by $\Phi_{\Lambda\dot{\Lambda}}(\omega, \dot{\omega})$, can be expressed as

$$\Phi_{\Lambda\dot{\Lambda}}(\omega, \dot{\omega}) = \prod_{m=1}^{M_T M_R} \Phi_{\zeta^2\dot{\zeta}^2}(\omega, \dot{\omega}) \tag{32}$$

where $\Phi_{\zeta^2\dot{\zeta}^2}(\omega, \dot{\omega})$ is given by (8). Hence, we obtain

$$\Phi_{\Lambda\dot{\Lambda}}(\omega, \dot{\omega}) = \frac{1}{(1 + 2\beta\omega^2 - j\dot{\omega})^{M_T M_R}}. \tag{33}$$

Using the inversion formula of the 2-D Fourier transforms, it follows that the joint PDF $p_{\Lambda\dot{\Lambda}}(z, \dot{z})$ of $\Lambda(t)$ and $\dot{\Lambda}(t)$ can be expressed as

$$p_{\Lambda\dot{\Lambda}}(z, \dot{z}) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{\Lambda\dot{\Lambda}}(\omega, \dot{\omega}) e^{-j(\omega z + \dot{\omega}\dot{z})} d\omega d\dot{\omega} \tag{34}$$

for $z \geq 0$ and $|\dot{z}| < \infty$. By substituting (33) in (34), we obtain, after some lengthy algebraic computations, the following expression:

$$p_{\Lambda\dot{\Lambda}}(z, \dot{z}) = \frac{z^{M_T M_R - 1} e^{-z - \dot{z}^2/(8\beta z)}}{2\Gamma(M_T M_R)\sqrt{2\pi\beta z}}, \qquad z \geq 0, \quad |\dot{z}| < \infty. \tag{35}$$

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SVD-Assisted Multiuser Transmitter and Multiuser Detector Design for MIMO Systems

W. Liu, L. L. Yang, and L. Hanzo

***Abstract*—A novel singular value decomposition (SVD)-based joint multiuser transmitter (MUT) and multiuser detector (MUD) aided multiple-input–multiple-output (MIMO) system is proposed, which takes advantage of the channel state information (CSI) of all users at the base station (BS), but only of the mobile station (MS)'s own CSI, to decompose the multiuser (MU) MIMO channels into parallel single-input–single-output (SISO) channels, where each SISO channel corresponds to the singular values of a particular MS's channel matrix. Based on the proposed scheme, the SVD-based transmission carried out in the context of a single user can readily be extended to the MU case for both the uplink (UL) and downlink (DL). As a beneficial application of the proposed scheme, we improve the system's achievable throughput and highlight its future applications.**

***Index Terms*—Multiple-input multiple-output (MIMO), postprocessing, preprocessing, singular value decomposition (SVD), space-division multiple access (SDMA), zero forcing (ZF).**

I. Introduction

In multiple-input–multiple-output (MIMO)-aided multiuser systems, both the uplink (UL) and downlink (DL) transmissions experience multiuser interference (MUI), also referred to as multiple access interference (MAI), as well as interantenna interference (IAI). The optimum maximum-likelihood (ML) receiver employed at the mobile station (MS) often imposes excessive computational complexity. To reduce the complexity of the MS, multiuser transmission (MUT) techniques can be invoked at the base station (BS) [1]–[5]. Widely used linear preprocessing techniques, such as the minimum mean square error (MMSE) and the zero-forcing (ZF) MUT arrangements, were

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detailed in [1] and [4]. However, the MMSE and ZF techniques only exploit the channel state information (CSI) at the BS. By contrast, in [2] and [3], the effective channel constituted by the transmitter, receiver, and the propagation channel of each user was determined by invoking the so-called block diagonalization technique at the BS, which removed the MUI. However, this scheme is only applicable for the DL transmission.

We note that the optimum ML receiver may be excessively complex for employment even in the UL at the BS. By contrast, the traditional MMSE or ZF UL receiver [6] is unable to take advantage of the CSI at each MS. Furthermore, in [7], an MMSE-based criterion was used for designing both the DL transmit preprocessing and DL receiver postprocessing matrices, where close cooperation of the MSs was required. As another design alternative, in [8], the so-called maximum ratio [9] UL transmission scheme was investigated, where not all, but only the dominant right-hand-side (RHS) and left-hand-side (LHS) singular eigenvectors were adopted as the preprocessing and postprocessing eigenvectors, hence increasing the achievable diversity gain at the cost of reducing the multiplexing gain.

It has been shown in [10] that when accurate and prompt CSI is available at both the transmitter and receiver, singular value decomposition (SVD)-based adaptive modulation (AM) techniques applied in the context of MIMO systems are capable of achieving a high average spectral efficiency (ASE). Moreover, both SVD-assisted space–time block coding (STBC)-based transmit diversity schemes and vertical Bell Laboratories layered space–time architecture (V-BLAST)-type spatial multiplexing arrangements have found numerous applications [11], [12]. However, these proposals were based on point-to-point communications. In the context of multiusers, SVD-based multiuser detection (MUD) was discussed in [13] and [14], when only the largest eigenvalue was invoked for the UL transmission, whereas in [15], multiple eigenvalues were invoked for the DL transmission, but only the IAI of the same user was cancelled with the aid of joint preprocessing and postprocessing.

In this paper, both SVD-based space-division multiple-access (SDMA) MUDs designed for UL reception and DL MUT are investigated. When using combined SVD-based preprocessing and postprocessing and assuming that the channel impulse responses (CIRs) of all users are perfectly known both at the MUT and MUD at the instant of transmission and reception, respectively, then the effect of both the MAI and IAI can perfectly be eliminated in both the UL and DL, since all signal links are uniquely and unambiguously identified by their CIRs. The proposed algorithm facilitates the employment of AM in the context of MIMO-aided multiusers and allows the extension of SVD-assisted STBC and V-BLAST to multiuser scenarios.

Against this background, the novel contributions of this paper are given in the list that follows.

1) Compared to traditional ZF or MMSE MUT and MUD techniques, the CSI is exploited at both the BS and MS.
2) Both the UL and DL processing can be constructed in the framework of the same structure.
3) The proposed SVD-aided SDMA MUT and MUD principles are sufficiently general to ensure that similar SVD-assisted closed-loop transmit diversity and BLAST-type transmit multiplexing schemes may also readily be created for multiuser scenarios.

This paper is structured as follows: In Section II, SVD-based joint preprocessing and postprocessing designed for MIMO-aided SDMA MUD in the UL is discussed. In Section III, SVD-based joint preprocessing and postprocessing conceived for the MIMO-assisted SDMA multiuser DL transmission is investigated. In Section IV, our simulation results are provided. Finally, our conclusions are offered in Section V.

II. SVD-Based UL Transmission and Detection

In this section, we consider both the UL transmission and detection in a multiuser MIMO system, where the BS supports multiple MSs. Although the extension of these principles to other types of MIMO systems is straightforward, the multiuser MIMO system considered here is in fact an SDMA system, where both the BS and MSs may employ multiple antennas both for reception and transmission. In our study, we assume that the BS is capable of acquiring the UL CIRs of all the UL users. By contrast, an MS is only capable of acquiring an estimate of the UL CIR of itself for its own future instant of transmission. Furthermore, we assume that there is no cooperation among the UL users.

The schematic of the UL multiuser MIMO system considered in this paper is shown in Fig. 1, where the BS employs M receive antennas, and the kth $(k = 1, 2, \ldots, K)$ MS uses N_k transmit antennas. In Fig. 1, $\boldsymbol{Q}_k$ $(k = 1, 2, \ldots, K)$ represents the UL MS transmitter preprocessing matrix formulated for the transmission of the kth MS's data $\boldsymbol{x}_k$. In Fig. 1, $\boldsymbol{T}_k$ $(k = 1, 2, \ldots, K)$ represents the receiver's postprocessing matrix formulated for detecting the UL data transmitted by the kth MS.

Let the N_k UL data symbols to be transmitted by the kth MS to the BS be hosted by a vector expressed as $\boldsymbol{x}_k = [x_{k1}, x_{k2}, \ldots, x_{kN_k}]^T$, $k = 1, 2, \ldots, K$ [7]. As shown in Fig. 1, $\boldsymbol{x}_k$ is preprocessed using the kth UL MS transmitter preprocessing matrix $\boldsymbol{Q}_k$, yielding the output[1] [7]

$$\boldsymbol{d}_k = \boldsymbol{Q}_k \boldsymbol{x}_k, \qquad k = 1, 2, \ldots, K. \tag{1}$$

Let the CIR matrix connecting the N_k UL transmit antennas of the kth MS with the M UL receive antennas at the BS be expressed as

$$\boldsymbol{H}_k = \begin{bmatrix} h_{11}^{(k)} & h_{12}^{(k)} & \cdots & h_{1N_k}^{(k)} \\ h_{21}^{(k)} & h_{22}^{(k)} & \cdots & h_{2N_k}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M1}^{(k)} & h_{M2}^{(k)} & \cdots & h_{MN_k}^{(k)} \end{bmatrix}, \qquad k = 1, 2, \ldots, K \tag{2}$$

which is an $(M \times N_k)$-component matrix. Then, the received length-M UL observation vector $\boldsymbol{y}$ at the BS can be expressed as [7]

$$\boldsymbol{y} = \sum_{k=1}^{K} \boldsymbol{H}_k \boldsymbol{d}_k + \boldsymbol{n} = \sum_{k=1}^{K} \boldsymbol{H}_k \boldsymbol{Q}_k \boldsymbol{x}_k + \boldsymbol{n} \tag{3}$$

where $\boldsymbol{n}$ is a length-M noise observation vector, which is assumed to be Gaussian distributed with zero mean and a covariance matrix given by $\sigma^2 \boldsymbol{I}_M$.

As shown in Fig. 1, at the BS's UL receiver, the kth MS's transmitted UL data are recovered by processing the observation vector $\boldsymbol{y}$ using an $(N_k \times M)$-component weight matrix $\boldsymbol{T}_k$, which can be expressed as

$$\hat{\boldsymbol{x}}_k = \boldsymbol{T}_k \boldsymbol{y}, \qquad k = 1, 2, \ldots, K. \tag{4}$$

Let us collect all the data estimates of the K UL users into a single vector $\hat{\boldsymbol{x}}$ as

$$\hat{\boldsymbol{x}} = \left[\hat{\boldsymbol{x}}_1^T, \hat{\boldsymbol{x}}_2^T, \ldots, \hat{\boldsymbol{x}}_k^T\right]^T = \boldsymbol{T}\boldsymbol{y} \tag{5}$$

[1]Note that, for the sake of simplifying our notation, in this paper, the variables without overbars are either related to the UL or are common for both the UL and DL, whereas the variables having an overbar specifically denote the DL.

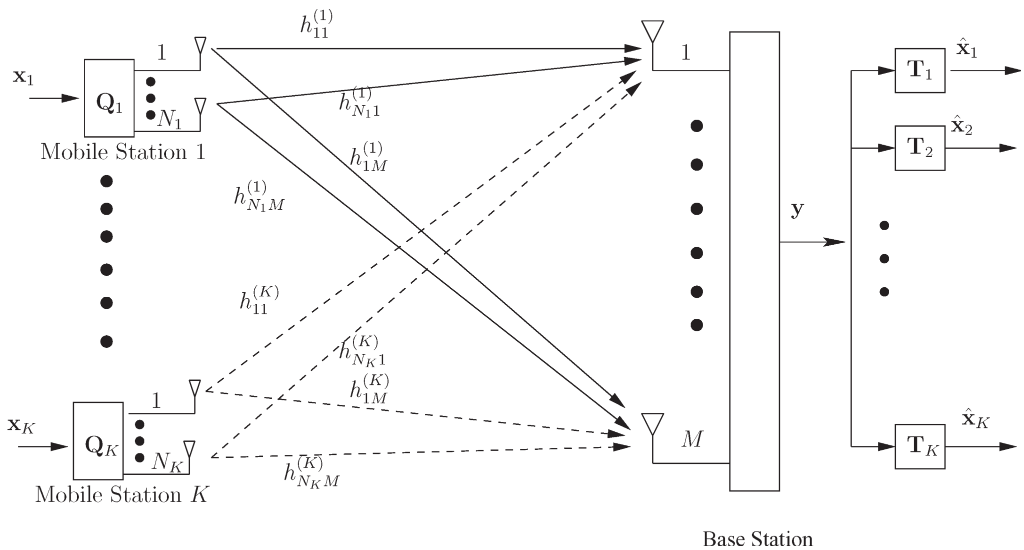

Fig. 1. Schematic of a UL multiuser MIMO system, where the BS employs M receive antennas, whereas the MSs may employ different numbers of transmit antennas.

where the overall $(\sum_{k=1}^{K} N_k \times M)$-component weight matrix is given by $\boldsymbol{T} = [\boldsymbol{T}_1^T, \boldsymbol{T}_2^T, \ldots, \boldsymbol{T}_K^T]^T$.

In our derivation, we assume that we have $M \geq \sum_{k=1}^{K} N_k$, which physically means that the number of antennas at the BS is equal to or higher than the sum of all antennas of all the K MSs. Let us assume that $\boldsymbol{H}_k$ of (2) satisfies rank $(\boldsymbol{H}_k) = N_k$. Then, the SVD of $\boldsymbol{H}_k$ can be expressed as

$$\boldsymbol{H}_k = \boldsymbol{U}_k \begin{bmatrix} \boldsymbol{\Lambda}_k^{1/2} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{V}_k^H = [\boldsymbol{U}_{ks} \quad \boldsymbol{U}_{kn}] \begin{bmatrix} \boldsymbol{\Lambda}_k^{1/2} \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{V}_k^H = \boldsymbol{U}_{ks} \boldsymbol{\Lambda}_k^{1/2} \boldsymbol{V}_k^H, \quad k = 1, 2, \ldots, K \tag{6}$$

where $\boldsymbol{U}_k$ and $\boldsymbol{V}_k$ are $(M \times M)$- and $(N_k \times N_k)$-component unitary matrices, respectively, whereas $\boldsymbol{\Lambda}_k = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{N_k}\}$ contains the N_k nonzero eigenvalues of $\boldsymbol{H}_k^H \boldsymbol{H}_k$ or $\boldsymbol{H}_k \boldsymbol{H}_k^H$. Furthermore, in (6), the columns of $\boldsymbol{U}_k$ are constituted by the eigenvectors of $\boldsymbol{H}_k \boldsymbol{H}_k^H$, $\boldsymbol{U}_{ks}$ consists of the N_k eigenvectors corresponding to the signal subspace of $\boldsymbol{H}_k \boldsymbol{H}_k^H$, whereas $\boldsymbol{U}_{kn}$ consists of the $(M - N_k)$ eigenvectors corresponding to the null subspace of $\boldsymbol{H}_k \boldsymbol{H}_k^H$. Similarly, the columns of $\boldsymbol{V}_k$ correspond to the eigenvectors of $\boldsymbol{H}_k^H \boldsymbol{H}_k$.

Upon substituting (6) into (3), the vector $\boldsymbol{y}$ of the UL received signal of Fig. 1 can be expressed as

$$\boldsymbol{y} = \sum_{k=1}^{K} \boldsymbol{U}_{ks} \boldsymbol{\Lambda}_k^{1/2} \boldsymbol{V}_k^H \boldsymbol{Q}_k \boldsymbol{x}_k + \boldsymbol{n} \tag{7}$$

where the channel matrix $\boldsymbol{H}_k$ of the kth user is replaced by its SVD. Let the transmitter preprocessing matrix $\boldsymbol{Q}_k$ of Fig. 1 be formulated as

$$\boldsymbol{Q}_k = \boldsymbol{V}_k, \qquad k = 1, 2, \ldots, K. \tag{8}$$

After substituting (8) into (7) and exploiting the property $\boldsymbol{V}_k^H \boldsymbol{V}_k = \boldsymbol{I}_{N_k}$, the vector $\boldsymbol{y}$ of the UL received signal shown in Fig. 1 can be simplified to

$$\boldsymbol{y} = \sum_{k=1}^{K} \boldsymbol{U}_{ks} \boldsymbol{\Lambda}_k^{1/2} \boldsymbol{x}_k + \boldsymbol{n} \tag{9}$$

where the RHS singular vectors of the channel matrix $\boldsymbol{H}_k$ of the kth UL transmitter has been cancelled out by the corresponding UL preprocessing matrix $\boldsymbol{Q}_k$ of Fig. 1 at the kth UL MS transmitter.

Equation (9) shows that the UL transmit preprocessing matrix $\boldsymbol{Q}_k$ of (8) decouples each of the antenna-specific transmitted data symbols of the kth MS from those of its other antennas.

Let us define

$$\boldsymbol{U}_s = [\boldsymbol{U}_{1s}, \boldsymbol{U}_{2s}, \ldots, \boldsymbol{U}_{Ks}]$$
$$\boldsymbol{\Lambda}^{1/2} = \text{diag}\left\{\boldsymbol{\Lambda}_1^{1/2}, \boldsymbol{\Lambda}_2^{1/2}, \ldots, \boldsymbol{\Lambda}_k^{1/2}\right\}. \tag{10}$$

Then, the received UL signal vector $\boldsymbol{y}$ of Fig. 1 can be expressed as

$$\boldsymbol{y} = \boldsymbol{U}_s \boldsymbol{\Lambda}^{1/2} \boldsymbol{x} + \boldsymbol{n}. \tag{11}$$

Note that although the columns of $\boldsymbol{U}_{ks}$ $(k = 1, 2, \ldots, K)$ are orthogonal, suggesting that there is no IAI, the columns of $\boldsymbol{U}_s$ in (11) corresponding to the different UL MS transmitters are nonorthogonal. Therefore, there is MAI, which should be cancelled by the BS's receiver.

Upon substituting (11) into (5), we arrive at

$$\hat{\boldsymbol{x}} = \boldsymbol{T} \boldsymbol{U}_s \boldsymbol{\Lambda}^{1/2} \boldsymbol{x} + \boldsymbol{T} \boldsymbol{n}. \tag{12}$$

It can be shown that there are many alternatives for the design of the BS's UL receiver postprocessing matrix $\boldsymbol{T}$, as discussed in [6]. As an example, in this paper, we focus our attention on the ZF UL MUD scheme, which is a linear detector and is capable of entirely eliminating the MAI, although at the cost of potential noise enhancement.

The ZF UL MUD solution encapsulated in $\boldsymbol{T}$ can readily be derived in the context of [6]

$$\boldsymbol{T} = [\boldsymbol{U}_s]^+ = \left(\boldsymbol{U}_s^H \boldsymbol{U}_s\right)^{-1} \boldsymbol{U}_s^H \tag{13}$$

where $[\cdot]^+$ denotes the pseudoinverse of the matrix $\boldsymbol{U}_s$. Upon substituting (13) into (12), we arrive at

$$\hat{\boldsymbol{x}} = \boldsymbol{\Lambda}^{1/2} \boldsymbol{x} + \boldsymbol{n}'. \tag{14}$$

Explicitly, the MAI is entirely removed. In (14), the noise term $\boldsymbol{n}' = \boldsymbol{T}\boldsymbol{n}$ still represents a Gaussian noise vector with zero mean, but its

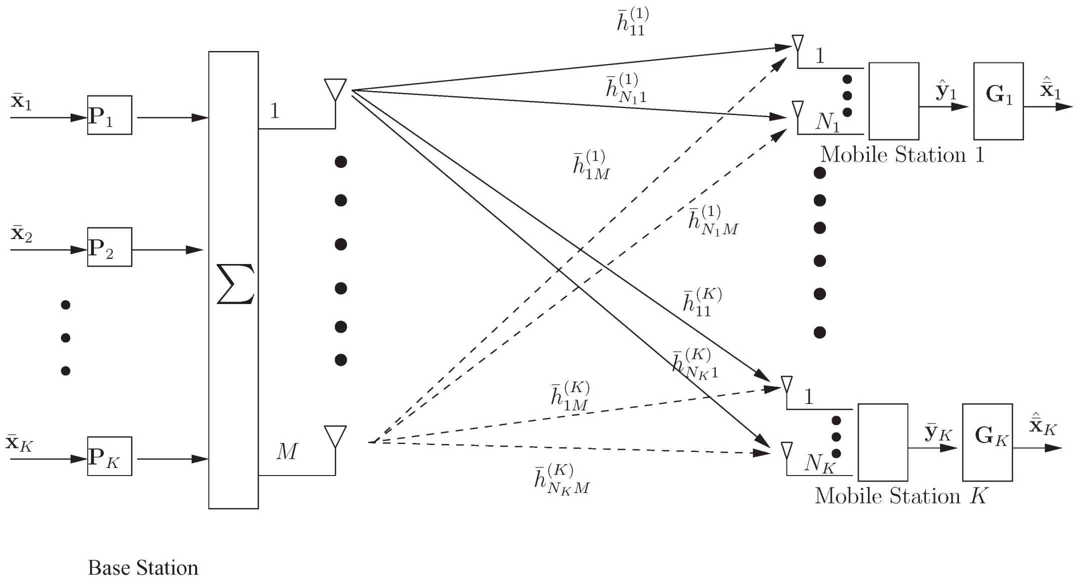

Fig. 2. Schematic of the SDMA DL transmission using both preprocessing and postprocessing.

covariance matrix is given by

$$E\left[\boldsymbol{n}'(\boldsymbol{n}')^H\right] = \sigma^2 \left(\boldsymbol{U}_s^H \boldsymbol{U}_s\right)^{-1} \tag{15}$$

which indicates that the noise observations become correlated after the ZF MUD receiver postprocessing.

Since the antenna-specific signals transmitted from a given MS are decoupled by its transmitter preprocessing, it can readily be shown that the diagonal entries of $\boldsymbol{U}_s^H \boldsymbol{U}_s$ are constituted by K unity matrices having the sizes of $(N_k \times N_k)$ for $k = 1, 2, \ldots, K$, respectively. Therefore, a given MS does not impose correlation on its own antenna-specific noise samples.

III. SVD-Based DL Transmission and Detection

Similarly to the UL, the DL system considered has a single BS supporting K MSs, as shown in Fig. 2. The BS is equipped with M DL transmit antennas, whereas the kth $(k = 1, 2, \ldots, K)$ MS has N_k receive antennas. Furthermore, we assume that the channel between any pair of transmit and receive antennas is flat fading. Let the N_k-component DL symbol vector $\bar{\boldsymbol{x}}_k = [\bar{x}_{k1}, \bar{x}_{k2}, \ldots, \bar{x}_{kN_k}]^T$ be transmitted to the kth MS. As shown in Fig. 2, $\bar{\boldsymbol{x}}_k$ is preprocessed before its transmission by premultiplying it with an $(M \times N_k)$-component DL preprocessing matrix $\boldsymbol{P}_k$, yielding

$$\bar{\boldsymbol{d}}_k = \boldsymbol{P}_k \bar{\boldsymbol{x}}_k, \qquad k = 1, 2, \ldots, K. \tag{16}$$

After DL transmitter preprocessing, the M-component signal broadcast by the BS to the K MSs can be expressed as

$$\bar{\boldsymbol{d}} = \sum_{k=1}^{K} \bar{\boldsymbol{d}}_k = \boldsymbol{P}\bar{\boldsymbol{x}} \tag{17}$$

where $\boldsymbol{P}$ is an $(M \times \sum_{k=1}^{K} N_k)$-component matrix given by

$$\boldsymbol{P} = [\boldsymbol{P}_1, \boldsymbol{P}_2, \ldots, \boldsymbol{P}_K] \tag{18}$$

and $\bar{\boldsymbol{x}}$ is a $(\sum_{k=1}^{K} N_k)$-component vector containing the transmitted DL data, which is given by

$$\bar{\boldsymbol{x}} = \left[\bar{\boldsymbol{x}}_1^T, \bar{\boldsymbol{x}}_2^T, \ldots, \bar{\boldsymbol{x}}_k^T\right]^T. \tag{19}$$

As shown in Fig. 2, the received N_k-component vector $\bar{\boldsymbol{y}}_k$ of the kth MS can be expressed as

$$\begin{aligned}\bar{\boldsymbol{y}}_k &= \bar{\boldsymbol{H}}_k \bar{\boldsymbol{d}} + \bar{\boldsymbol{n}}_k = \bar{\boldsymbol{H}}_k \boldsymbol{P} \bar{\boldsymbol{x}} + \bar{\boldsymbol{n}}_k \\ &= \bar{\boldsymbol{H}}_k \boldsymbol{P}_k \bar{\boldsymbol{x}}_k + \sum_{i=1, i\neq k}^{K} \bar{\boldsymbol{H}}_k \boldsymbol{P}_i \bar{\boldsymbol{x}}_i + \bar{\boldsymbol{n}}_k, \qquad k = 1, 2, \ldots, K\end{aligned} \tag{20}$$

where $\bar{\boldsymbol{n}}_k$ is an N_k-length additive white Gaussian noise (AWGN) vector having zero mean and a covariance matrix of $E[\bar{\boldsymbol{n}}_k \bar{\boldsymbol{n}}_k^H] = \sigma^2 \boldsymbol{I}_{N_k}$, whereas $\bar{\boldsymbol{H}}_k$ is an $(N_k \times M)$-component channel transfer matrix connecting the M DL transmit antennas of the BS with the kth MS's N_k receive antennas, which can be expressed as

$$\bar{\boldsymbol{H}}_k = \begin{bmatrix} \bar{h}_{11}^{(k)} & \bar{h}_{12}^{(k)} & \cdots & \bar{h}_{1M}^{(k)} \\ \bar{h}_{21}^{(k)} & \bar{h}_{22}^{(k)} & \cdots & \bar{h}_{2M}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ \bar{h}_{N_k 1}^{(k)} & \bar{h}_{N_k 2}^{(k)} & \cdots & \bar{h}_{N_k M}^{(k)} \end{bmatrix} \tag{21}$$

where $\bar{h}_{ij}^{(k)}$ represents the CIR coefficients between the jth DL BS transmit antenna and the ith DL receive antenna of the kth MS. As we can see from (20), the received DL signals at the MSs experience MUI.

Let us assume that the rows of $\bar{\boldsymbol{H}}_k$ $(k = 1, 2, \ldots, K)$ have full rank, i.e., we have rank $(\bar{\boldsymbol{H}}_k) = N_k$, and that $M \geq \sum_{k=1}^{K} N_k$. Then, upon carrying out the SVD of $\bar{\boldsymbol{H}}_k$, we arrive at

$$\bar{\boldsymbol{H}}_k = \bar{\boldsymbol{U}}_k \left[\bar{\boldsymbol{\Lambda}}_k^{1/2}, \boldsymbol{0}\right] \bar{\boldsymbol{V}}_k^H = \bar{\boldsymbol{U}}_k \left[\bar{\boldsymbol{\Lambda}}_k^{1/2}, \boldsymbol{0}\right] \begin{bmatrix} \bar{\boldsymbol{V}}_{ks}^H \\ \bar{\boldsymbol{V}}_{kn}^H \end{bmatrix} = \bar{\boldsymbol{U}}_k \bar{\boldsymbol{\Lambda}}_k^{1/2} \bar{\boldsymbol{V}}_{ks}^H \tag{22}$$

where $\bar{\boldsymbol{U}}_k$ and $\bar{\boldsymbol{V}}_k$ are $(N_k \times N_k)$- and $(M \times M)$-component unitary matrices, respectively, and $\bar{\boldsymbol{\Lambda}}_k$ is an $(N_k \times N_k)$-component diagonal matrix containing the eigenvalues of $\bar{\boldsymbol{H}}_k \bar{\boldsymbol{H}}_k^H$, i.e., we have $\bar{\boldsymbol{\Lambda}}_k = \text{diag}\{\bar{\lambda}_{k1}, \bar{\lambda}_{k2}, \ldots, \bar{\lambda}_{kN_k}\}$. Furthermore, in (22), $\bar{\boldsymbol{V}}_{ks}$ is an $(M \times N_k)$-component matrix, which is constituted by the eigenvectors corresponding to the nonzero eigenvalues of $\bar{\boldsymbol{H}}_k^H \bar{\boldsymbol{H}}_k$. By contrast, $\bar{\boldsymbol{V}}_{kn}$ is an $[M \times (M - N_k)]$-component matrix, which is constituted by the eigenvectors corresponding to the zero eigenvalues of $\bar{\boldsymbol{H}}_k^H \bar{\boldsymbol{H}}_k$. Similarly, $\bar{\boldsymbol{U}}_k$ consists of the eigenvectors of $\bar{\boldsymbol{H}}_k \bar{\boldsymbol{H}}_k^H$.

Upon substituting (22) into the first line in (20), the received DL signal $\bar{\boldsymbol{y}}_k$ of the kth MS shown in Fig. 2 may be expressed as

$$\bar{\boldsymbol{y}}_k = \bar{\boldsymbol{U}}_k \bar{\boldsymbol{\Lambda}}_k^{1/2} \bar{\boldsymbol{V}}_{sk}^H \boldsymbol{P}\bar{\boldsymbol{x}} + \bar{\boldsymbol{n}}_k, \qquad k = 1, 2, \ldots, K. \tag{23}$$

Let us now collect all the K received DL signal vectors $\{\bar{\boldsymbol{y}}_k\}$ of (20) into a vector $\bar{\boldsymbol{y}} = [\bar{\boldsymbol{y}}_1^T, \bar{\boldsymbol{y}}_2^T, \ldots, \bar{\boldsymbol{y}}_K^T]^T$. Then, according to (23), it can be shown that the overall DL received signal vector $\bar{\boldsymbol{y}}$ of all the K MSs can be expressed as

$$\bar{\boldsymbol{y}} = \bar{\boldsymbol{U}} \bar{\boldsymbol{\Lambda}}^{1/2} \bar{\boldsymbol{V}}_s^H \boldsymbol{P}\bar{\boldsymbol{x}} + \bar{\boldsymbol{n}} \tag{24}$$

where we introduced the following definitions:

$$\begin{aligned} \bar{\boldsymbol{U}} &= \mathrm{diag}\{\bar{\boldsymbol{U}}_1, \bar{\boldsymbol{U}}_2, \ldots, \bar{\boldsymbol{U}}_k\} \\ \bar{\boldsymbol{\Lambda}} &= \mathrm{diag}\{\bar{\boldsymbol{\Lambda}}_1, \bar{\boldsymbol{\Lambda}}_2, \ldots, \bar{\boldsymbol{\Lambda}}_k\} \\ \bar{\boldsymbol{V}}_s &= [\bar{\boldsymbol{V}}_{1s}, \bar{\boldsymbol{V}}_{2s}, \ldots, \bar{\boldsymbol{V}}_{Ks}] \\ \bar{\boldsymbol{n}} &= \left[\bar{\boldsymbol{n}}_1^T, \bar{\boldsymbol{n}}_2^T, \ldots, \bar{\boldsymbol{n}}_k^T\right]^T. \end{aligned} \tag{25}$$

In (25), $\bar{\boldsymbol{U}}$ and $\bar{\boldsymbol{\Lambda}}$ are $(\sum_{k=1}^K N_k \times \sum_{k=1}^K N_k)$-component matrices, $\bar{\boldsymbol{V}}_s$ is an $(M \times \sum_{k=1}^K N_k)$-component matrix, and $\bar{\boldsymbol{n}}$ is an AWGN vector having zero mean and a covariance matrix of $\sigma^2 \boldsymbol{I}_{\sum_{k=1}^K N_k}$.

The DL BS transmit preprocessing matrix $\boldsymbol{P}$ is designed so that the DL MUI can efficiently be suppressed. As shown in (24), the MUI can fully be removed when the DL preprocessing matrix $\boldsymbol{P}$ is chosen to satisfy

$$\bar{\boldsymbol{V}}_s^H \boldsymbol{P} = \bar{\boldsymbol{\beta}} \tag{26}$$

where the power allocation regime of $\bar{\boldsymbol{\beta}} = \mathrm{diag}\{\bar{\beta}_1, \bar{\beta}_2, \ldots, \bar{\beta}_{\sum_{k=1}^K N_k}\} = \mathrm{diag}\{\bar{\beta}_{11}, \ldots, \bar{\beta}_{1N_1}; \ldots; \bar{\beta}_{K1}, \ldots, \bar{\beta}_{KN_K}\}$ represents our transmission power constraint, which will be considered later on.

To satisfy (26), $\boldsymbol{P}$ can be set to

$$\boldsymbol{P} = \left[\bar{\boldsymbol{V}}_s^H\right]^+ \bar{\boldsymbol{\beta}} = \tilde{\boldsymbol{P}} \bar{\boldsymbol{\beta}} \tag{27}$$

where $[\bar{\boldsymbol{V}}_s^H]^+$ denotes the pseudoinverse of the matrix $\bar{\boldsymbol{V}}_s^H$, and $\tilde{\boldsymbol{P}} = [\bar{\boldsymbol{V}}_s^H]^+ = \bar{\boldsymbol{V}}_s [\bar{\boldsymbol{V}}_s^H \bar{\boldsymbol{V}}_s]^{-1}$.

When substituting the overall DL preprocessing matrix of (26) into (24), the overall received signal vector $\bar{\boldsymbol{y}}$ of all K MSs can be simplified to

$$\bar{\boldsymbol{y}} = \bar{\boldsymbol{U}} \bar{\boldsymbol{\Lambda}}^{1/2} \bar{\boldsymbol{\beta}} \bar{\boldsymbol{x}} + \bar{\boldsymbol{n}}. \tag{28}$$

To be more specific, the N_k-length observation vector of the kth MS can be expressed as

$$\bar{\boldsymbol{y}}_k = \bar{\boldsymbol{U}}_k \bar{\boldsymbol{\Lambda}}_k^{1/2} \bar{\boldsymbol{\beta}}_k \bar{\boldsymbol{x}}_k + \bar{\boldsymbol{n}}_k, \qquad k = 1, 2, \ldots, K \tag{29}$$

where we have $\bar{\boldsymbol{\beta}}_k = \mathrm{diag}\{\bar{\beta}_{k1}, \bar{\beta}_{k2}, \ldots, \bar{\beta}_{kN_k}\}$. Explicitly, the kth user endures no MAI imposed by the other users. However, there may exist IAI among the antenna-specific symbols transmitted by the BS to the kth MS. This IAI can be suppressed with the aid of the SVD-based matrices $\{\bar{\boldsymbol{U}}_k\}$ of (22). Consequently, after DL receiver postprocessing of the received signal vectors $\{\bar{\boldsymbol{y}}_k\}$ by $\{\boldsymbol{G}_k = \bar{\boldsymbol{U}}_k^H\}$ according to Fig. 2, the user-specific decision variables can individually be expressed as

$$\hat{\bar{\boldsymbol{x}}}_k = \bar{\boldsymbol{\Lambda}}_k^{1/2} \bar{\boldsymbol{\beta}}_k \bar{\boldsymbol{x}}_k + \bar{\boldsymbol{U}}_k^H \bar{\boldsymbol{n}}_k, \qquad k = 1, 2, \ldots, K \tag{30}$$

or jointly as

$$\hat{\bar{\boldsymbol{x}}} = [\hat{\bar{\boldsymbol{x}}}_1^T, \ldots, \hat{\bar{\boldsymbol{x}}}_K^T]^T = \bar{\boldsymbol{\Lambda}}^{1/2} \bar{\boldsymbol{\beta}} \bar{\boldsymbol{x}} + \bar{\boldsymbol{U}}^H \bar{\boldsymbol{n}}. \tag{31}$$

TABLE I
PARAMETERS FOR THE SDMA TRANSMISSION BASED ON SVD

Number of antennas at BS M	8
Number of users K	2
Number of antenna each user N_k	2
Normalized maximum Doppler frequency	0.001
Modulation scheme for largest eigenvalue	4QAM
Modulation scheme for second largest eigenvalue	BPSK

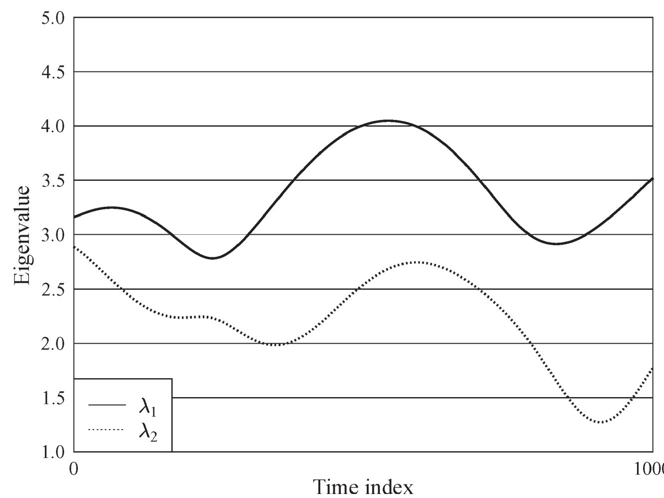

Fig. 3. Singular values for the UL transmission of the first user. The remaining parameters are assumed to be the same as in Table I.

An important constraint for preprocessing may be to keep the transmitted power for all users unchanged before and after the preprocessing, i.e.,

$$E\left[\|\mathbf{P}\bar{\mathbf{x}}\|_2^2\right] = E\left[\|\bar{\mathbf{x}}\|_2^2\right] = \sum_{k=1}^K N_k. \tag{32}$$

A natural power allocation scheme is to allocate the same power to each data stream [1], [4]. In this case, the coefficients $\bar{\beta}_i$ are set according to $\bar{\beta}_1 = \cdots = \bar{\beta}_{\sum_{k=1}^K N_K} = \bar{\beta}$ [1], [4], where $\bar{\beta}$ is a constant, given by [1], [4]

$$\bar{\beta} = \sqrt{\frac{\sum_{k=1}^K N_k}{\mathrm{trace}\left(\left[\bar{\boldsymbol{V}}_s^H \bar{\boldsymbol{V}}_s\right]^{-1}\right)}}. \tag{33}$$

IV. PERFORMANCE RESULTS

In this section, simulation results are provided for characterizing the attainable performance of the proposed algorithm in the context of the system parameters summarized in Table I.

The evolution of two singular values λ_1 and λ_2 are plotted in Fig. 3 as a function of time for the UL transmission of the first user. The remaining parameters are assumed to be the same as in Table I. We can see in Fig. 3 that both singular values fluctuate, and sometimes, the singular value λ_1 is much larger than λ_2, which results in a higher signal-to-noise ratio (SNR) gain.

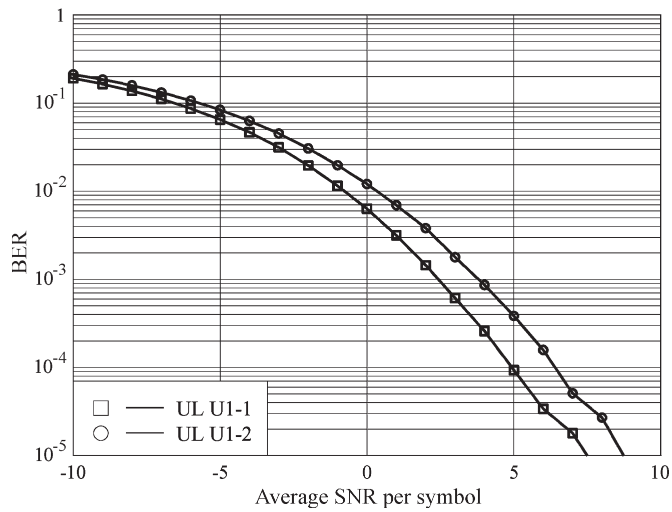

Fig. 4. BER versus average SNR per symbol performance for the UL transmission. The remaining parameters are assumed to be the same as in Table I.

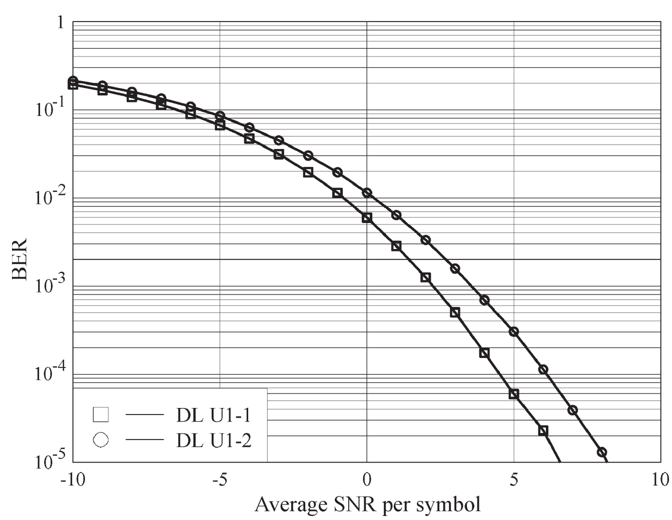

Fig. 5. BER versus average SNR per symbol performance for the DL transmission. The remaining parameters are assumed to be the same as in Table I.

The attainable bit error rate (BER) versus average SNR per symbol performance of both the UL and DL transmissions of the first user are portrayed in Figs. 4 and 5, respectively, when 4-ary quadrature amplitude modulation (4QAM) and binary phase-shift keying modulation are used corresponding to the largest and second largest singular values, respectively, whereas the other parameters are summarized in Table I. We can see in Figs. 4 and 5 that the BER performance corresponding to the largest singular value is better than that corresponding to the second largest singular value, despite the fact that the higher throughput, and hence more vulnerable, 4QAM scheme is used corresponding to the largest singular value. This is because having a higher singular value results in a higher SNR, as may be surmised in Fig. 3.

V. Conclusion

In this paper, SVD-based SDMA algorithms have been proposed for both the UL and DL transmissions, where the MU MIMO channels were decomposed into parallel SISO channels corresponding to their singular values. Based on the proposed algorithm, different modulation schemes can be adopted for different SISO channels, which can potentially improve the system's throughput. Furthermore, AM schemes can be invoked in the context of multiusers for both the UL and DL transmissions by adjusting the related parameters, for example, the power or the transmission rate, to maximize the throughput or minimize the transmission power and so on [10], [16]. A typical application of this scheme is found in multimedia communication, where different modulation schemes can be chosen to satisfy the different quality-of-service requirements [17], [18].

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