Block-based precoding for serially concatenated codes

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***Abstract*—A block-based precoder is designed for serially concatenated schemes, which facilitates iterative decoding convergence towards the Maximum Likelihood (ML) error ratio performance at reduced channel Signal to Noise Ratios (SNRs) in practical schemes, where the affordable implementational and computational complexity is limited.**

Introduction: Recursive convolutional precoders [1] have been shown to assist the iterative decoding convergence of serially concatenated schemes. This may be attributed to their infinite impulse response, which facilitates the efficient spreading of extrinsic information in the receiver. In this letter, we propose an alternative precoder that is block-based rather than convolutional. Since it allows each block to be decoded independently, our approach facilitates a more efficient implementation, which can benefit from parallel processing, pipelining and a reduced memory requirement. Furthermore, *we will demonstrate that our precoder requires 50% fewer computational operations per decoding iteration than even the simplest of convolutional precoders, enabling it to outperform this benchmarker by about 2 dB, when the affordable decoding complexity is limited.*

The Algorithm: In the transmitter of Figure 1, the block-based precoder encodes the bit sequence $\mathbf{c}$ provided by the outer interleaver π_o in order to generate an input sequence $\mathbf{e}$ for the inner interleaver π_i, similarly to a convolutional precoder. In the receiver, the various decoders of Figure 1 iteratively exchange increasingly reliable extrinsic Logarithmic Likelihood Ratio (LLR) sequences $\tilde{\mathbf{e}}^\mathrm{e}$, which pertain to the bits of $\mathbf{e}$. The decoders are aided by the *a priori* LLR sequences $\tilde{\mathbf{e}}^\mathrm{a}$, which are generated by summing the extrinsic LLRs provided by the other decoders, as shown in Figure 1.

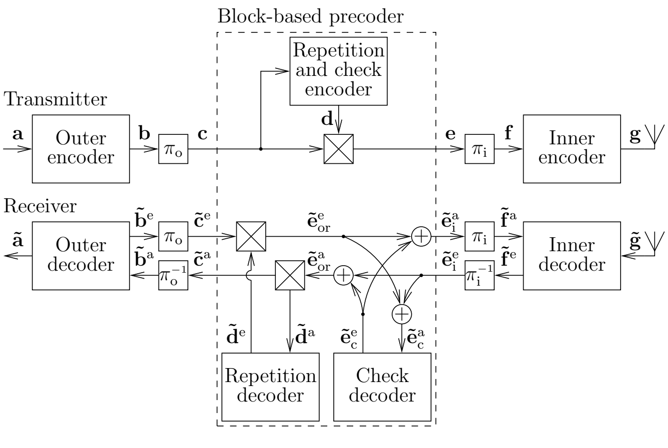

Fig. 1. Block-based precoder schematic.

If both the outer code and the precoder of Figure 1 support iterative decoding convergence towards the ML error ratio performance, then this can be achieved when the channel SNR is sufficiently high. A sufficient condition for the outer code to support this is satisfied if the possible permutations of its output bit sequence $\mathbf{b}$ have a free distance of at least two [2]. Similarly, desirable iterative decoding convergence is supported by the proposed block-based precoder, since it is designed to generate bit sequences $\mathbf{e}$ having a free distance of two, as we shall now detail.

The repetition and check encoder generates the $N_\mathbf{d}$-bit sequence $\mathbf{d}$ of Figure 1 by first decomposing the $N_\mathbf{c}$-bit sequence $\mathbf{c}$ into $(N_\mathbf{d}-1)/2$ equal-length sub-sequences $\{\mathbf{c}_j\}_{j=1}^{(N_\mathbf{d}-1)/2}$, where $N_\mathbf{d}$ is odd. The bits of $\mathbf{d} = \{d_i\}_{i=1}^{N_\mathbf{d}}$ having an index $i \in \{1, 3, 5, \ldots, N_\mathbf{d}-4, N_\mathbf{d}-2\}$ are obtained as the modulo-2 sum of the preceeding bit d_{i-1} and the bits in the sub-sequence $\mathbf{c}_{(i+1)/2}$, where we employ $d_0 = 0$. Meanwhile, the bits having an index $i \in \{2, 4, 6, \ldots, N_\mathbf{d}-3, N_\mathbf{d}-1, N_\mathbf{d}\}$ are set equal to the preceeding bit d_{i-1}. As indicated by the crossed block in Figure 1, the bit sequence $\mathbf{e}$ is obtained by multiplexing the bits of $\mathbf{c}$ and $\mathbf{d}$ in the order exemplified for $N_\mathbf{d} = 9$ in Figure 2.

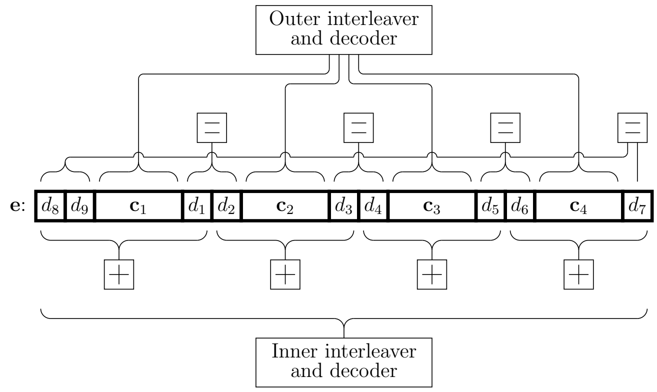

Fig. 2. Block-based precoder factor graph. Braces indicate for which bits each decoder can generate extrinsic LLRs.

The repetition decoder of Figure 1 generates extrinsic LLRs $\tilde{\mathbf{d}}^\mathrm{e}$ pertaining to the blocks of bits in $\mathbf{d}$ that have equal values, as indicated by the boxed equal signs in Figure 2. The extrinsic LLR pertaining to each bit in a particular block of $\mathbf{d}$ is obtained as the sum of the *a priori* LLRs in $\tilde{\mathbf{d}}^\mathrm{a}$ that pertain to the other bits in the block. Since most blocks contain only two bits, the repetition decoder has a negligible computational complexity. Note that the repetition of the bits in $\mathbf{d}$ during check and repetition encoding results in a free distance of at least two for the blocks shown in Figure 2, which is desirable as described above.

Similarly, the check decoder of Figure 1 employs the forward-backward algorithm [3] to generate extrinsic LLRs $\tilde{\mathbf{e}}^\mathrm{e}_\mathrm{c}$ pertaining to the blocks of bits in $\mathbf{d}$ that have even Hamming weights, as indicated by the boxed plus signs in Figure 2. Since the forward-backward algorithm does not consider multiple states, the check decoder has a 50% lower computational complexity than even the simplest of convolutional precoders, which employs two states [1]. Note again that the modulo-2 sums employed during check and repetition encoding are motivated, since an even Hamming weight is associated with a free distance of two.

Results: Let us now compare the performance of the scheme shown in Figure 1 to that of a benchmarker, in which the block-based precoder is replaced by a convolutional precoder. In both schemes, the source sequence $\mathbf{a}$ comprised 16-ary source symbols, having values that occur with unequal probabilities, resulting in an entropy of $E = 3.77$ bits per symbol. Furthermore, the inner code was provided by set partitioned $M_\mathrm{i} = 16$-ary Quadrature Amplitude Modulation

(16QAM) in both schemes. In the ‘block-based precoder’ scheme, a coding rate of $R_{\rm rc} = N_{\mathbf{c}}/(N_{\mathbf{c}} + N_{\mathbf{d}}) = 5/6$ was employed, while the outer code was provided by an $L_{\rm o} = 5$-bit Fixed Length Code (FLC), having a coding rate of $R_{\rm o} = E/L_{\rm o} = 0.754$ and a free distance of two. By contrast, the two-state convolutional precoder of the benchmarker scheme employed a coding rate of $R_{\rm p} = 1$. In order to maintain a throughput of $\eta = R_{\rm o} R_{\rm rc} \log_2(M_{\rm i}) = 2.51$ bits per channel use and to facilitate a fair comparison, a more error resilient $L_{\rm o} = 6$-bit FLC having a coding rate of $R_{\rm o} = 0.628$ and a free distance of two was employed in the ‘convolutional precoder’ scheme.

We elected to consider the transmission of $N_{\rm a} = 100$-symbol source frames, since this short frame length is typical in the challenging audio, speech and wireless sensor network scenarios. Since the affordable decoding complexity is typically limited in these scenarios, we considered the Symbol Error Ratio (SER) performance that can be achieved without exceeding a particular complexity limit. This limit was chosen to facilitate at least three decoding iterations in both of the schemes and at every channel SNR considered. Our SER performance results are plotted in Figure 3 for the case of transmitting over an uncorrelated narrowband Rayleigh fading channel having a range of channel SNRs per bit E_b/N_0. As shown in Figure 3, the ‘block-based precoder’ scheme offers a gain of approximately 2 dB compared to the ‘convolutional precoder’ benchmarker. Owing to their precoders, both schemes facilitate iterative decoding convergence towards the ML SER performance, as indicated by the similar gradients of the SER curves seen in the high-SNR region of Figure 3.

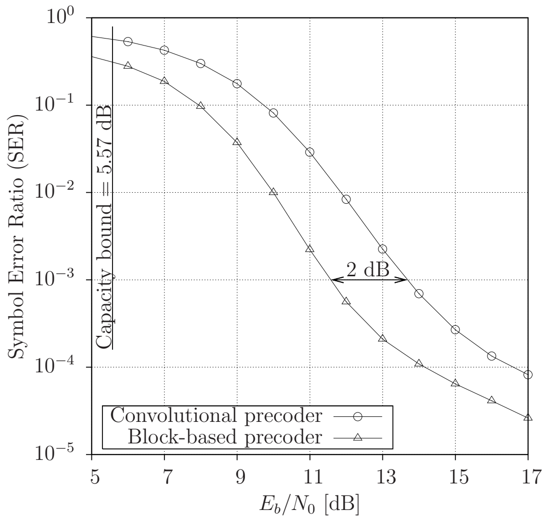

Fig. 3. SER performance of the ‘block-based precoder’ and the ‘convolutional precoder’ schemes.

References

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