Abstract—Recently, the turbo-detected Sphere Packing (SP) aided Space-Time Block-Coding (STBC) STBC-SP scheme was demonstrated to provide useful performance improvements over conventionally-modulated orthogonal design based STBC schemes in the context of single-user multiple-input multiple-output (SU-MIMO) systems. Hence, in this treatise we extend its employment to the multi-user MIMO (MU-MIMO) scenario. For the sake of achieving a near Maximum A Posteriori (MAP) performance while imposing a moderate complexity, we specifically design the $K$-best sphere detection (SD) scheme for supporting the operation of the SP-modulated system, since the conventional SD cannot be directly applied to such a system. Consequently, with the aid of our modified SD, a significant performance gain can be achieved by the SP-modulated system over its conventionally-modulated counterpart in the context of MU-MIMO systems. For example, a performance gain of 3.5 dB can be achieved by the proposed 256-SP over the identical-throughput 16-QAM benchmark system in the scenario of a four-receive-antenna SDMA uplink (UL) system supporting two users both employing Alamouti’s $G_2$ space-time code, given a target BER of $10^{-4}$.

I. INTRODUCTION

The family of multiple-input-multiple-output (MIMO) transmission schemes typically fall into two main categories, i.e. spatial multiplexing and spatial diversity schemes, which aim for maximizing the data rate and for minimizing the Bit Error Ratio (BER), respectively. Research efforts have also been invested in striking a flexible compromise between the achievable rate and the BER [1]. To be more specific, in uplink (UL) multiuser-MIMO (MU-MIMO) scenarios, Spatial Division Multiple Access (SDMA) [2] significantly increases the overall system’s throughput in a given bandwidth. In other words, SDMA substantially increases the achievable spectral efficiency by exploiting the unique, user-specific channel impulse responses (CIR) between the transmit and receive antenna pairs. Although traditionally only the BS is equipped with more than one transmit/receive antenna elements, laptop computers and sophisticated multimedia communicators can be equipped with multiple UL transmit antennas in the interest of improving their performance. Orthogonal space-time code (STC) designs have recently attracted considerable research interests, which was inspired by the two-transmit-antenna scenario proposed by Tarokh et al. in [3]. This concept was then further generalised for an arbitrary number of transmit antennas by Tarokh et al. in [4], which facilitate linear detection at the receiver, while achieving full transmit diversity.

The concept of combining orthogonal transmit designs with the principle of sphere packing (SP) [5] was introduced by Su et al. in [6] in order to maximise the achievable coding advantage, demonstrating that the proposed SP-aided STBC (STBC-SP) scheme, was capable of outperforming the conventional orthogonal design based STBC schemes of [3,4,7] in the single-user MIMO (SU-MIMO) scenario. In parallel, sphere detection (SD) techniques [8,9] designed for conventionally-modulated MIMO systems, have received wide interest in both the research and industrial communities, which are capable of achieving a near-MAP performance at a significantly lower computational complexity compared to the classic MAP detectors.

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Against this background, we design a STBC-SP scheme for a MU-MIMO SDMA/OFDM scenario with the aid of the SD technique. Our main contribution in this paper is the challenging design of the $K$-best SD for SP-modulated systems, which extends the employment of STBC-SP schemes to MU-MIMO scenarios, while approaching the MAP performance at a moderate complexity: 1) We improve the STBC performance by jointly designing the space-time signals of the two time slots of an SDMA/OFDM scheme using SP modulation, while existing designs make no attempt to do so owing to its complex detection. 2) We solve this complexity problem by further developing the $K$-best SD for the detection of SP modulation, because SP offers a substantial SNR reduction at a potentially excessive complexity, which is reduced by the new SD.

Notation: The bold uppercase variables, such as $X$ and the lowercase variables $x$ denote matrices and vectors, respectively. The element in the $i$th row and $j$th column of a matrix $X$ is represented by $X_{ij}$, while the $i$th element of a vector $x$ is denoted by $x_i$. Furthermore, $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ and $| \cdot |$ denote the transposition, the conjugate, the Hermitian transposition and the Frobenius norm operations, respectively. Finally, $\mathbb{R}\{\cdot\}$ and $\mathbb{I}\{\cdot\}$ denote the real and imaginary components of a complex number.

II. J OINT ORTHOGONAL SPACE-TIME SIGNAL DESIGN OF TWO ANTENNAS USING SPHERE PACKING

A. Orthogonal Design Using Sphere Packing Modulation

Conventionally, the orthogonal design of STBCs [3,4,7] is based on conventional PSK/QAM modulated symbols. Without loss of generality, we take the Alamouti $G_2$ scheme as a simple example, where the corresponding Space-Time (ST) signaling matrix is constructed as [3]:

$$G_2(x_1, x_2) = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}, \quad (1)$$

and the rows as well as columns respectively represent the spatial and temporal dimensions. The inputs $(x_1, x_2)$ of the $G_2$ encoder are chosen independently from the constellation corresponding to a specific modulation scheme, then mapped to two transmit antennas, and finally transmitted during two consecutive symbol periods. Therefore, no efforts are made to jointly design the input symbols $(x_1, x_2)$. However, it was shown by Su et al. in [6] that combining the orthogonal design with sphere packing (SP) [5] is capable of attaining extra coding gains by maximizing the diversity product of the STBC signals in the presence of temporally correlated fading channels. The diversity product expression for the square-shaped orthogonal STBC matrix $G_{2k}$ [7] in the context of time-correlated fading channels is given by [6]:

$$\zeta_{2k} = \frac{1}{2\sqrt{k+1}} \min_{(x_1, \ldots, x_{k+1})} \left( \sum_{i=1}^{k+1} |x_i - \tilde{x}_i|^2 \right)^{1/2}, \quad (2)$$

where $x_i$ and $\tilde{x}_i$ are the elements of two distinct space-time signalling matrices $C$ and $\tilde{C}$, respectively. From Eq.(2) we can observe that the diversity product is actually determined by the Minimum

$\zeta_{2k}$.
Euclidean Distance (MED) among all the possible ST signal vectors \((x_1, x_2, \ldots, x_{k+1})\). Thus, the idea of combining the individual antenna signals into a joint ST design using SP is both straightforward and desirable, since the SP modulated symbols have the best known MED in the \(2(k + 1)\)-dimensional real-valued Euclidean space \(\mathbb{R}^{2(k+1)}\) [5]. Hence, the system becomes capable of maximizing the achievable diversity product of STBC codes, which in turn minimizes the transmission error probability.

Again, we consider the \(G_2\) scheme as an example. Let us define the lattice \(D_2\) as a SP having the best MED from all other \((L-1)\) legitimate constellation points in \(\mathbb{R}^4\) [5], which may be also defined as a lattice that consists of all legitimate SP constellation points having integer coordinates \([a_1, a_2, a_3, a_4]\). These coordinates uniquely and unambiguously describe the legitimate combinations of the two time-slots’ modulated symbols in the \(G_2\) scheme, while obeying the SP constraint of \(a_1 + a_2 + a_3 + a_4 = p\), where \(p\) is an even integer [10]. Furthermore, each input vector of the \(G_2\) scheme, i.e. \((x_1, x_2)\), in the two-dimensional complex-valued space \(C^2\), can be represented by four real numbers, which as a whole corresponds to the coordinates of a four-dimensional real-valued phasor in the \(R^4\) space represented in the following way:

\[
(x_1, x_2) \iff (\mathcal{R}\{x_1\}, \mathcal{I}\{x_1\}, \mathcal{R}\{x_2\}, \mathcal{I}\{x_2\}).
\]

Hence, the joint design of \((x_1, x_2)\) in \(C^2\) is readily transformed into \(R^4\). Consequently, with the aid of the above-mentioned SP scheme, the joint ST signal design of the individual transmit antennas can be achieved by maximizing the coding advantage of \(G_2\) by maximizing the Euclidean distance of the four-tuples [10]:

\[
(x_l, x_{l+1}, x_{l+2}, x_{l+3}) = F_4(a_{1, l}, a_{1, l+1}, a_{1, l+2}, a_{1, l+3}),
\]

\[
= (a_{1, l} + ja_{2, l}, a_{1, l+1} + ja_{2, l+1}).
\]

Upon choosing \(L\) legitimate constellation points from the lattice \(D_4\) to construct a set denoted by \(A = \{\tilde{a}_l = [a_{1, l}, a_{2, l}, a_{3, l}, a_{4, l}]^T \in \mathbb{R}^4 : 0 \leq l \leq L - 1\}\), the \(L\) resultant energy-normalised codewords given by:

\[
C_l = \sqrt{\frac{2L}{E_{total}}} G_2(F_4(\tilde{a}_l)), \quad l = 0, 1, \ldots, L - 1,
\]

where \(E_{total} = \sum_{l=0}^{L} ([a_{1, l}]^2 + |a_{2, l}|^2 + |a_{3, l}|^2 + |a_{4, l}|^2)\), constitutes the ST signal space \(C_{G2}\), whose diversity product \(\zeta_{G2}\) is determined by the MED of the set \(A\) formulated in Eq.(2).

### III. SYSTEM OVERVIEW

Let us now construct the generalised system equations for an STBC-aided UL MU-MIMO scenario, where the SDMA/OFDM system supports a total of \(U\) UL users and employs \(N\) number of receive antennas at the BS. For the sake of simplicity, the \(G_2\) scheme using two transmit antennas is employed by each user. The overall equivalent MU-MIMO system equation can be derived in complete analogy to the case of STBC-assisted SU-MIMO systems (e.g. [11, 12]) with the aid of the so-called equivalent channel matrix. After straightforward manipulations, under the assumption that the CIR taps between each of the two transmit antennas of a specific user and the \(n\)th receive antenna at the BS remain constant during two consecutive symbol periods, we have:

\[
\tilde{y} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_N \end{bmatrix}_{2N \times 1} = \sum_{u=1}^{U} \tilde{H}^{(u)} \cdot \begin{bmatrix} x^{(u)} \\ x^{(u)} \end{bmatrix}_{2 \times 1} + \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \vdots \\ \tilde{w}_N \end{bmatrix}_{2N \times 1}
\]

\[
= \tilde{H} \cdot x + \tilde{w},
\]

where the overall equivalent channel matrix \(\tilde{H}\) can be expressed as:

\[
\tilde{H} = \begin{bmatrix} \tilde{H}_1^{(1)} & \tilde{H}_2^{(1)} & \cdots & \tilde{H}_N^{(1)} \\ \tilde{H}_1^{(2)} & \tilde{H}_2^{(2)} & \cdots & \tilde{H}_N^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{H}_1^{(U)} & \tilde{H}_2^{(U)} & \cdots & \tilde{H}_N^{(U)} \end{bmatrix},
\]

with each submatrix \(\tilde{H}_n^{(u)}\) being defined as

\[
\tilde{H}_n^{(u)} = \begin{bmatrix} h_{n_1}^{(u)} & h_{n_2}^{(u)} \\ h_{n_3}^{(u)} & h_{n_4}^{(u)} \end{bmatrix},
\]

where \(h_{n_1}^{(u)}\) and \(h_{n_2}^{(u)}\) are the UL CIR coefficients between the first and second transmit antennas of the \(u\)th user to the \(n\)th BS receive antenna, respectively. Additionally, the transmitted symbol vector \(x\) of the entire MU-MIMO system is a column vector created by concatenating each user’s transmitted symbol vector \(x^{(u)} = F_4(a^{(u)}) = [x_1^{(u)} \ x_2^{(u)}]^T\) which is given by:

\[
x = \begin{bmatrix} F_4((\tilde{a}^{(1)})^T) & F_4((\tilde{a}^{(2)})^T) & \cdots & F_4((\tilde{a}^{(U)})^T) \end{bmatrix}^T
\]

\[
= \begin{bmatrix} (\tilde{a}^{(1)})^T & (\tilde{a}^{(2)})^T & \cdots & (\tilde{a}^{(U)})^T \end{bmatrix}^T
\]

\[
= \begin{bmatrix} (x^{(1)})^T & (x^{(2)})^T & \cdots & (x^{(U)})^T \end{bmatrix}^T.
\]

Thus, by defining \(a = [(\tilde{a}^{(1)})^T \ (\tilde{a}^{(2)})^T \ \cdots \ (\tilde{a}^{(U)})^T]^T\), we have:

\[
x = F_4(a).
\]

Moreover, as observed in Eq.(6), the equivalent received noise-contaminated signal vector \(\tilde{y}\) and the equivalent noise vector \(\tilde{w}\) is formed by concatenating the \(N\) number of two-elements sub-vector \(\tilde{y}_n\) and \(\tilde{w}_n\) respectively, which can be written as:

\[
\tilde{y}_n = [y_{1,n} \ y_{2,n}]^T,
\]

\[
\tilde{w}_n = [w_{1,n} \ w_{2,n}]^T,
\]

where the first element \(y_{1,n}\) corresponds to the signal received by the \(n\)th antenna during the first symbol period and the second element \(y_{2,n}\) is the conjugate of the signal received at the same antenna during the second symbol period, while

\[
w_{1,n} = [w_{1,n} \ w_{2,n}]^T,
\]

where again \(w_{1,n}\) denotes the AWGN imposed on the \(n\)th receive antenna during the first symbol period and \(w_{2,n}\) is the conjugate of the AWGN inflicted during the second symbol period. The AWGN encountered during each symbol period has a zero mean and a variance of \(\sigma_w^2\).

According to [11], an orthogonal STBC scheme eliminates the Multi-Stream-Interference (MSI) among the MIMO elements of a specific user, due to the orthogonality of the equivalent channel matrix \(\tilde{H}_n^{(u)}\) in Eq.(9). Therefore, the receiver is capable of performing Maximum-Likelihood (ML) detection based on low-complexity linear processing to achieve full transmit diversity by imposing a negligible extra encoding complexity at the MS in the STBC-SP-assisted SU-MIMO UL scenario [10]. However, in the context of a MU-MIMO system, the resultant overall equivalent channel matrix \(\tilde{H}\) of Eq.(8) is no longer orthogonal, therefore imposing the Multi-User-Interference (MUI). Hence, in order to avoid using the traditional brute-force ML detector, we intend to further develop the \(K\)-best SD to be used at the BS in the STBC-SP-assisted SDMA/OFDM UL scenario, in order to achieve a near-MAP performance at a moderate complexity.

The schematic of the system is depicted in Fig.1, where the transmitted source bits of the \(u\)th user are channel encoded and then interleaved by a random bit-interleaver. The \(B\) interleaved bits \(b^{(u)} = b^{(u)}_{0, \ldots, B-1} \in \{0, 1\}\) are mapped to an SP modulated symbol \(\tilde{a}^{(u)} \in A\) by the SP modulator/mapper of Fig. 1, where \(B = \log_2 L\).
B. Sphere Detection Design for Sphere Packing Modulation

1) Transformation of the ML Metric: The well-known ML solution of Eq.(7) is:

\[
\hat{a}_{ML} = \arg\min_{a \in \mathbb{A}^U} ||\mathbf{y} - \mathbf{\tilde{h}} H F_1(\hat{a})||^2,
\]  

(19)

where \( F_1(\cdot) \) is defined in Eq.(13) in the context of our multi-user system. By comparing the unconstrained least-square (LS) solution of \( \hat{a}_U = F_1^{-1}(\hat{a}_U) = F_1^{-1}(\mathbf{H}^H \mathbf{\tilde{H}}^H \mathbf{\tilde{H}}) \mathbf{y} \) to all legitimate constrained/sliced solution, namely \( \hat{a} \in \mathbb{A}^U \), the ML solution of Eq.(19) can be transformed into:

\[
\hat{a}_{ML} = \arg\min_{a \in \mathbb{A}^U} F_4(\hat{a} - \hat{a}_U) H^H H F_3(\hat{a} - \hat{a}_U),
\]  

(20)

2) Channel Matrix Triangulation: Let us now generate the \((2U \times 2U)\)-dimensional upper-triangular matrix \( \mathbf{U} \), which satisfies \( \mathbf{U}^H \mathbf{U} = \mathbf{H}^H \mathbf{H} \) with the aid of, for example, the ubiquitous Cholesky factorization [8]. Then, upon defining a matrix \( Q \) with elements \( Q_{i,i} = \mathbf{U}_{i,i} \) and \( Q_{i,j} = \mathbf{U}_{i,j} / \mathbf{U}_{i,i} \) we can rewrite Eq.(20) as:

\[
\hat{a}_{ML} = \arg\min_{a \in \mathbb{A}^U} F_4(\hat{a} - \hat{a}_U) H^H \mathbf{U} F_4(\hat{a} - \hat{a}_U),
\]  

\[
= \arg\min_{a \in \mathbb{A}^U} \sum_{u=1}^{U} Q_{2u-1,2u-1} e_1^{(u)} + \sum_{u=1}^{U} Q_{2u-2,2u-2} e_2^{(u)}
\]  

\[
+ \sum_{u=1}^{U} Q_{2u-1,2u-1} e_1^{(u)} + \sum_{u=1}^{U} Q_{2u-2,2u-2} e_2^{(u)},
\]  

(21)

where \( e_1^{(u)} \) is the \( u \)-th two-element sub-vector of the multi-user vector \( e = [(e_1^{(1)})^T, \ldots, (e_1^{(U)})^T, (e_2^{(1)})^T, \ldots, (e_2^{(U)})^T] \), corresponding to the \( u \)-th user, and is given by:

\[
e_1^{(u)} = \mathbf{x}_1^{(u)} - \mathbf{x}_2^{(u)},
\]  

(22)

where \( \mathbf{x}_1^{(u)} = [x_1^{(u)}, x_2^{(u)}]^T = F_4(a_1^{(u)}) \), \( a_1^{(u)} \in A \) and \( \mathbf{x}_2^{(u)} = [x_1^{(u)}, x_2^{(u)}]^T = F_4(a_2^{(u)}) \). Hence, the sum in \( \{ \cdot \} \) is the user-based accumulated Partial Euclidean Distance (PED) between the tentative symbol vector \( \mathbf{x} = [(x_1^{(1)})^T, (x_2^{(1)})^T, \ldots, (x_1^{(U)})^T, (x_2^{(U)})^T] \) and the search center \( \mathbf{x}_1^{(u)} = [(x_1^{(1)})^T, (x_2^{(1)})^T, \ldots, (x_1^{(U)})^T, \ldots, (x_1^{(u)})^T] \).

3) User-Based Tree Search: Let us now consider the tree search carried out by the K-best SD for conventional modulation schemes, such as BPSK, where each tree level represents an independent data stream corresponding to a certain transmit antenna of a specific user [9], whilst each tree node corresponds to a legitimate complex-valued BPSK symbol \( 2 \) in the constellation of domain \( \mathbb{C} \). Consequently, in the absence of joint ST signal design for the \( M_u = 2 \) transmit antennas, the BPSK constellation of the two adjacent tree levels corresponding to a specific user are independent and identical. However, when the joint ST signals are transmitted from the \( M_u = 2 \) transmit antennas of the \( u \)-th user, they are combined into a joint ST design with the aid of the SP scheme as discussed in Section II-A. The corresponding SP-symbol based search tree structure is depicted in Fig. 2, where the search tree of the modified K-best SD is exemplified in the context of an UL SDMA system supporting \( U = 2 \) G_2-SP-aided users, where \( K = 2 \) and a 4-point STBC-SP data streams of a specific user should be considered together.

\( ^2\)In this treatise, we consider complex-valued BPSK symbols having zero imaginary parts.
in the tree search process, resulting in multi-dimensional/multi-layer tree nodes in the $\mathbb{C}^2$ SP-symbol domain, which we refer to as user-based tree search. The resultant 2-D complex-valued tree node is constituted of two complex-valued BPSK symbols, which are the constituent components of a transformed SP symbol $F_k(\hat{\theta})$. On the other hand, due to the joint consideration of the two adjacent BPSK tree levels, the number of effective search tree levels is reduced by a factor of two, whilst each symbols becomes quaternary, instead of being binary.

As observed in Fig. 2, since a 4-SP scheme is employed and the number of candidate tree nodes retained at each tree level is $K = 2$, each of the two selected tree nodes having the smallest two accumulated PED values at the previous search-tree level of the survivor path has to be expanded into four child nodes at the current level. Consequently, in analogy to the conventional $K$-best algorithm [9], both the calculation of the user-based accumulated PEDs as well as the tree pruning process continues in the downward direction of Fig. 2 all the way along the tree, until it reaches the tree-leaf level, producing a candidate list of $L_{cand} \in A^K$. This list contains $N_{cand} = K$ number of SP symbol candidate solutions, which are then used for the extrinsic bit LLR calculation using Eq.(18). Having a reduced candidate list size assists us in achieving a substantial complexity reduction. Explicitly, after the max-log-approximation, we arrive at:

\[
L_E(b_k | \tilde{y}) = \max_{\mathbf{a} \in L_{cand} \mathcal{A}^K} \left[ -\frac{1}{2\sigma_\mathbf{a}^2} ||\tilde{y} - \hat{\mathbf{H}} \cdot F_k(\mathbf{a})||^2 + \sum_{j=1,j\neq k}^{B/4} b_j L_A(b_j) \right] - \max_{\mathbf{0} \in L_{cand} \mathcal{A}^K} \left[ -\frac{1}{2\sigma_\mathbf{a}^2} ||\tilde{y} - \hat{\mathbf{H}} \cdot F_k(\mathbf{a})||^2 + \sum_{j=1,j\neq k}^{B/4} b_j L_A(b_j) \right].
\]

(23)

The $K$-best SD algorithm designed for $N_D = 4$-dimensional SP modulation scheme is summarized in the Appendix.

![Fig. 2. The Search Tree of $K$-Best SD in the Scenario of an STBC-SP Aided Uplink SDMA System: the number of users is $U = 2$, the number of transmit antennas per user is $M_u = 2$ and the number of candidates retained at each search tree level is $K = 2$.](image)

**V. RESULTS AND DISCUSSION**

For the sake of investigating the performance of the STBC-SP-assisted multi-user SDMA/OFDM UL system, we compare the SP-modulated system with its conventionally-modulated counterpart in the two scenarios using the system parameters summarised in Table I. Fig. 3(a) and 3(b) depict, respectively, the corresponding EXIT charts [15] used as a convenient visualization technique for analyzing the convergence characteristics of turbo receivers. This technique computes the mutual information (MI) of the output extrinsic and input a priori components, which are denoted by $I_E$ and $I_A$ respectively, corresponding to the associated bits for each of the iterative SISO blocks of Fig. 1, namely, to the SD and the RSC(2,1,3).

![Fig. 3. EXIT charts of STBC-SP-aided iterative receiver of Fig. 1 employing the modified $K$-best SD and the parameters of Table I. The overall system throughput is 8 bits/symbol.](image)

**TABLE I SUMMARY OF SYSTEM PARAMETERS**

<table>
<thead>
<tr>
<th>Modulation</th>
<th>Scenario I</th>
<th>Scenario II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4-QAM/16-SP</td>
<td>16-QAM/256-SP</td>
</tr>
<tr>
<td>Users Supported</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>System</td>
<td>SDMA/OFDM</td>
<td>G2</td>
</tr>
<tr>
<td>Sub-Carriers</td>
<td>1024</td>
<td>4</td>
</tr>
<tr>
<td>STBC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rx at BS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CIR Model</td>
<td>$P(\tau_b) = [0.5, 0.3, 0.2]$</td>
<td>$K$-Best List-SD</td>
</tr>
<tr>
<td>Detector/MAP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>List Length</td>
<td>$N_{cand} = K$</td>
<td></td>
</tr>
<tr>
<td>Channel Code</td>
<td>Half-Rate RSC(2,1,3)</td>
<td>(5/7)</td>
</tr>
<tr>
<td>BW Efficiency</td>
<td>4 bits/sec/Hz</td>
<td></td>
</tr>
</tbody>
</table>

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on the Grammian matrix $G$ and the antennas supported by the system.

For example, a SP-modulated system over its conventionally-modulated identical-counterpart in MU-MIMO scenarios. For example, a SP scheme to the MU-MIMO scenario, while achieving a near-MAP

Fig. 4. BER performance of the system of Fig. 1 in Scenario I and Scenario II of Table I. The overall system throughput is $8$ bits/symbol.

SP scheme to the MU-MIMO scenario, while achieving a near-MAP performance at a moderate complexity. Consequently, with the aid of our $K$-best SD, a significant performance gain can be attained by the SP-modulated system over its conventionally-modulated identical-throughput counterpart in MU-MIMO scenarios. For example, a performance gain of $3.5$ dB can be achieved over a 16-QAM benchmarker by the 256-SP scheme in the scenario of a four-receive-antenna SDMA UL system supporting $U = 2G_2$-assisted users, given a target BER of $10^{-4}$.

**APPENDIX**

**K-BEST SD ALGORITHM FOR $N_D = 4$-DIMENSIONAL SP**

The preprocessing phase:

1) Obtain the upper-triangular matrix $U$ via Cholesky factorization on the Grammian matrix $G = \bar{H}^T \bar{H}$, namely, we have $U = \text{Chol}(G)$.

2) Calculate the search center $\hat{x}_{ls}$ by:

$$\hat{x}_{ls} = G^{-1} \bar{H}^T y.$$  \hspace{1cm} (24)

The tree search phase:

The first stage:

1) $m = M, d_M = \hat{x}_{lsM}$, where $M$ is the total number of transmit antennas supported by the system.

2) Calculate the corresponding PED for each SP symbol $(\hat{x}_{1,1}, \hat{x}_{1,2})$, $l = 1, 2, \ldots, L$ in the constellation of $C^2$ domain as follows:

$$e_M = \hat{x}_{lsM} - \hat{x}_{l1,1},$$  \hspace{1cm} (25)

$$d_{M-1} = \hat{x}_{lsM-1} + \frac{U_{M-1,M-1}}{U_{M-1,M-1}} e_M,$$  \hspace{1cm} (26)

$$PED = U_{M-1,M-1}^2 (d_{M-1} - \hat{x}_{l1,2}).$$  \hspace{1cm} (27)

3) Choose $K$ number of SP symbols $(\hat{x}_{k,1}, \hat{x}_{k,2})$, $k = 1, 2, \ldots, K$ that have the $K$ smallest PEDs.

4) For each chosen SP symbol, compute

$$e_{M-1} = \hat{x}_{lsM-1} - \hat{x}_{k,2},$$  \hspace{1cm} (28)

$$d_{M-2} = \hat{x}_{lsM-1} + \frac{U_{M-2,M-1}}{U_{M-2,M-2}} e_{M-1} + \frac{U_{M-1,M-1}}{U_{M-1,M-1}} e_M$$  \hspace{1cm} (29)

The $m$th stage:

1) $m = m - 2$.

2) For each survived search tree path from the previous tree level, calculate the corresponding PED for each SP symbol $(\hat{x}_{l1,1}, \hat{x}_{l1,2})$ in the constellation of $C^2$ domain:

$$e_m = \hat{x}_{lsM} - \hat{x}_{l1,1},$$  \hspace{1cm} (30)

$$d_{m-1} = \hat{x}_{lsM} - \hat{x}_{l1,1} + \sum_{j=m}^{M} \frac{U_{m-1,j}}{U_{m-1,m-1}} e_j,$$  \hspace{1cm} (31)

$$PED = U_{m-1,m-1}^2 (d_{m-1} - \hat{x}_{l1,2}).$$  \hspace{1cm} (32)

3) Choose $K$ number of SP symbols $(\hat{x}_{k,1}, \hat{x}_{k,2})$, $k = 1, 2, \ldots, K$ that have the $K$ smallest PEDs.

4) For each chosen SP symbol, compute

$$e_{m-1} = \hat{x}_{lsM} - \hat{x}_{k,2},$$  \hspace{1cm} (33)

$$d_{m-2} = \hat{x}_{lsM} - \hat{x}_{k,2} + \frac{U_{m-2,m-1}}{U_{m-2,m-2}} e_{m-1} + \frac{U_{m-1,m-1}}{U_{m-1,m-1}} e_m$$  \hspace{1cm} (34)

5) If $m - 1 = 1$, obtain the solution by backtracing from the tree leaf having the largest accumulated PED to the tree root. Otherwise, go to step 1 of the $m$th stage.

**REFERENCES**


