Particle Swarm Optimisation Aided Minimum Bit Error Rate Multiuser Transmission

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***Abstract*— We consider the downlink of multiuser system from a transmitter equipped with multiple antennas to multiple non-cooperative single-antenna mobile receivers. Particle Swarm Optimisation (PSO) algorithm is invoked to solve the constrained nonlinear optimisation problem for the Minimum Bit Error Rate (MBER) Multiuser Transmission (MUT). The proposed PSO aided MBER-MUT scheme provides much better performance over the conventional minimum mean-square-error MUT scheme, and it achieves a much lower complexity compared to the state-of-the-art sequential quadratic programming based MBER MUT. Simulation results also show that the proposed MBER MUT scheme is capable of supporting more users than the number of transmit antennas available.**

I. Introduction

For the downlink of a Space-Division Multiple-Access (SDMA) system with decentralised non-cooperative mobile devices at the receive end, the mobile users are unable to do cooperative Multiuser Detection (MUD). In order to achieve better performance, the signals should be pre-processed at the Base Station (BS), leading to Multiuser Transmission (MUT). MUT techniques provide the possibility to implement low-complexity and high power-efficiency Mobile Terminals (MTs) for mobile broadcast channels when channel state information is available at the transmitter [1]. The assumption that the downlink channel impulse response is known at the BS is valid in Time Division Duplex (TDD) systems due to the channel reciprocity. Transmit preprocessing is then possible when the channel coherence time is large compared to one transmission interval. However, for Frequency Division Duplex (FDD) systems, where the uplink and downlink channels are not reciprocal, feedback from the MT receivers to the BS transmitter is necessary.

Most of the MUT techniques are based on the Minimum Mean-Square-Error (MMSE) criterion [2], [3]. Since the Bit Error Rate (BER) is the ultimate system performance indicator, interests on Minimum BER (MBER) based MUT techniques have increased recently. A MBER-MUT scheme was proposed in [4] for the TDD Code-Division Multiple-Access (CDMA) downlink over frequency-selective channels, and this work was extended to multiple transmit and receive antennas in [5]. A chip level MBER-MUT scheme was proposed in [6]. The MBER-MUT techniques mentioned so far are designed exactly for the given transmit symbol vector and, therefore, the coefficients of the precoder have to be calculated for every transmit symbol vector. A true MBER-MUT design was proposed and investigated for Binary Phase Shift Keying (BPSK) modulation [7] and Quadrature Phase Shift Keying (QPSK) modulation [8], where the coefficients of the precoder only need to be re-calculated when the channel coefficients are changed. The MBER-MUT design is a constrained nonlinear optimisation [7], [8], and the Sequential Quadratic Programming (SQP) algorithm [9] is typically used to obtain the precoder's coefficients for the MBER-MUT [7], [8], [10], and the computational complexity of the SQP based MBER-MUT solution may be too high to be implemented in a real-time system [10].

In this contribution, we invoke the Particle Swarm Optimisation (PSO) algorithm [11] to find the precoder's coefficients in order to reduce the computational complexity of the MBER-MUT. PSO is a population based stochastic optimisation technique [12] inspired by social behaviour of bird flocking or fish schooling. The algorithm starts with random initialisation of a population of individuals, called particles, within the problem search space. It finds the global best solution by simply adjusting the trajectory of each individual toward its own best location and toward the best particle of the entire swarm at each time. The PSO method is becoming very popular due to its simplicity in implementation, ability to quickly converge to a reasonably good solution and its robustness against local minima. It has been applied to wide-ranging optimisation problems successfully. In particular, many researchers have applied PSO techniques to Multiuser Detection (MUD) [13]–[17], and the experimental results obtained have shown that the PSO aided MUD achieves better performance with lower computational complexity, compared with the Genetic Algorithm (GA) assisted MUD [16]. We will show that the proposed PSO approach achieves the optimal MBER MUT solution at a much lower complexity, compared to the existing state-of-the-art SQP based MBER MUT method.

The rest of this contribution is structured as follows. In Section II, the signal model of the downlink SDMA system is introduced. The MBER-MUT strategy is summarised in Section III, while Section IV outlines our proposed PSO assisted MBER-MUT algorithm. Our simulation study is given in Section V, and we conclude the paper in Section VI.

II. System Model

The downlink of a SDMA system with decentralised non-cooperative mobile devices at the receive end is considered here. The BS is equipped with N transmit antennas and communicates over flat fading channels with K MTs, each

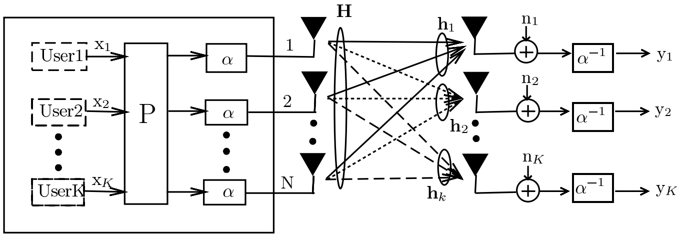

Fig. 1. Schematic diagram of the downlink of a SDMA system using preprocessing at the BS. The system employs N transmit antennas to communicate with K decentralised non-cooperative mobile devices.

employing only one receive antenna. Frequency selective channels can be made narrowband using for example the Orthogonal Frequency Division Multiplexing (OFDM) technique [18]. This system model is illustrated in Fig. 1. The vector of information symbols for transmission is given by $\mathbf{x} = [x_1\ x_2 \cdots x_K]^T$, where x_k denotes the transmitted symbol for the kth MT and the symbol energy is given by $E[|x_k|^2] = \sigma_x^2$, for $1 \leq k \leq K$, with $E[\bullet]$ denoting the expectation operator. The $N \times K$-dimensional precoder matrix $\mathbf{P}$ is defined by

$$\mathbf{P} = [\mathbf{p}_1\ \mathbf{p}_2 \cdots \mathbf{p}_K] \tag{1}$$

where $\mathbf{p}_k$, $1 \leq k \leq K$ is the precoder's coefficient vector for the kth user's data stream. Given a fixed total transmit power E_T at the BS, a scaling factor should be used to fullfill this transmit power constraint, which is defined as

$$\alpha = \sqrt{\mathrm{E}_T / E[\|\mathbf{P}\mathbf{x}\|^2]}. \tag{2}$$

At the receive end, the inverse of the scaling factor, α^{-1}, is multiplied with the received signal to ensure unit gain transmission. The channel matrix $\mathbf{H}$ is given by

$$\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2 \cdots \mathbf{h}_K] \tag{3}$$

where $\mathbf{h}_k = [h_{1,k}\ h_{2,k} \cdots h_{N,k}]^T$, $1 \leq k \leq K$, is the kth user's spatial signature. The channel taps $h_{i,k}$ for $1 \leq k \leq K$ and $1 \leq i \leq N$ are independent with each other and obey the complex-valued Gaussian distributions associated with $E[|h_{i,k}|^2] = 1$. The additive Gaussian white noise vector $\mathbf{n}$ is defined by $\mathbf{n} = [n_1\ n_2 \cdots n_K]^T$, where n_k, $1 \leq k \leq K$ is a complex-valued Gaussian random process with zero mean and a variance of $\sigma_n^2 = 1/2\mathrm{SNR}$ per real dimension, and SNR stands for the signal-to-noise ratio of the downlink. Thus, the baseband model of the system can be described as

$$\mathbf{y} = \mathbf{H}^T \mathbf{P}\mathbf{x} + \alpha^{-1}\mathbf{n}, \tag{4}$$

where $\mathbf{y} = [y_1\ y_2 \cdots y_K]^T$ denotes the received signal vector, and y_k, $1 \leq k \leq K$, is a sufficient statistics for the kth MT to detect the transmitted data symbol x_k.

III. MBER Multiuser Transmission

Two MBER-MUT strategies exist. The first design can be referred to as the symbol-specific MBER MUT [10], and the other one the true MBER MUT [7], [8]. These two methods are outlined in the following.

A. Symbol-Specific MBER-MUT

This approach was developed based on the fact that the information symbols to be transmitted are known exactly at the transmitter and the precoding matrix can be chosen specifically for the given symbol vector so that the BER is minimised. Given the symbol vector $\mathbf{x}$ for transmission, the average BER of the in-phase component of $\mathbf{y}$ at the receiver is

$$P_{e_I,\mathbf{x}} = \frac{1}{K}\sum_{k=1}^{K} Q\left(\frac{\mathrm{sgn}(\Re[x_k])\Re[\mathbf{h}_k^T\mathbf{P}\mathbf{x}]}{\sigma_n}\right), \tag{5}$$

where $Q(\bullet)$ is the standard Gaussian error function and $\Re[\bullet]$ denotes the real part. Thus, the BER for BPSK signalling is given by

$$P_{e,\mathbf{x}} = P_{e_I,\mathbf{x}}. \tag{6}$$

Similarly, the average BER of the quadrature-phase component of $\mathbf{y}$ given $\mathbf{x}$ is

$$P_{e_Q,\mathbf{x}} = \frac{1}{K}\sum_{k=1}^{K} Q\left(\frac{\mathrm{sgn}(\Im[x_k])\Im[\mathbf{h}_k^T\mathbf{P}\mathbf{x}]}{\sigma_n}\right), \tag{7}$$

where $\Im[\bullet]$ denotes the imaginary part. Thus, the BER for QPSK signalling is

$$P_{e,\mathbf{x}} = \frac{1}{2}(P_{e_I,\mathbf{x}} + P_{e_Q,\mathbf{x}}). \tag{8}$$

Therefore, the solution of the symbol-specific MBER-MUT is defined as

$$\begin{aligned} \mathbf{P}_{\mathrm{TxMBER},\mathbf{x}} &= \arg\min_{\mathbf{P}} P_{e,\mathbf{x}} \\ &\quad \text{s.t.} \quad E[\|\mathbf{P}\mathbf{x}\|^2] = \mathrm{E}_T. \end{aligned} \tag{9}$$

The problem associated with this approach is that for every transmitted symbol vector $\mathbf{x}$, the precoder matrix $\mathbf{P}$ must be calculated by solving the constrained optimisation problem (9).

B. True MBER-MUT

To avoid the computational complexity associated with the previous symbol-specific MBER MUT scheme, we should determine the precoder matrix that remains optimal for all the legitimate transmission symbol vectors. The average BER of the in-phase component of $\mathbf{y}$ at the receiver can be shown to be [19]

$$P_{e_I} = \frac{1}{KM^{K-1}}\sum_{q=1}^{M^{K-1}}\sum_{k=1}^{K} Q\left(\frac{\mathrm{sgn}(\Re[x_k^{(q)}])\Re[\mathbf{h}_k^T\mathbf{P}\mathbf{x}^{(q)}]}{\sigma_n}\right). \tag{10}$$

Here M^{K-1} is the number of equiprobable legitimate transmit symbol vectors $\mathbf{x}^{(q)}$, given $x_k = +1 + j$, for the M-ary PSK signalling, where $1 \leq q \leq M^{K-1}$ and $j = \sqrt{-1}$. Thus, the BER for BPSK signalling is

$$P_e = P_{e_I} \tag{11}$$

TABLE I

COMPUTATIONAL COMPLEXITY PER ITERATION OF TWO MBER MUT DESIGNS FOR QPSK SIGNALLING, WHERE N IS THE NUMBER OF TRANSMIT ANTENNAS, K THE NUMBER OF MOBILE USERS, $M = 4$ IS THE SIZE OF SYMBOL CONSTELLATION AND S IS THE PARTICLE SIZE.

Algorithm	Flops
SQP	$K \times (8 \times N^2 \times K^2 + 6 \times N \times K + 6 \times N + 8 \times K + 4) \times M^{K-1} + 8 \times N \times K \times (N^2 \times K^2 + N \times K + 2 \times K + N)$ $+12 \times N \times K + 6 \times K^2 - 2 \times N^2 + N - 2 \times K + 11$
PSO	$((4 \times N \times K + 2 \times N + 12) \times K \times M^{K-1} + 24 \times N \times K + K) \times S + 8$

with $M^{K-1} = 2^{K-1}$. Similarly, the average BER of the quadrature-phase component of $\mathbf{y}$ is give by

$$P_{e_Q} = \frac{1}{KM^{K-1}} \sum_{q=1}^{M^{K-1}} \sum_{k=1}^{K} Q\left(\frac{\text{sgn}(\Im[x_k^{(q)}])\Im[\mathbf{h}_k^T \mathbf{P}\mathbf{x}^{(q)}]}{\sigma_n}\right), \tag{12}$$

and the BER for QPSK signalling is defined by

$$P_e = \frac{1}{2}(P_{e_I} + P_{e_Q}), \tag{13}$$

with $M^{K-1} = 4^{K-1}$.

Hence, the solution of the true MBER MUT is defined as

$$\begin{aligned} \mathbf{P}_{\text{TxMBER}} &= \arg\min_{\mathbf{P}} P_e \\ &\text{s.t.} \quad E[\|\mathbf{P}\mathbf{x}\|^2] = \text{E}_T. \end{aligned} \tag{14}$$

The optimisation problem (14) is a constrained nonlinear optimisation one, and is typically solved by an iterative gradient based optimisation algorithm known as the SQP [7], [8], [10]. The computational complexity per iteration of the SQP based MBER MUT scheme is listed in Table I for QPSK modulation. Detailed derivation of this complexity can be found in [8], and we assume that the complexity of a real-valued multiplication is equal to a real-valued addition. The total computational complexity equals the number of iterations that the algorithm required to arrive at a global optimal solution multiplied by this complexity per iteration.

IV. PSO ASSISTED MBER MUT

We invoke the PSO algorithm to solve the MBER-MUT design problem (14). In the original PSO algorithm [12], a group of particles that represent potential solutions are initialised over the whole search space randomly. Each particle has a fitness value F associated with it, based on the related cost function of the optimisation problem, and its F value is evaluated at each iteration. Each particle knows its best position, **pbest**, which provides the *cognitive information*, and the best position so far among the entire group, **gbest**, which provides the *social information*. The **pbest**s and **gbest** are updated at each iteration. Each particle also has its own velocity to direct its flying, which relies on its previous speed as well as its cognitive and social information. In each iteration, the velocity and the position of the particle is updated based on the following equations

$$\begin{aligned} \mathbf{v}_i^{l+1} &= w * \mathbf{v}_i^l + rand() * c_1 * (\mathbf{pbest}_i^l - \mathbf{s}_i^l) \\ &\quad + rand() * c_2 * (\mathbf{gbest}^l - \mathbf{s}_i^l), \end{aligned} \tag{15}$$

$$\mathbf{s}_i^{l+1} = \mathbf{s}_i^l + \mathbf{v}_i^{l+1}, \tag{16}$$

where l is iteration index, $1 \le i \le S$ and S is the particle size,

$\mathbf{v}_i^l$	the velocity of the *i*th particle at *l*th iteration. The elements of $\mathbf{v}_i^l$ are in the range $[-V_{max},\ V_{max}]$
w	inertia weight
c_m	the acceleration coefficients, $m = 1, 2$
$rand()$	uniform random number between 0 and 1
$\mathbf{s}_i^l$	the position of the *i*th particle at *l*th iteration. The elements of $\mathbf{s}_i^l$ are in the range $[-S_{max},\ S_{max}]$
$\mathbf{pbest}_i^l$	the best position that the *i*th particle has visited at *l*th iteration
$\mathbf{gbest}^l$	the best position that all the particles have visited at *l*th iteration

Many enhancements and modifications have been made to this original PSO. For a conventional PSO, the acceleration coefficients are kept constant for all iterations. However, it was reported in [20] that using Time Varying Acceleration Coefficient (TVAC) can enhance the performance of PSO. The reason is that at the initial stages, a large cognitive component and a small social component help particles to wander around the search space and to avoid local minima. In the later stages, a small cognitive component and a large social component help particles to converge quickly to the global minima. We adopt the TVAC mechanism as suggested in [20], in which c_1 for the cognitive component is reduced from 2.5 to 0.5 and the c_2 for the social component varies from 0.5 to 2.5 during the iterative procedure

$$\begin{aligned} c_1 &= (0.5 - 2.5)\frac{l}{\text{Max Iteration}} + 2.5, \\ c_2 &= (2.5 - 0.5)\frac{l}{\text{Max Iteration}} + 0.5, \end{aligned} \tag{17}$$

where Max Iteration denotes the maximum number of iterations. The second modification suggested in [20] is to remove the influence of the previous velocity by setting the inertia weight $w = 0$. Then, when the velocity in equation (15) approaches zero, it is reinitialised to proportional to V_{max} with a factor γ. This modification is found to be beneficial in our problem although it was reported to be only helpful in certain cases in [20]. The third modification is in the way of dealing with the particle $\mathbf{s}_i$ that exceeds the range $[-S_{max},\ S_{max}]$. In our approach, the particle is moved inside the search space randomly, instead of forcing it to stay at the border. A similar procedure can be found in [21].

Since the $N \times K$ precoder matrix $\mathbf{P}$ is complex-valued, the optimisation problem (14) is a $2 \times N \times K$-dimensional one and the length or dimension of each particle $\mathbf{s}_i$ is $2 \times N \times K$. The search space is set to $[-1,\ 1]^{2\times N\times K}$ with $S_{max} = 1$. Elements of each initial particle position $\mathbf{s}_i^0$, $\mathbf{s}_i^0|_q$ for $1 \le q \le 2\times N\times K$, are uniformly randomly chosen within $[-1,\ 1]$, where $\mathbf{s}_i^0|_q$ denotes the qth element of $\mathbf{s}_i^0$. A limit of $V_{max} = 1$ is used in our algorithm for velocity. It is also found by experiments that $\gamma = 0.1$ provides us excellent performance and convergence speed. We define the following penalty function to take into account the power constraint

$$G(\mathbf{P}) = \begin{cases} 0, & E[\|\mathbf{Px}\|^2] - \mathrm{E}_T \le 0, \\ E[\|\mathbf{Px}\|^2] - \mathrm{E}_T, & E[\|\mathbf{Px}\|^2] - \mathrm{E}_T > 0, \end{cases} \quad (18)$$

and introduce the cost function

$$F = P_e + \lambda G(\mathbf{P}) \quad (19)$$

to convert (14) into an unconstrained optimisation

$$\mathbf{P}_{\text{TxMBER}} = \arg\min_{\mathbf{P}} \{P_e + \lambda G(\mathbf{P})\}, \quad (20)$$

where λ is the penalty factor and its value should be chosen appropriately to ensure fast convergence. The pseudocode of the PSO algorithm we adopted is summarised as follows.

Initialise the positions of the particles, $\{\mathbf{s}_i^0\}_{i=1}^S$, set all the $\{F(\mathbf{pbest}_i^0)\}_{i=1}^S$ and $F(\mathbf{gbest}^0)$ to a large positive number;
For ($l = 0$; $l <$ Max Iteration; l++)
  Evaluate $\{F_i^l\}_{i=1}^S$ for all the particles;
  For ($i = 1$; $i \le S$; i++)
    If ($F_i^l < F(\mathbf{pbest}_i^l)$)
      $F(\mathbf{pbest}_i^l) = F_i^l$;
      $\mathbf{pbest}_i^l = \mathbf{s}_i^l$;
    End if;
  End for;
  $i^* = \arg\min_{1\le i\le S} F(\mathbf{pbest}_i^l)$;
  If ($F(\mathbf{pbest}_{i^*}^l) < F(\mathbf{gbest}^l)$)
    $F(\mathbf{gbest}^l) = F(\mathbf{pbest}_{i^*}^l)$;
    $\mathbf{gbest}^l = \mathbf{pbest}_{i^*}^l$;
  End if;
  For ($i = 1$; $i \le S$; i++)
    $\mathbf{v}_i^{l+1} = rand() * c_1 * (\mathbf{pbest}_i^l - \mathbf{s}_i^l)$
      $+ rand() * c_2 * (\mathbf{gbest}^l - \mathbf{s}_i^l)$;
    If ($\mathbf{v}_i^{l+1}|_q == 0$)
      If ($rand() < 0.5$)
        $\mathbf{v}_i^{l+1}|_q = rand() * \gamma * V_{max}$;
      Else
        $\mathbf{v}_i^{l+1}|_q = -rand() * \gamma * V_{max}$;
      End if;
    End if;
    If ($\mathbf{v}_i^{l+1}|_q > V_{max}$)
      $\mathbf{v}_i^{l+1}|_q = V_{max}$;
    Else if ($\mathbf{v}_i^{l+1}|_q < -V_{max}$)
      $\mathbf{v}_i^{l+1}|_q = -V_{max}$;
    End if;
    $\mathbf{s}_i^{l+1} = \mathbf{s}_i^l + \mathbf{v}_i^{l+1}$;
    If ($\mathbf{s}_i^{l+1}|_q > S_{max}$)
      $\mathbf{s}_i^{l+1}|_q = rand() * S_{max}$;
    Else if ($\mathbf{s}_i^{l+1}|_q < -S_{max}$)
      $\mathbf{s}_i^{l+1}|_q = -rand() * S_{max}$;
    End if
  End for;
End for;

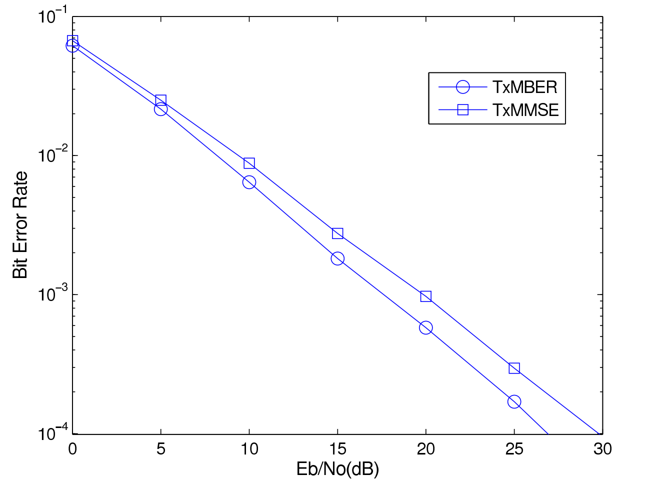

Fig. 2. BER versus SNR performance of the transmit MMSE and transmit MBER schemes, communicating over flat Rayleigh fading channels using 4 transmit antennas to support 4 QPSK users.

The computational complexity per iteration of this PSO based MBER MUT scheme is also listed in Table I.

V. Simulation Results

A multiuser downlink broadcast system employing 4 transmit antennas at the BS was considered. Full channel state

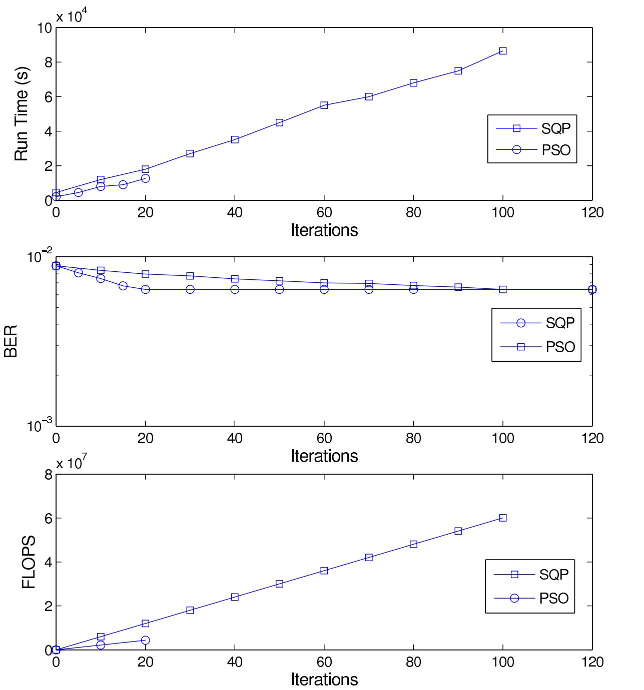

Fig. 3. Complexity and convergence performance of the SQP and PSO based MBER-MUT schemes for the system employing 4 transmit antennas to support 4 QPSK users over flat Rayleigh fading channels. Eb/No=10 dB, and 2×10^6 symbols are used to test the run time.

information was assumed at the BS. Transmission was over flat Rayleigh fading channel. First the system was used to support 4 QPSK users. The particle size was chosen to be $S = 20$ for the PSO algorithm. Fig. 2 compares the uncoded BER performance of the transmit MMSE scheme with that of the transmit MBER scheme using the PSO algorithm. It can be seen from Fig. 2 that the PSO aided MBER-MUT achieved a SNR gain of 4 dB over the MMSE-MUT at the target BER of 10^{-4}. Complexity and convergence speed of the PSO aided MBER-MUT were investigated, using the SQP based MBER-MUT as the benchmark. Given SNR= Eb/No $= 10$ dB and under the identical computational platform, Fig. 3 compares the convergence performance and computational complexity of the SQP and PSO based MBER MUT schemes. It is clear that the SQP converged to the MBER-MUT solution after 100 iterations, which took 86532.3 seconds at a cost of 56807100 flops, while the PSO arrived at the MBER-MUT solution with 20 iterations, which took 12678.6 seconds with a cost of 8756960 flops. The PSO algorithm is approximately 7 times faster than the SQP algorithm for this case.

The system was next used to support $K > 4$ BPSK users, and Fig. 4 shows the BER performance of the MMSE-MUT and the PSO aided MBER-MUT in this rank deficient scenarios. The MMSE-MUT is unable to differentiate the users, when the number of downlink transmit antennas is less than the number of users at the receive end and exhibits high residual BER floor as can be seen in Fig. 4. By contrast, the MBER-MUT is capable of supportting more downlink users than the number of transmit antennas.

VI. Conclusions

We have proposed a PSO assisted MBER-MUT algorithm, which offers a much lower computational complexity than the existing SQP based MBER-MUT algorithm. Our future research will investigate the robustness of the PSO based MBER-MUT solution under the condition of channel estimation error.

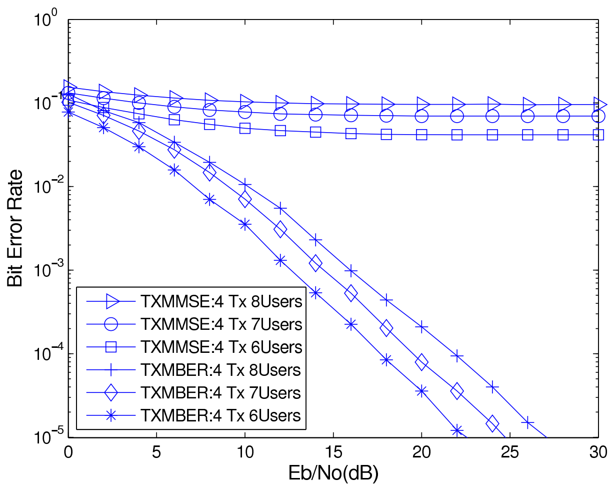

Fig. 4. BER versus SNR performance of the transmit MMSE and transmit MBER schemes, communicating over flat Rayleigh fading channels using 4 transmit antennas to support $K > 4$ BPSK users.

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