mathbf{c}_A^l\}:c_{A,m}=1} d(\hat{\mathbf{x}}_A)p(\mathbf{c}_A^l,\Delta\mathbf{c}_A^l)$$
$$=\sum_{\{\mathbf{c}_{A,2}^l,\Delta\mathbf{c}_{A,2}^l,\hat{c}_{A,1}\}:c_{A,m}=1,\hat{c}_{A,1}=0} d(\hat{\mathbf{x}}_A)\{(1-q)p(c_{A,1}=0)$$
$$+qp(c_{A,1}=1)\}\prod_{k=2}^{N_tM_c} p(\Delta c_{A,k})p(c_{A,k})$$
$$+\sum_{\{\mathbf{c}_{A,2}^l,\Delta\mathbf{c}_{A,2}^l,\hat{c}_{A,1}\}:c_{A,m}=1,\hat{c}_{A,1}=1} d(\hat{\mathbf{x}}_A)\{qp(c_{A,1}=0)$$
$$+(1-q)p(c_{A,1}=1)\}\prod_{k=2}^{N_tM_c} p(\Delta c_{A,k})p(c_{A,k}), \quad (9)$$

where we have $\mathbf{c}_{A,k}^l=\{c_{A,k},\ c_{A,k+1},\cdots,c_{A,N_tM_c}\}$ and $\Delta\mathbf{c}_{A,k}^l=\{\Delta c_{A,k},\ \Delta c_{A,k+1},\cdots,c_{A,N_tM_c}\}$. Here, it is worth noting that the a-priori probability of $\hat{c}_{A,k}$ is expressed as

$$p(\hat{c}_{A,k})=\begin{cases}(1-q)p(c_{A,k}=0)+qp(c_{A,k}=1), \text{ if } \hat{c}_{A,k}=0\\ qp(c_{A,k}=0)+(1-q)p(c_{A,k}=1), \text{ if } \hat{c}_{A,k}=1\end{cases}. \quad (10)$$

Employing (9) and (10) can be simplified as

$$\sum_{\{\mathbf{c}_A^l,\Delta\mathbf{c}_A^l\}:c_{A,m}=1} d(\hat{\mathbf{x}}_A)p(\mathbf{c}_A^l,\Delta\mathbf{c}_A^l)=$$
$$\sum_{\{\mathbf{c}_{A,2}^l,\Delta\mathbf{c}_{A,2}^l,\hat{c}_{A,1}\}:c_{A,m}=1} d(\hat{\mathbf{x}}_A)p(\hat{c}_{A,1})\prod_{k=2}^{N_tM_c} p(\Delta c_{A,k})p(c_{A,k}). \quad (11)$$

Applying similar operations to those in (8)-(11) for $\{c_{A,2},c_{A,3},\cdots,c_{A,N_tM_c}\}$ except for $c_{A,m}$, we arrive at

$$\sum_{\{\mathbf{c}_A^l,\Delta\mathbf{c}_A^l\}:c_{A,m}=1} d(\hat{\mathbf{x}}_A)p(\mathbf{c}_A^l,\Delta\mathbf{c}_A^l)$$
$$=\sum_{\{c_{A,m},\Delta c_{A,m},\hat{\mathbf{c}}_{A,\overline{m}}^l\}:c_{A,m}=1} d(\hat{\mathbf{x}}_A)p(\Delta c_{A,m})p(c_{A,m}=1)$$
$$\cdot\prod_{k=1,k\neq m}^{N_tM_c} p(\hat{c}_{A,k})$$
$$=p(c_{A,m}=1)\Bigg\{(1-q)\sum_{\hat{\mathbf{c}}_A^l:\hat{c}_{A,m}=1} d(\hat{\mathbf{x}}_A)\prod_{k=1,k\neq m}^{N_tM_c} p(\hat{c}_{A,k})$$
$$+q\sum_{\hat{\mathbf{c}}_A^l:\hat{c}_{A,m}=0} d(\hat{\mathbf{x}}_A)\prod_{k=1,k\neq m}^{N_tM_c} p(\hat{c}_{A,k})\Bigg\}, \quad (12)$$

where $\hat{\mathbf{c}}_{A,\overline{m}}^l$ denotes the estimated bit stream vector obtained by excluding $\hat{c}_{A,m}$ from $\hat{\mathbf{c}}_A$. In a similar manner, we simplify the likelihood function for $c_{A,m}=0$ and obtain the LLR for $c_{A,m}$ in the form of (13), where the last term corresponds to the extrinsic information.

B. MIMO decoder for soft decision relaying

The HDF generally imposes a lower computational complexity than the SDF at the cost of a performance degradation, because it discards the soft information by slicing the signal. In this subsection, we derive a MIMO decoder for the SDF system for the sake of attaining a better performance. We assume that the RS transmits the expectation values of the symbols [8]. For example, when BPSK modulation is assumed, the kth element of $\hat{\mathbf{x}}_A^s$ is formulated as

$$\hat{x}_{A,k}^s=p(c_{A,k}=1|\mathbf{y}_{MR},\mathbf{y}_{BR})-p(c_{A,k}=0|\mathbf{y}_{MR},\mathbf{y}_{BR}), \quad (14)$$

where $\hat{\mathbf{x}}_A^s$ is the soft estimate of $\mathbf{x}_A$.

By transmitting the expectation value of a symbol rather than the sliced value, we minimize the mean squared error of the relayed signals and preserve the soft information. In [8], the error of $\hat{x}_{A,k}^s$ was modeled as a Gaussian distributed random variable. However, to obtain a more accurate error model, we divide the error into two terms: the hard-decision error and the soft-decision error. The error vector $\Delta\mathbf{x}_A=\hat{\mathbf{x}}_A^s-\mathbf{x}_A$ is expressed as

$$\Delta\mathbf{x}_A=\Delta\mathbf{x}_A^h+\Delta\mathbf{x}_A^s, \quad (15)$$

where $\Delta\mathbf{x}_A^h = M(\hat{\mathbf{c}}_A^l) - \mathbf{x}_A$ is the hard-valued error vector and $\Delta\mathbf{x}_A^s$ represents the soft-valued error vector. Here, $M(\cdot)$ denotes the constellation mapper. We assume that the elements of $\Delta\mathbf{x}_A^s$ are independent zero-mean Gaussian distributed random variables.

The received signal at the MS is expressed as

$$\mathbf{y}_{RM}^s = \mathbf{H}_{RM}\hat{\mathbf{x}}_A^s + \mathbf{v}_{RM}. \tag{16}$$

Considering that we have $\hat{\mathbf{x}}_A^s = \mathbf{x}_A + \Delta\mathbf{x}_A^h + \Delta\mathbf{x}_A^s$, (16) can be rewritten as

$$\mathbf{y}_{RM}^s = \mathbf{H}_{RM}(\mathbf{x}_A + \Delta\mathbf{x}_A^h) + \mathbf{H}_{RM}\Delta\mathbf{x}_A^s + \mathbf{v}_{RM}. \tag{17}$$

Furthermore, exploiting that $\hat{\mathbf{x}}_A^h = \mathbf{x}_A + \Delta\mathbf{x}_A^h$, we obtain

$$\mathbf{y}_{RM}^s = \mathbf{H}_{RM}\hat{\mathbf{x}}_A^h + \mathbf{H}_{RM}\Delta\mathbf{x}_A^s + \mathbf{v}_{RM}, \tag{18}$$

where $\mathbf{H}_{RM}\Delta\mathbf{x}_A^s$ can be regarded as an interference term. Comparing (2) and (18), we can see that the only difference between them is that the interference term of $\mathbf{H}_{RM}\Delta\mathbf{x}_A^s$ is added in (18). Hence we can apply the proposed MIMO decoder in Section III-A to the SDF system by considering $\tilde{\mathbf{v}} = \mathbf{H}_{RM}\Delta\mathbf{x}_A^s + \mathbf{v}_{RM}$ as a new interference-plus-noise term. Since $\Delta\mathbf{x}_A^s$ and $\mathbf{v}_{RM}$ are Gaussian distributed random vectors, we know that $\tilde{\mathbf{v}}$ is also a Gaussian distributed random vector of $\tilde{\mathbf{v}} \sim \mathcal{CN}(\mathbf{0}_{N_r}, \mathbf{R}_{\tilde{v}})$, where $\mathbf{R}_{\tilde{v}} = \sigma_v^2\mathbf{I}_{N_r} + \sigma_e^2\mathbf{H}_{RM}\mathbf{H}_{RM}^H$. Here, $(\cdot)^H$ denotes the complex conjugate transpose of a matrix and σ_e^2 is the variance of soft-valued errors.

Therefore, the LLR of $c_{A,m}$ can be computed as

$$L_{RM}^{d,1}(c_{A,m}) = \log\frac{\sum_{\{\mathbf{c}_A^l, \Delta\mathbf{c}_A^l\}:c_{A,m}=1} \bar{d}(\hat{\mathbf{x}}_A)p(\mathbf{c}_A^l, \Delta\mathbf{c}_A^l)}{\sum_{\{\mathbf{c}_A^l, \Delta\mathbf{c}_A^l\}:c_{A,m}=0} \bar{d}(\hat{\mathbf{x}}_A)p(\mathbf{c}_A^l, \Delta\mathbf{c}_A^l)}, \tag{19}$$

where

$$\bar{d}(\hat{\mathbf{x}}_A) = e^{-(\mathbf{y}_{RM}^s - \mathbf{H}_{RM}\hat{\mathbf{x}}_A)^H\mathbf{R}_{\tilde{v}}^{-1}(\mathbf{y}_{RM}^s - \mathbf{H}_{RM}\hat{\mathbf{x}}_A)/4}. \tag{20}$$

Since $\mathbf{R}_{\tilde{v}}$ does not depend on $\Delta\mathbf{x}_A^h$, we can simplify the computation of the extrinsic information as we did in (4)-(13) of Section III-A. Explicitly, when computing $L_{RM}^{d,1}(c_{A,m})$ of (19), all we have to do is to change $d(\hat{\mathbf{x}}_A)$ in (13) to $\bar{d}(\hat{\mathbf{x}}_A)$ of (20).

We note that the likelihood function of (20) considers the effects of both soft-decision errors as well as of the Additive White Gaussian Noise (AWGN), while the HDF decoder only considers the AWGN. When the soft-decision error of a codeword is low, i.e. $\sigma_e^2 \approx 0$, the likelihood function of (20) is reduced to that of the HDF system.

C. Parameter estimation

To apply the proposed decoders described in the previous subsections, we need the knowledge of the bit error ratio q as well as the the variance of the soft error σ_e^2. These parameters are estimated at the RS and forwarded to both the BS and the MS. In this subsection, we derive methods to estimate q and σ_e^2.

Employing the LLRs of $c_{B,k}$ and $c_{M,k}$ we can compute $p(c_{A,k}|\mathbf{y}_{MR}, \mathbf{y}_{BR})$, which is the probability of the composite bit. Since the specific logical bit value having a higher probability becomes the estimate of the coded bit in HDF, the error probability of $c_{A,k}$ is formulated as

$$p_e(c_{A,k}) = \min\{p(c_{A,k}=0|\mathbf{y}_{MR}, \mathbf{y}_{BR}), p(c_{A,k}=1|\mathbf{y}_{MR}, \mathbf{y}_{BR})\}.$$

Therefore, q is estimated by averaging $p_e(c_{A,k})$ over the entire codeword of length TN_tM_c.

By contrast, to obtain the estimate of σ_e^2 in SDF, we first compute $\hat{x}_{A,k}^h$, which is the hard-decision estimate of the kth element of $\mathbf{x}_A$. The corresponding soft error $\Delta x_{A,k}^s$ is the difference between the soft-decision estimate $\hat{x}_{A,k}^s$ and the hard-decision estimate $\hat{x}_{A,k}^h$. Hence the estimate of σ_e^2 is calculated by averaging $|\hat{x}_{A,k}^s - \hat{x}_{A,k}^h|^2$ over the entire codeword.

IV. Simulation Results

Computer simulations have been performed to characterise the proposed decoding algorithms. We employed a turbo code having the rate $R = 1/2$ and length of $TN_tM_c = 1024$, which is constituted by two recursive systematic convolutional (RSC) codes with the octal generators (7,5). Throughout our simulations, we used QPSK signaling and the exact log-MAP channel decoding algorithm at the receiver. The number of decoding iterations in the turbo channel decoder was set to five. We have assumed that all the elements of MIMO channel matrices are independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variables having a variance of 1/2 per dimension, which are fixed in a TS. We define the SNR as the ratio of the average power per information bit arriving at the receiver to the spectral density of the noise. In the IDD of the MIMO receiver, all possible symbol combinations are considered in the computation of (13). The number of MIMO detection/decoding iterations was selected to be four.

The SNRs of the paths arriving from the BS and the MS to the RS are assumed to be the same and are denoted as γ_{RS}. This implies that the RS is somewhere half-way between the BS and MS, where the associated path loss and fading parameters are the same, although in practice the path loss between the BS and the RS may be expected to be lower than that between the RS and MS. Similarly, we assume that γ_{MR}, the SNR of the path from the RS to the MS, is the same as γ_{RS}. The SNR of the direct link from the BS to the MS is denoted as γ_{BM}. We also assume that the relay channel has the same or a higher SNR compared to the direct channel (i.e. $\gamma_{MR} \geq \gamma_{BM}$) and we denote the ratio between these SNRs as $\gamma_{diff}(> 1)$.

Figs. 2-4 characterize the packet error ratio (PER) performance of various decoders, which is defined synonymously to the codeword error rate. The *perfect DF* denotes the DF relaying scheme that transmits the perfectly decoded packet from the RS. More specifically, it does not suffer from decoding errors at the RS and hence it naturally outperforms the realistic relaying schemes. By contrast, the *conventional HDF* represents the specific HDF scheme, which does not consider the potential presence of decoding errors at the RS. More explicitly, the decoder at the BS or the MS of conventional HDF relaying always assumes the presence of perfectly decoded data, which are transmitted from the RS, regardless whether the forwarded packet actually contains errors. The idealized HDF and SDF decoders assuming the perfect knowledge of q and σ_e are referred to as *Proposed HDF 1* and *Proposed SDF 1* schemes, respectively. The proposed decoders that generate realistic estimates of q and σ_e as described in Section III-C are referred to as *Proposed HDF 2* and *Proposed SDF 2*. The *SDF using the Gaussian model* represents the relaying system, where the expectation values of the symbols are transmitted from the RS to both the MS and BS to minimize the mean square error of the forwarded signal. Then again, the Gaussian error model is assumed for supporting the decoder's operation at the destination nodes, as proposed in [8].

Fig. 2 illustrates the PER performance of NCR systems using $N_t = 2$, $N_r = 2$, and $\gamma_{diff} = 0$ dB. It is observed that the proposed decoders using the proposed error model have a better performance compared to the decoder assuming the Gaussian error model. Interestingly, the proposed HDF decoders also have a better performance than the SDF decoder of [8] despite its lower complexity. This is because the proposed HDF decoder employs an accurate hard error model. The SDF reduces the mean square error of the symbol to be forwarded, but the relatively inaccurate error model results in the observed performance degradation. It is also seen in Fig. 2 that the proposed SDF decoders have a slightly better performance than the proposed HDF decoders. In the low-SNR region, the decoders

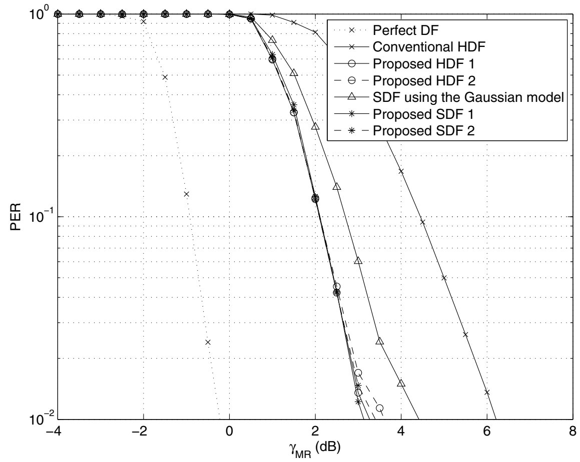

Fig. 2. PER versus SNR performance over a frequency-flat channel using $N_t = 2,\ N_r = 2,\ \gamma_{diff}$=0 dB.

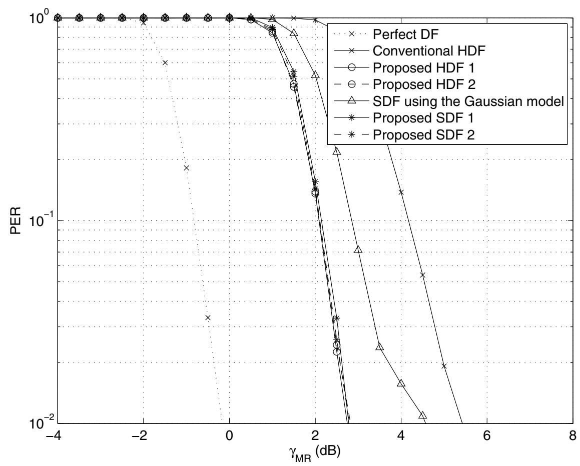

Fig. 3. PER versus SNR performance over a frequency-flat channel using $N_t = 4,\ N_r = 4,\ \gamma_{diff}$=0 dB.

using the estimated parameters exhibit a similar performance to those associated with the perfect knowledge of the parameters q and σ_e. In the high-SNR region, they perform slightly worse, but their SNR disadvantage is less than 0.4 dB.

In Fig. 3, the performance of the $N_t = 4$, $N_r = 4$, and $\gamma_{diff} = 0$ dB scenario is characterised. Fig. 3 demonstrates that the proposed decoders provide a substantial performance improvement compared to both the conventional HDF decoder and to the SDF decoder using the Gaussian error model. At PER=10^{-2}, the proposed decoders achieve an approximately 1.8 dB SNR gain over the SDF decoder assuming the Gaussian error model.

Fig. 4 characterises the PER versus SNR performance of NCR systems associated with $N_t = 4$, $N_r = 4$, and $\gamma_{diff} = 3$ dB, where the proposed decoders outperform the conventional HDF decoder and the SDF decoder using the Gaussian model although the SNR gains of the proposed decoders decrease, as γ_{diff} increases.

V. Conclusion

In this paper, we have derived error models for NCR systems using multiple antennas. First, we considered the HDF relaying scheme and proposed a new decoding algorithm, which takes the estimated bit error rate of the forwarded packet into account, when performing iterative detection and decoding at both the BS and the MS. More

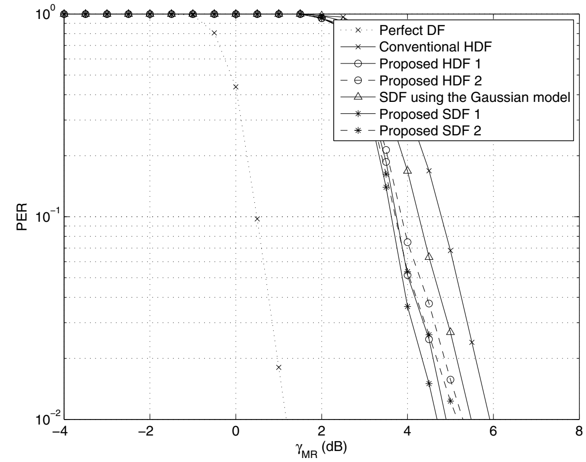

Fig. 4. PER versus SNR performance over a frequency-flat channel using $N_t = 4,\ N_r = 4,\ \gamma_{diff}$=3 dB.

explicitly, we obtained a simplified expression for the extrinsic LLR at the output of the MIMO detector. Secondly, a new decoder has been derived for SDF relaying. In (15), we modeled the error of the forwarded signal as the combination of the hard and soft errors, and developed the HDF scheme's decoder for employment in SDF in conjunction with a modified cost function. Finally, we proposed realistic estimation algorithms to acquire the parameters necessary for the operation of the proposed decoders.

The simulation results of Figs. 2-4 show that both of proposed HDF and SDF decoders achieve a better performance compared to the decoders previously proposed in the open literature. In the NCR system using $N_t = 4$, $N_r = 4$, and $\gamma_{diff} = 0$, the proposed decoders provide around 1.8 dB SNR gain at PER=10^{-2} with respect to their benchmarker using the Gaussian error model.

References

[1] L. Hanzo, C. H. Wong, and M. S. Yee, *Adaptive Wireless Transceivers*. New York: Wiley; IEEE Press, 2002.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part I: System description," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.

[3] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[4] R. Pabst, B. H. Walke, D. C. Schultz, P. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. D. Falconer, and G. P. Fettweis, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Commun. Mag.*, vol. 42, pp. 80–89, Sept. 2004.

[5] P. Larsson, N. Johansson, and K.-E. Sunell, "Coded bi-directional relaying," in *Proc. IEEE Vehicular Technology Conf.*, Melbourne, Australia, May 2006, pp. 851–855.

[6] L. Xiao, T. Fuja, J. Kliewer, and D. J. Costello, "A network coding approach to cooperative diversity," *IEEE Trans. Inform. Theory*, vol. 53, pp. 3714–3722, Oct 2007.

[7] H. H. Sneessens and L. Vandendorpe, "Soft decode and forward improves cooperative communications," in *IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing*, Puerto Vallarta, Mexico, Dec. 2005, pp. 157–160. .

[8] Y. Li, B. Vucetic, T. F. Wong, and M. Dohler "Distributed turbo coding with soft information relaying in multihop relay networks," *IEEE J. Select. Areas Commun.*, vol. 24, pp. 2040–2050, Nov. 2006.

[9] B. M. Hochwald and S. ten Brink, "Achieving near-capacity on a multiple-antenna channel," *IEEE Trans. Inform. Theory*, vol. 51, pp. 389–399, March 2003.