

Generic z-domain discrete-time transfer function estimation for ultra-wideband systems

R.A. Riaz, M.F.U. Butt, S. Chen and L. Hanzo

Generic z-domain discrete-time transfer function estimation is proposed for ultra-wideband channels, which requires no channel sounding sequence transmission and hence constitutes a blind technique. This is achieved by estimating the channel impulse response with the aid of the information signalling pulses and then equalising the effects of the channel by the corresponding inverse system.

Introduction: Ultra-wideband (UWB) communication systems are characterised by distinct clusters of dense multipath faded received components. In [1], channel estimation techniques were proposed for assisting the receiver in recovering the transmitted UWB signal and hypothesising that an inverse system may be found for signal recovery. Hence the novel contribution of this Letter is that this complete inverse system is indeed found and the corresponding recursive equation is provided for channel estimation. The system does not require any extra channel sounding sequence for the recovery of the received channel-contaminated UWB pulses and so avoids any channel-sounding-related throughput reduction and therefore may be viewed as a blind technique.

UWB channel model: The statistical channel model specified by the IEEE 802.15 standard is employed for link-budget design. The corresponding wide sense stationary uncorrelated scattering (WSSUS) based channel impulse response (CIR) exhibiting clusters of similar delay paths may be modelled as [2]:

$$h_j(t) = \Theta_j \sum_{l=1}^L \sum_{w=1}^W \beta_{w,l}^j \delta(t - T_l^j - \tau_{w,l}^j) \quad (1)$$

where $h_j(t)$ is the j th CIR realisation, $\beta_{w,l}$ is the w th ray-magnitude of the l th CIR-tap-cluster, while T_l is the delay of the l th cluster. Moreover, $\tau_{w,l}$ is the delay of the w th ray relative to the l th cluster's arrival time T_l . Finally, Θ_j represents the magnitudes of the corresponding cluster components. When assuming perfect symbol synchronisation, (1) may be simplified to:

$$h(t) = \sum_{l=1}^L \beta_l \delta(t - \tau_l) \quad (2)$$

because as stated in [1], perfect symbol synchronisation removes the effects of cluster-overlapping. Furthermore, it was validated by Kolmogorov-Smirnov distribution-fitting to measurements at a significance level of 1% that the fading amplitudes can be modelled as Nakagami- m distributed [2].

UWB channel estimation: The UWB pulse transmission regime based upon prolate spheroidal wave functions (PSWFs) [3] is employed for transmitting the UWB-PSWF pulse $\phi_p(t)$ of width T , $t \in [\tau, \tau + T]$ over the UWB channel characterised by (2). The received signal $r_p(t)$ is given by

$$r_p(t) = \phi_p(t) * h(t) + n(t) \quad (3)$$

where $*$ represents the convolution operation, and $n(t)$ is the additive white Gaussian noise (AWGN) having a zero mean and a double-sided power spectral density of $N(s)$. Moreover, when using the orthonormal basis of the Paley-Wiener space [4] for PSWFs, (2) can be expressed as

$$h(t) = \sum_{n=0}^{\infty} a_n \phi_n(t) \quad (4)$$

The s -transform of (4) becomes

$$H(s) = \sum_{q=0}^{\infty} a_q \xi_q \Phi_q(s) \quad (5)$$

where ξ_q represents the specific fraction of energy in the interval $q = 0 \dots \infty$. Taking the s -transform of (2) and equating it with (5) we obtain the value for β_l with UWB-PSWF pulse duration and energy T

and E , respectively [1]:

$$\beta_l = \sum_{m=0}^{\infty} \sqrt{\frac{E \xi_m}{T}} \xi_m \phi_m(\tau_l) \quad (6)$$

Substituting (6) into the s -transformed version of (2) yields

$$H(s) = \sum_{l=1}^L \sum_{m=0}^{\infty} \sqrt{\frac{E \xi_m}{T}} \xi_m \phi_m(\tau_l) e^{-s \tau_l} \quad (7)$$

The Tustin transformation (TT) $\{z \rightarrow e^{sT_d}\}$ is a conformal mapping employing the sampling time T_d , in order to convert the s -domain channel transfer function (CHTF) $\{H(s)\}$ to the z -domain $\{H(z)\}$. Using the following approximation of the TT

$$s = \frac{1}{T_d} \ln(z) = \frac{2}{T_d} \left[\frac{z-1}{z+1} + \frac{1}{3} \left(\frac{z-1}{z+1} \right)^3 + \dots \right] \quad (8)$$

$$\simeq \frac{2}{T_d} \frac{1-z^{-1}}{1+z^{-1}}$$

and substituting (8) into (7) to convert $H(s)$ to $H(z)$ yields

$$H(z) = H(s) \Big|_{s=\frac{2}{T_d} \frac{1-z^{-1}}{1+z^{-1}}} \quad (9)$$

$$= \sum_{l=1}^L \sum_{m=0}^{\infty} \sqrt{\frac{E \xi_m}{T}} \xi_m \phi_m(\tau_l) e^{-\frac{2\tau_l}{T_d} \frac{1-z^{-1}}{1+z^{-1}}}$$

When applying the power series expansion of

$$\frac{e^{-\frac{2\tau_l}{T_d} \frac{1-z^{-1}}{1+z^{-1}}}}{n} = \sum_{n=0}^{\infty} \frac{\left(-\frac{2\tau_l}{T_d} \frac{1-z^{-1}}{1+z^{-1}} \right)^n}{n} \simeq 1 - \frac{2\tau_l}{T_d} \frac{1-z^{-1}}{1+z^{-1}} \quad (10)$$

and exploiting that τ_l is in the range of nanoseconds, the higher-order components of the power series in (10) can be neglected. Since τ_l is in range of nanoseconds for different range channel models as explained in [2], (10) holds for any value of τ_l between 3–7 ns. Then substituting (10) into (9) yields a general expression for the UWB-channel's z -domain discrete-time transfer function (DTTF) having L number of CIR-tap clusters as follows:

$$H(z) = \sum_{l=1}^L \sum_{m=0}^{\infty} \sqrt{\frac{E \xi_m}{T}} \xi_m \phi_m(\tau_l) \left(1 - \frac{2\tau_l}{T_d} \frac{1-z^{-1}}{1+z^{-1}} \right) \quad (11)$$

where ξ_m is a statistically independent positive random variable having a Nakagami- m probability density function (PDF) with unity fading parameter m for rays within a cluster given by

$$p(\xi_m) = \left(\frac{2\xi_m}{\xi_m^2} \right) e^{-\frac{2\xi_m}{\xi_m^2}} \quad (12)$$

Application example: We demonstrate the accuracy of our model using an $L = 2$ -cluster scenario, where each cluster obey a Nakagami- m distribution with fading parameter of $m = 1$ for ξ_m in (12). After a few algebraic steps we arrive from (11) at the DTTF:

$$H_{(L=2,m)}^*(z) = \Psi^\dagger \left(\frac{1-z^{-1}}{1+z^{-1}} \right) \quad (13)$$

where

$$\Psi^\dagger = \left[\left(\sqrt{\frac{E \xi_m}{T}} \xi_m \right) \{ (\phi_m(\tau_1) + \phi_m(\tau_2)) - \left(\frac{2}{T_d} (\tau_1 \phi_m(\tau_1) + \tau_2 \phi_m(\tau_2)) \right) \} \right] \quad (14)$$

Upon taking the inverse z -transform of (13) over the stable region of convergence, the resultant recursive equation can be expressed as

$$h_{(L=2,m)}^*(n) = \Psi^\dagger 1^n u[-n-1] - \Psi^\dagger 1^n u[-n-1] \delta[n-1] \quad (15)$$

Finally, upon substituting Ψ^\dagger from (14) into (15), the resultant two-cluster Nakagami- m distributed CIR model becomes:

$$\begin{aligned}
 h_{(L=2,m)}^*(n) = & \left[\left(\sqrt{\frac{E\xi_m}{T}} \xi_m \right) \{(\phi_m(\tau_1) + \phi_m(\tau_2)) \right. \\
 & \left. - \left(\frac{2}{T_d} (\tau_{1\phi_m}(\tau_1) + \tau_{2\phi_m}(\tau_2)) \right) \right] 1^n u[-n-1] \\
 & - \left[\left(\sqrt{\frac{E\xi_m}{T}} \xi_m \right) \{(\phi_m(\tau_1) + \phi_m(\tau_2)) \right. \\
 & \left. - \left(\frac{2}{T_d} (\tau_{1\phi_m}(\tau_1) + \tau_{2\phi_m}(\tau_2)) \right) \right] 1^n u[-n-1] \delta[n-1]
 \end{aligned} \tag{16}$$

The CIR of (13) was restricted to retain only those multipath components, which are less than 12 dB below the maximum multipath component. The UWB-PSWF pulse duration was 0.15 ns, which was transmitted every 120 ns and the channel's inverse system was estimated for each new pulse. Finally, a threshold-detector is used to recover the transmitted data. The BER against signal-to-noise ratio (SNR) performance is shown in Fig. 1, demonstrating that using $W = 15$ taps in the proposed model approaches the full complexity performance-evaluation curve of (19) in [1].

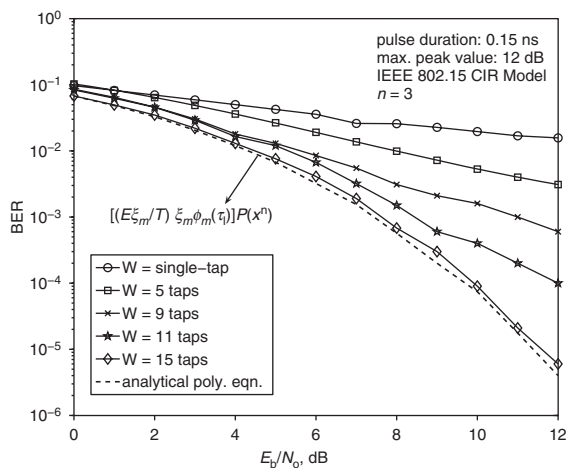


Fig. 1 Accuracy of proposed z -domain DTF estimation of (16) for $L = 2$ and $W = 1, 5, 9, 11, 15$ CIR taps

Conclusion: The generic z -domain DTF estimation proposed for UWB systems facilitates accurate performance evaluation and requires no channel sounding sequence.

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