Pilot Symbol Assisted Coding

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Abstract

We propose a novel technique, hereby referred to as pilot symbol assisted coding (PSAC), where a predetermined fraction of binary pilot symbols is interspersed with the channel-coded bits at the channel coding stage, instead of multiplexing the pilots with the data symbols at the modulation stage, as in classic pilot symbol assisted modulation (PSAM). We subsequently demonstrate that the PSAC succeeds in gleaning more benefits from the pilot overhead investment, than just simply the capability of channel estimation such as in the PSAM technique.

Introduction: There are two dominant approaches that are frequently employed to estimate the channel; namely that of either estimating the channel blindly or using reference/pilot symbols. Typically blind channel estimation techniques impose a high complexity and suffer from a performance degradation as well as from a slow rate of convergence. On the other hand, the insertion of known pilot symbols into the transmitted data stream using PSAM [1] potentially achieves an improved better bit error ratio (BER) performance, at the expense of an unavoidable reduction of the effective throughput due to the associated pilot overhead. In this letter, we will assume that the receiver estimates the channel's amplitude and phase using known pilots; however, instead of inserting pilots at the modulation stage as in classic PSAM [1], the proposed pilot symbol assisted coding (PSAC) technique appropriately intersperses a predetermined fraction of pilot bits with the codeword bits. This provides the iterative decoder with additional extrinsic information, hence improving the achievable decoding performance. Our technique is generic, since it can be applied to any iterative decoding (ID) aided channel coding scheme. In fact we recently discovered that a somewhat similar technique was employed in the context of low-density parity-check codes (LDPC) [2]. However, in this letter we generalise the technique to also include non-systematic codes. We believe that the benefits of the proposed technique are more pronounced in non-systematic codes, whose ID process typically requires the employment of code doping for triggering their ID convergence.

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Pilot Symbol Assisted Transmitter: We elected to consider the application of the proposed PSAC technique in the context of left-regular, right-irregular non-systematic repeat-accumulate (RA) codes [3]. Firstly, the pilot overhead δ_1^p quantifying the required number of pilots K_p is determined by ensuring the effective sampling of the channel's complex-valued envelope at a sufficiently high rate that is higher than the Nyquist rate. This pilot symbol assisted encoder of rate $R > \delta_1^p$ is represented in Figure 1, which will then map a K-bit (input) information sequence represented by $\mathbf{a} = [a_1, a_2, \ldots, a_K]$ into a $(K^{'}R^{-1})$-bit output sequence $\mathbf{c}$, where $K^{'} = K + K_p$, by first attaching a known pilot-bit sequence $\mathbf{p} = [p_1, p_2, \ldots, p_{K_p}]$ to the beginning of the K-bit input stream $\mathbf{a}$, so that the modified $K^{'}$-bit input sequence becomes $\mathbf{a}' = [\mathbf{p}\ \mathbf{a}]$. Each bit in $\mathbf{a}'$ is then repeated d_v times. The value of each bit in the intermediate (check) bit sequence represented by $\mathbf{b}$ is then calculated by randomly selecting the check node degree $d_c \in \mathbf{d}$ bits from the repeated $\mathbf{a}'$ bits, where $\mathbf{d}$ denotes the set of all possible degrees, and then combined using modulo-2 addition. As it can be observed from Figure 1, K_p number of bits denoting the pilot-bit sequence $\mathbf{p}$ are left unselected and uncombined, however these are again attached to the beginning of $\mathbf{b}$ thus creating $\mathbf{b}' = [\mathbf{p}\ \mathbf{b}]$. Finally, the value of the each codeword bit $c_i \in \mathbf{c}$, $i = 1, \ldots, K^{'}R^{-1}$ is determined by calculating the values of $c_1 = b_1^{'}$ and of $c_i = b_i^{'} \oplus c_{i-1}^{'}$ for $i = 2, \ldots, K^{'}R^{-1}$, where $b_i^{'} \in \mathbf{b}^{'}$ and $\oplus$ represents the modulo-2 addition operation. Figure 1 also depicts what we refer to as the the pilot position interleaver Π_p, which spaces a pair of pilots every $(\eta - 1)$ data bits, where η denotes the pilot spacing. The data bits are separated by a pair of pilot bits instead of a single pilot, since we are going to consider a single-user multiple-input multiple-output (MIMO) system employing two transmit and two receive antennas. Hence a total of four channels have to be estimated using the two pilots. The interleaved codeword $\pi_p(\mathbf{c})$ is then modulated and re-encoded using a rate-one Alamouti space-time block code (STBC).

Pilot Symbol Assisted Receiver: The channel estimates calculated from the known pilots are then up-sampled and interpolated using a low-pass interpolator. Armed with this MIMO channel estimate, the received signal is then detected using a soft-input soft-output (SISO) maximum a-posteriori probability (MAP) detector. The detected signal is de-interleaved using Π_p and then passed to the decoder, which employs the classic belief propagation algorithm to estimate the original information bit sequence.

Recall that in a channel-coded PSAM scheme, the pilot bits are only beneficial for estimating the channel, which are removed after channel estimation. By contrast, the pilot bits in PSAC are also valuable for enhancing the performance of the decoder, hence they are retained and further processed by the decoder. Our argument is strengthened by examining the combined detector, accumulator (ACC) and check node decoder's (CND) extrinsic information transfer (EXIT) function, which can be approximated by

$$I_{E,D\&A\&C}(\cdot) \approx \frac{\delta_1^p}{\overline{d}_c} + \sum_{\forall d_c \in \mathbf{d} \backslash d_1} \Delta_{d_c}[1 - J\left(\sqrt{(d_c - 1) \cdot [J^{-1}(1 - I_A)]^2 + [J^{-1}(1 - I_E)]^2}\right)], \tag{1}$$

where $\overline{d}_c$, Δ_{d_c}, I_A, I_E and $J(\cdot)$ denote the average check node degree, the fraction of edges emanating from the check node of degree $d_c \in \mathbf{d}$, the a-priori CND information input, the extrinsic information ACC output and the mutual information function. The fraction $\overline{\delta} := \delta_1^p/\overline{d}_c$ in (1) contributed by the pilot bits is independent of both I_A and I_E, thus making the pilot bits indispensable for guaranteeing the reliable triggering of this iterative decoding process. Additionally, the pilot bits also contribute to the widening of the open EXIT tunnel between the $I_{E,D\&A\&C}(\cdot)$ curve and the corresponding outer decoder's EXIT function, thus potentially reducing the computational complexity. This occurs due to the fact that this $\overline{\delta}$-fraction is positive and proportional to δ_1^p.

Simulation results: We considered binary phase shift keying (BPSK) modulated transmission of $K = 10000$ bits over a (2×2) correlated Rayleigh MIMO channel. We assumed a high mobile velocity of 100 mph. The data signalling rate and the carrier frequency were those from the 3G standard, and were set to 15 kbps and 2 GHz, respectively. The proposed PSAC scheme described above was compared to its RA-coded PSAM counterpart, where both schemes had an effective throughput of 0.82 bits/s/Hz and a 10% pilot overhead.

Figure 2 compares the performance of the two schemes in terms of the achievable BER and the computational complexity, where the latter was evaluated in terms of the number of message-passing updates per decoded bit, given by $I_{\text{avg}}|E|/K$, where I_{avg} represents the average number of iterations required for finding a legitimate codeword and $|E|$ represents the number of edges in the Tanner graph. It can be observed that the proposed PSAC scheme exhibits small but noticeable BER performance gains. Besides this advantage, the computational complexity of the PSAC is also significantly lower, requiring 45% fewer message-passing updates per decoded bit than the benchmarker scheme at $E_b/N_0 = 2$ dB.

Conclusions: It is hence more beneficial to appropriately intersperse pilots at the channel code stage as in PSAC rather than at the modulation stage as in classic PSAM, since the pilot bits are not only useful for channel estimation but also for enhancing the decoder's performance. From another point-of-view, we can also regard pilot symbol assisted codes, as a family of codes, which are specifically designed for systems that require pilot-aided channel estimation.

References

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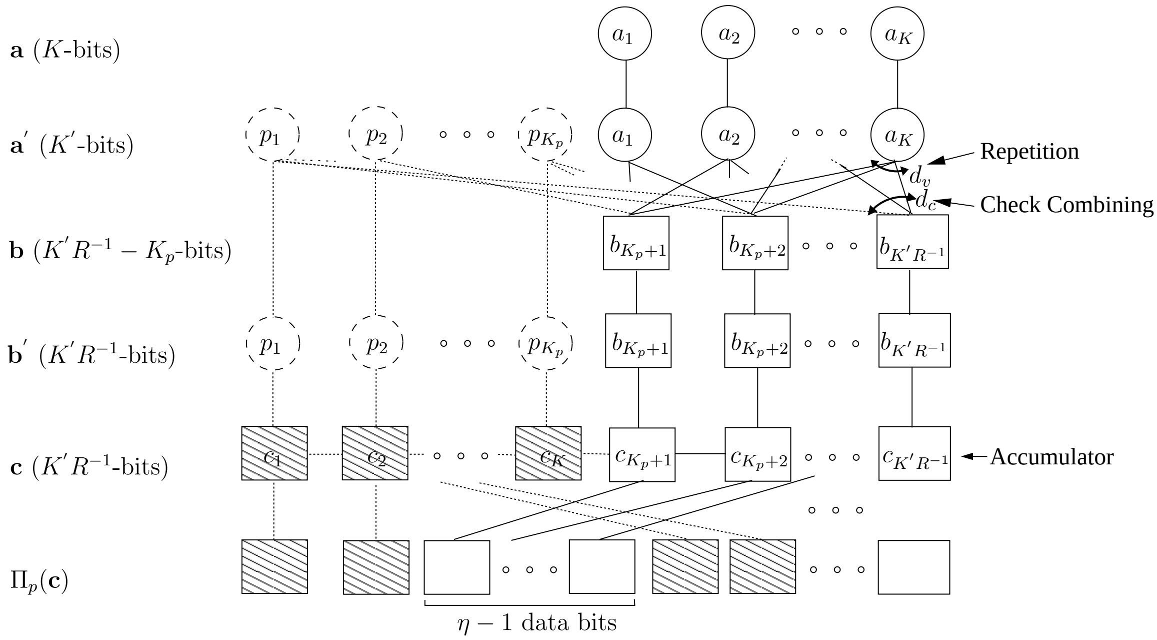

Figure 1: A graph-based representation of a specific pilot symbol assisted encoder of rate R.

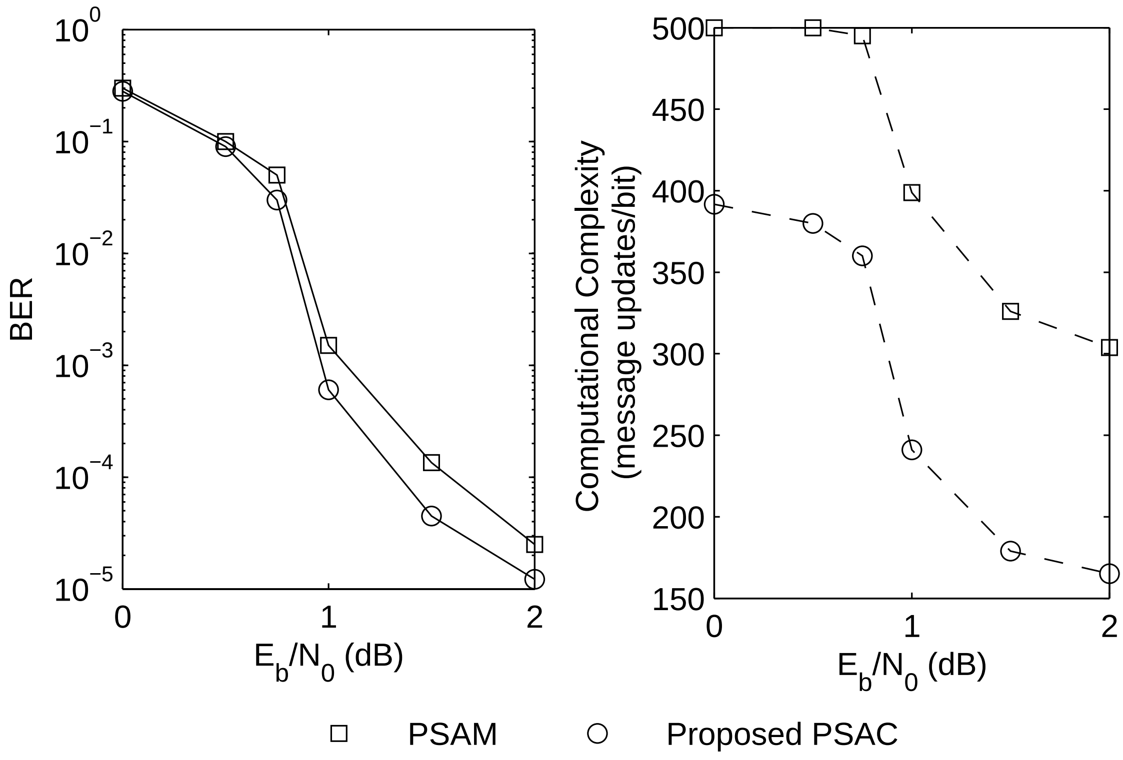

Figure 2: Performance comparison in terms of the achievable BER and the computational complexity.