

Calculation of photonic band-gap of one dimensional photonic crystal

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Abstract. Photonic crystal is a composite dielectric with periodic modulation of refractive index that brings about unique photonic band-gap effect by which light could be totally reflected on the surface of such material regardless of any incident angle or mode. Many applications based on this mechanism have been developed and introduced into new waveguides. Such band-gap effect could be understood by investigating the optical performance of one dimensional photonic crystal. In this paper, the principle of band-gap effect of one dimension photonic crystal is firstly explained by Bragg reflection, the band-gap centre is obtained; then the modelling of band-gap of one dimension photonic crystal is conducted as well, the result is in agreement with the Bragg results and shows that the centre of photonic band-gap shifts to larger wavelength with higher refractive index (RI) ratio or dimension ratio between the two component dielectrics, but the involved boundary condition in simulation leads to higher onset RI ratio (3.5) responsible for the presence of photonic band-gap. This work would be useful to the design and fabrication of photonic crystal devices.

1. Introduction

The photonic crystal consists of two dielectrics with different permittivity assumed to be constant [1]. These two dielectrics are arranged alternately in three dimensions. While the space between the two components is reduced to be as small as $\sim 1\mu\text{m}$, or even smaller, light beam with a particular frequency (or wavelength) that radiates on the crystal could be totally reflected. The energy responding to such frequency is right located in a forbidden region of its energy band and the light is forbidden transmitting. Such forbidden band is named the photonic band-gap. Materials with photonic band-gap attract increasingly both fundamental and technological interest around the world. The introduction of photonic crystal structure into optical fibres to create superb lower-loss optical fibres and nonlinear optical devices is one classic application [2], which is bringing growing attention.

Understanding of such photonic band-gap effect is of great importance. To that point, the basic one dimensional photonic crystal is chosen to be investigated for simplicity in this paper. The mechanism of photonic band-gap is illustrated by Bragg reflection and then the photonic band-gap is calculated through numerical modelling by the finite difference time domain (FDTD) method.

2. Bragg reflection in photonic crystal

A schematic light flow into one dimensional photonic crystal is presented in figure 1. A and B indicate two dielectrics constructing one dimensional photonic crystal, and their thickness and refractive indices are a , b and n_1 , n_2 respectively. Now the situation of perpendicular propagation of the incident light L_i is considered particularly.

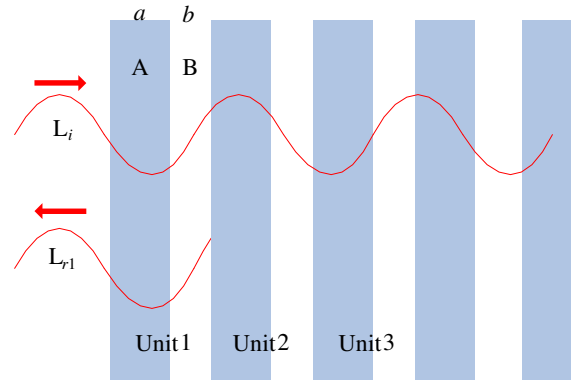


Figure 1. A light flow into one dimensional photonic crystal.

The two dielectrics are assumed to be loss-less within the operational frequency range, so the light beam reaching the internal interfaces undergoes only reflection and transmission without any absorption and scattering. As light energy goes off with each reflection, the eventual light energy that goes out the whole crystal will attenuate to be zero after infinite layers. That means that light could be totally reflected after infinite periodic units. Actually the total reflection can be realised within finite layers if the frequency of light is right in certain band-gap.

As for the photonic crystal in figure 1, the light beam should be reflected at each interface, however, the reflection at the interface of every periodic unit is considered to be dominantly responsible for its unique properties. When the incident light L_i with wavelength λ propagates into one dimensional photonic crystal, it will be reflected at the interface between the first two units and there comes the reflect light L_{r1} ; likewise, Light L_{ri} occurs at the interface between following units.

The difference of optical path between the incident light L_i and the reflect light L_{r1} is expressed as,

$$d = 2(n_1 \times a + n_2 \times b) \quad (1)$$

If the optical path difference d satisfies the condition in equation (2), L_i will superpose with L_{r1} and produce a standing wave, the other reflect lights L_{ri} will also join in the standing wave. Then the incident light L_i with wavelength λ is totally reflected, namely the Bragg reflection, and forbidden transmitting through the crystal. The corresponding frequency $f=c/\lambda$ (c is the velocity of light in vacuum) is right in an energy region which is forbidden by the photonic crystal and thus the incident light could be totally reflected. Such wavelength λ (or the frequency f) can be named as the centre of the band-gap.

$$d = 2(n_1 \times a + n_2 \times b) = m\lambda \quad m = 1, 2, 3... \quad (2)$$

The central wavelength λ_c of the band-gap is expressed in equation (3) and it is decided by the refractive index and dimension parameters. When considering the dimensionless parameters, n_1/n_2 , a/b , and the crystal lattice constant of the photonic crystal is $l=a+b$, the central wavelength can be expressed as follows,

$$\lambda_c = \frac{2(n_1 \times a + n_2 \times b)}{m} = 2n_2 \frac{l}{1 + \frac{a}{b}} \left(\frac{n_1}{n_2} \times \frac{a}{b} + 1 \right) \frac{1}{m} \quad m = 1, 2, 3... \quad (3)$$

2.1 Influence of refractive index ratio on central wavelength

Given that one dimensional photonic crystal is composed of dielectric A and air ($n_2 \approx 1.0$), the crystal lattice constant is $l=10\mu\text{m}$, the dependence of central wavelength on the refractive index ratio $n_1/n_2=n_1$ is shown in figure 2 ($m=2$).

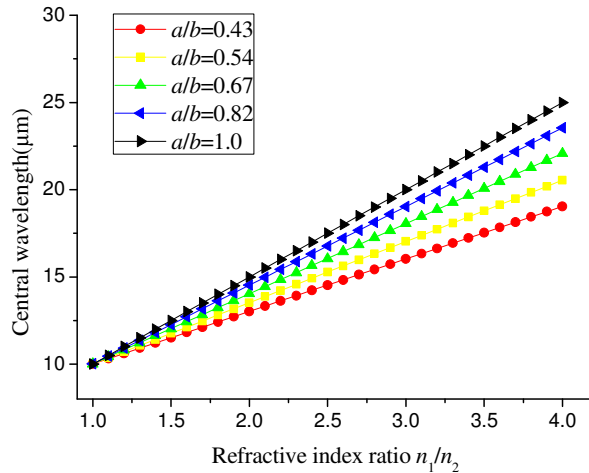


Figure 2. Dependence of central wavelength on refractive index ratio.

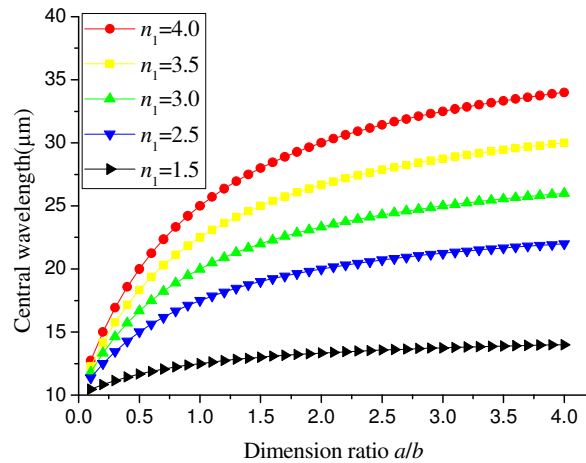


Figure 3. Dependence of central wavelength on dimension ratio.

As indicated in figure 2, the central wavelength increases with the rise of refractive index ratio n_1/n_2 , even when the ratio reaches 4.0, that is possibly the highest RI of silicon [3]. This suggests that photonic crystals can be made of materials ranging from transparent polymers to semiconductors.

2.2 Influence of dimension ratio on central wavelength

Given that the crystal lattice constant $l=10\mu\text{m}$, i.e. $a+b=10\mu\text{m}$; the refractive index of dielectric A is $n_1=1.5, 2.5, 3.0, 3.5, 4.0$, and the other dielectric is still air ($n_2 \approx 1.0$), $m=2$. The dependence of central wavelength on the dimension ratio a/b is shown in figure 3, which clearly expresses a non-linear increasing with dimension ratio a/b . It indicates that the thinner layer dielectric A with higher index has much influence than B on the central wavelength.

3. Simulated photonic band-gap

To verify the results obtained above, numerical simulation of the band-gap of one dimensional photonic crystal based on the FDTD method is conducted. The layout of one dimensional photonic crystal is shown in figure 4. The photonic crystal consists of dielectric A and B (air), and it is set to the right side of the excitation source in horizontal direction. The thickness of the dielectric A and air is a , b respectively, the lattice constant is $l=10\mu\text{m}$. a is increased from $0.5\mu\text{m}$ by $0.5\mu\text{m}$ step until $a/b=1.0$. The refractive index of the dielectric A n_1 varies from 1.0 to 4.0.

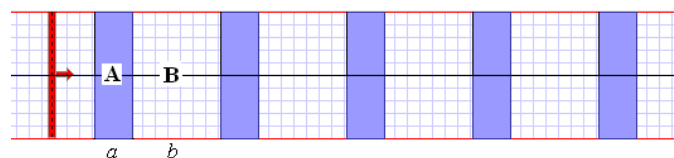


Figure 4. The layout of one dimensional photonic crystal in simulation.

The band-gap properties (indicated by wavelength not frequency) of one dimensional photonic crystal is calculated and shown in figure 5, figure 6. The width of band-gap increases nearly linearly with the refractive index ratio between the dielectric A and air; but the band-gap start to appear when the index ratio is above 3.5, which is different from the prediction by Bragg reflection. In figure 6, the

width of band-gap shows a maximum value when the dimension ratio is around 0.6, which apparently does not change at various refractive index ratios. And it goes higher with larger index ratio.

In addition, the centre of band-gap is also calculated. As shown in figure 7 and figure 8, the numeric results indicate same trend of dependence of centre of band-gap on the refractive index ratio and dimension ratio as in figure 2 and figure 3, except that band-gap starts to turn up at $n_1=3.5$, and the band-gap goes off at lower dimension ratio.

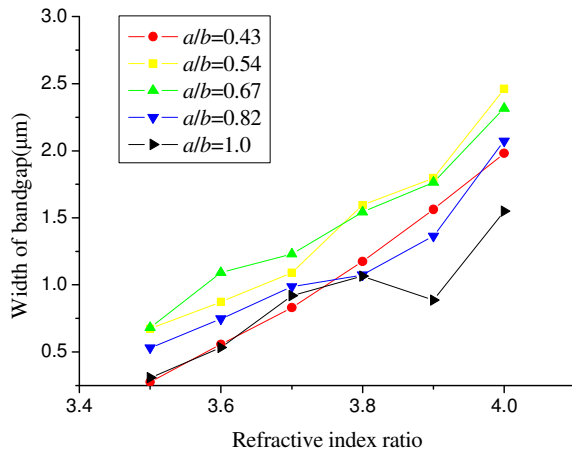


Figure 5. Width of band-gap vs. index ratio.

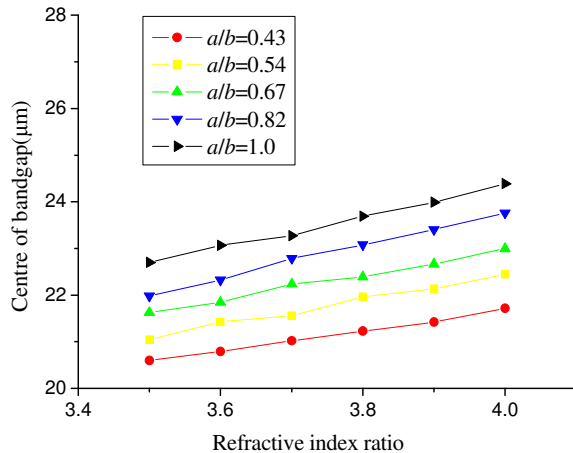


Figure 7. Centre of band-gap vs. index ratio.

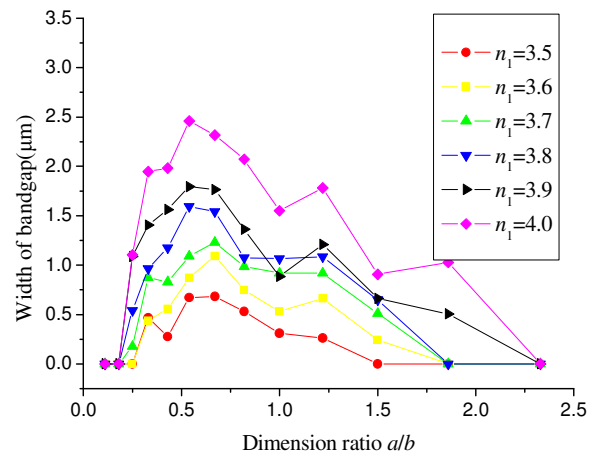


Figure 6. Width of band-gap vs. dimension ratio.

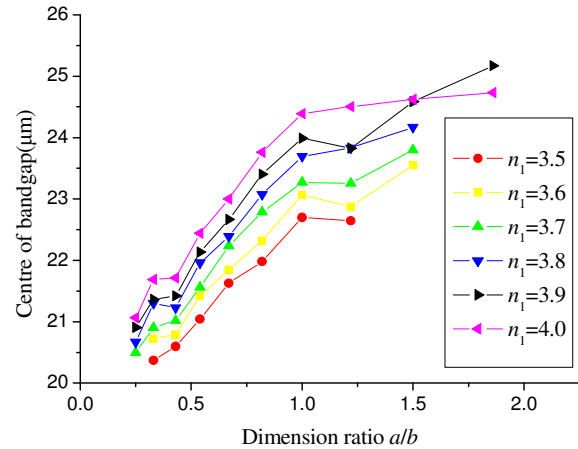


Figure 8. Centre of band-gap vs. dimension ratio.

4. Conclusion

Numeric results validate the analysis on the formation of photonic band-gap through Bragg reflection within one dimensional photonic crystal and verify that the equation of central wavelength can predict the possible band-gap centre even though indicated by wavelength not frequency, as well as the appropriate value of $m=2$ for the selected model. However the numerical analysis presents rigorous result that higher refractive index is required to obtain band-gap.

5. References

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