Particle Swarm Optimisation Aided Minimum Bit Error Rate Multiuser Transmission

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Motivations

- **Multiuser transmission** preprocessing technique is attractive for system with noncooperative mobile receivers
  \[\Rightarrow\] Low-complexity high power-efficiency mobile terminals

- **Minimum mean square error** MUT design has appealing simplicities but is limited in achievable bit error rate

- **Minimum bit error rate** MUT design enhances achievable BER performance
  \[\Rightarrow\] Standard MBER design based on *sequential quadratic programming* imposes high complexity

- **Particle swarm optimisation** aided MBER MUT design to significantly reduce complexity
- $N$ transmit antennas and $K$ mobile users
- Transmit symbol vector $\mathbf{x} = [x_1 \ x_2 \cdots x_K]^T$
- Precoder $\mathbf{P} = [p_1 \cdots p_K]$
- Flat fading MIMO channel $\mathbf{H} = [h_1 \ h_2 \cdots h_K]$

Transmit power constraint $E_T \Rightarrow$ scaling factor $\alpha = \sqrt{E_T/E[\|\mathbf{P}\mathbf{x}\|^2]}$

Receive signal vector $\mathbf{y} = [y_1 \ y_2 \cdots y_K]^T$ is given by

$$\mathbf{y} = \mathbf{H}^T\mathbf{P}\mathbf{x} + \alpha^{-1}\mathbf{n}$$

$y_k$, $1 \leq k \leq K$, are sufficient statistics for detecting $x_k$, $1 \leq k \leq K$
Bit Error Rate

- Average **bit error rate** for QPSK signalling is given by

\[ P_e(P) = \left( P_{eI}(P) + P_{eQ}(P) \right) / 2 \]

- Average BER of **in-phase** component of \( y \)

\[ P_{eI}(P) = \frac{1}{K M^K} \sum_{q=1}^{M^K} \sum_{k=1}^{K} Q \left( \frac{\text{sgn}(\Re[x_k^{(q)}]) \Re[h_k^T P x^{(q)}]}{\sigma_n} \right) \]

where \( M^K = 4^K \) is the number of equiprobable legitimate transmit symbol vectors \( x^{(q)} \) for QPSK signalling, \( x_k^{(q)} \) the \( k \)th element of \( x^{(q)} \), \( 1 \leq q \leq M^K \), and \( Q(\bullet) \) the standard Gaussian error function

- Average BER of **quadrature-phase** component of \( y \)

\[ P_{eQ}(P) = \frac{1}{K M^K} \sum_{q=1}^{M^K} \sum_{k=1}^{K} Q \left( \frac{\text{sgn}(\Im[x_k^{(q)}]) \Im[h_k^T P x^{(q)}]}{\sigma_n} \right) \]
MBER MUT Design

- MBER MUT solution is defined as

\[
P_{\text{TxMBER}} = \arg \min_P P_e(P)
\]

s.t. \( E[\|Px\|^2] = E_T \)

- Solving this optimisation by SQP algorithm has total complexity

\[
C_{T_{\text{SQP}}} = I_{\text{SQP}} \times C_{1_{\text{SQP}}}
\]

where \( I_{\text{SQP}} \) is number of iterations and \( C_{1_{\text{SQP}}} \) complexity per iteration

Complexity per iteration \( C_{1_{\text{SQP}}} \) of SQP-MBER-MUT for QPSK signalling, where \( N \) is the number of transmit antennas, \( K \) the number of mobile users, \( M = 4 \) is the size of symbol constellation

<table>
<thead>
<tr>
<th>Flops</th>
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<tbody>
<tr>
<td>( K \times (8 \times N^2 \times K^2 + 6 \times N \times K + 6 \times N + 8 \times K + 4) \times M^K + \mathcal{O}(8 \times N^3 \times K^3) + 8 \times N^2 \times K^2 + 16 \times N \times K^2 + 8 \times N^2 \times K + 12 \times N \times K + 6 \times K^2 - 2 \times N^2 + N - 2 \times K + 11 )</td>
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Particle Swarm Optimisation

- **Optimisation** for MBER-MUT
  \[ \mathbf{P}_{\text{TxMBER}} = \arg \min_{\mathbf{P} \in S_{N \times K}} F(\mathbf{P}) \]

- **Cost** function
  \[ F(\mathbf{P}) = P_e(\mathbf{P}) + G(\mathbf{P}) \]
  with penalty function
  \[ G(\mathbf{P}) = 0 \]
  if \( E[\|\mathbf{P}\mathbf{x}\|^2] - E_T \leq 0 \);
  \[ G(\mathbf{P}) = \lambda( E[\|\mathbf{P}\mathbf{x}\|^2] - E_T ) \]
  if \( E[\|\mathbf{P}\mathbf{x}\|^2] - E_T > 0 \)

- **Search space** \( S_{N \times K} \) with search range for each precoder coefficient
  \[ S = [-P_{\text{max}}, P_{\text{max}}] + j[-P_{\text{max}}, P_{\text{max}}] \]
PSO algorithm

PSO is a population based stochastic optimisation method inspired by social behaviour of bird flocking or fish schooling: a swarm of particles \( \{ \hat{P}_i^{(l)} \}_{i=1}^S \) evolves in search space \( S^{N \times K} \), where \( S \) is swarm size and index \( l \) denotes iteration.

- **a) Swarm initialisation** With iteration index \( l = 0 \), set \( \hat{P}_1^{(l)} \) to MMSE solution and randomly generate rest of initial particles, \( \{ \hat{P}_i^{(l)} \}_{i=2}^S \), in search space \( S^{N \times K} \).

- **b) Swarm evaluation** Particle \( \hat{P}_i^{(l)} \) has cost \( F(\hat{P}_i^{(l)}) \), based on which cognitive information \( \text{Pb}_i^{(l)} \), \( 1 \leq i \leq S \), and social information \( \text{Gb}_i^{(l)} \) are updated.

- **c) Swarm update** Each particle has a velocity, \( V_i^{(l)} \in V^{N \times K} \), to direct its flying:
  \[
  V_i^{(l+1)} = w \cdot V_i^{(l)} + \text{rand}() \cdot c_1 \cdot (\text{Pb}_i^{(l)} - \hat{P}_i^{(l)}) + \text{rand}() \cdot c_2 \cdot (\text{Gb}_i^{(l)} - \hat{P}_i^{(l)})
  \]
  \[
  \hat{P}_i^{(l+1)} = \hat{P}_i^{(l)} + V_i^{(l+1)}
  \]

- **d) Termination condition check.** If maximum number of iterations, \( I_{\text{PSO}} \), is reached, terminate with solution \( \text{Gb}_{(I_{\text{PSO}})} \); otherwise, \( l = l + 1 \) and go to **b).**
Complexity of PSO Based Design

- There are well-defined rules for choosing PSO algorithmic parameters.
- PSO based MBER-MUT design has total complexity

\[ C_{T_{PSO}} = I_{PSO} \times C_{1_{PSO}} \]

where \( I_{PSO} \) is number of iterations and \( C_{1_{PSO}} \) complexity per iteration.

Complexity per iteration \( C_{1_{PSO}} \) of PSO-MBER-MUT for QPSK signalling, where \( N \) is the number of transmit antennas, \( K \) the number of mobile users, \( M = 4 \) is the size of symbol constellation, \( S \) is the swarm size.

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<tr>
<td>(((16 \times N \times K + 7 \times K + 6 \times N + 1) \times M^K + 20 \times N \times K + 2) \times S + 8 )</td>
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- Since \( C_{1_{PSO}} < C_{1_{SQP}} \) and typically \( I_{PSO} \ll I_{SQP} \), PSO-MBER-MUT design imposes lower complexity than SQP-MBER-MUT design, i.e.

\[ C_{T_{PSO}} \ll C_{T_{SQP}} \]
**Simulation Results**

- BS with $N = 4$ transmit antennas communicated over $4 \times 4$ flat Rayleigh fading channels with $K = 4$ single-receive-antenna QPSK mobile users
- $S = 20$ and $I_{PSO} = 20$ to 40, depending on SNR value, which were adequate as PSO algorithm arrived at MBER solution with lowest $CT_{PSO}$
- Results obtained by averaging over 100 channel realisations
- Channel taps $h_{i,k}$: uncorrelated complex-value Gaussian with $E[|h_{i,k}|^2] = 1$
- In the case of channel estimation error, AWGN with variance 0.01 was added to each tap
Convergence Comparison

- Given SNR=Eb/No=10 dB, PSO-MBER-MUT converged after $I_{PSO} = 20$ iterations while SQP-MBER-MUT converged after $I_{SQP} = 100$ iterations.
- Given SNR=Eb/No=15 dB, PSO-MBER-MUT converged after $I_{PSO} = 40$ iterations while SQP-MBER-MUT converged after $I_{SQP} = 140$ iterations.
Complexity Comparison

- SNR=10 dB: Complexity in terms of (a) total Flops required, and (b) run times recorded, both yielding a complexity ratio $C_{T_{SQP}} : C_{T_{PSO}} \approx 7 : 1$

(a)

- SNR=15 dB: yielding a complexity ratio $C_{T_{SQP}} : C_{T_{PSO}} \approx 5 : 1$

(b)
Swarm size $S = 20$ was found to be appropriate

SNR=15 dB: (a) convergence performance, and (b) total complexity required

(a) (b)

- **Graph (a):** BER vs. iterations for different swarm sizes.
- **Graph (b):** FLOPS vs. swarm size.

![Graph (a): BER vs. iterations](image)

![Graph (b): FLOPS vs. swarm size](image)
Conclusions

We have proposed PSO assisted MBER-MUT algorithm

- Which offers a much lower computational complexity than existing SQP based MBER-MUT algorithm

Simulating system of four transmit antennas and four QPSK mobile users over flat Rayleigh fading channels has confirmed

- PSO-based MBER-MUT imposes approximately five to seven times lower complexity than SQP-based MBER-MUT