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FREQUENCY DOMAIN MODELLING OF HIGH VOLTAGE TRANSFORMERS USING A NONLINEAR LEAST SQUARE ESTIMATION TECHNIQUE

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Abstract: The development of transformer models can be achieved based on experimental frequency response measurements providing access to the windings is available and the physical dimensions are known in order to calculate R, L and C parameter values. Of interest are methods that allow the generation of a suitable model if the R, L and C parameters are unknown and access is restricted to the external terminals of the winding only. Using a lumped parameter model and the measured frequency response across the whole winding, it is possible to estimate the intermediary winding responses. Knowledge of the intermediary winding frequency responses facilitates the development of condition monitoring tools that are capable of locating the source of partial discharge activity or winding deformation within a faulty transformer [1]. This paper includes a full description of the developed modeling technique, along with experimental results from a model winding system that validates the proposed approach.

1. INTRODUCTION

Transformer modelling has been widely researched in order to understand transient behaviours. A large number of transformer failures are due to high frequency transients that are unpredictable and may cause catastrophic failure during live operation. The ability to estimate the effect of transients may assist in predicting transformer asset health and lifetime.

The modelling of propagation of high frequency signals within a transformer winding may demonstrate attenuation or be loss free depending on the model adopted [2]. Prior to application of any modelling technique, experimental investigations can be undertaken to determine the most suitable model to be employed.

The model and estimation technique described in this paper are employed based on a limited knowledge of winding dimension, which in turn, means that the resistance, inductance, capacitance and mutual inductance parameters are unknown and must be replaced by harmonic decay components in frequency domain [4]. By using a lumped parameter model along with the measured frequency response across the whole winding, it is possible to estimate the intermediary winding responses.

2. THE TRANSFORMER EXPERIMENTAL MODEL

The experimental transformer model used was developed and manufactured by Alstom and includes an interleaved disc winding and a plain disc winding. It has been further developed by the Tony Davies High Voltage Laboratory at the University of Southampton. One of its main characteristics is that it can see high voltages of up to 30kV without discharging.

The structure of experimental model has two types of winding (interleaved and plain disc winding). Figure 1(a) shows the half cross section view of the two windings wrapped around a central iron core. The interleaved disc winding is above the plain disc winding. The two windings have the same construction size and use identical materials. Every pair of discs of either winding provides a terminal as a measurement point. A metal cylinder connected to earth is placed inside the windings to represent an iron core.

2.1. Transformer equivalent circuit model

In the transformer experimental model, both windings are disc-type and each winding includes 14 discs. A pair of discs is called a 'section' where each section can be accessed externally with the direct connection from its terminals. The end connection of each winding are both connected to an end plate which noted as EP. Figure 1(a) also shows the arrangement of capacitance and inductance for both windings.

At high frequency, the lumped parameter circuit model is a series connection of capacitances and inductances arranged in parallel and series respectively [3]. This is shown in Figure 1(b). At high frequencies the voltage oscillations arising in a winding are damped by the winding resistance and the core losses. If the damping is also to be considered, the model of Figure 1(a) has to be completed with resistances. The resistance of the winding is taken into account by a resistance put in series with the inductances, whereas the core loss is represented by a resistance connected in parallel with them. C_{dx} and G_{dx} are the series of capacitance and shunt conductance to model the damping behaviour

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from winding to the ground. The relationship between each winding is modelled by its mutual inductance denoted as a matrix of inductance *[L]*. A section of such model is shown in Figure 1(b), where r is the resistance of a section of unit length of the winding, and g is the conductance corresponding to the iron loss in a unit length section of the core [1, 2].

2.2. The Corresponding Equivalent Equation

The lumped parameter circuit model consists of winding parameters which are simplified to derive a partial differential solution. The usual procedure in solving a partial differential equation is to assume the form of the solution and try it by direct substitution in the differential equation and the boundary conditions. The partial differential equation of a winding section can be described as follows [1]:

$$rK\frac{\partial^5 e}{\partial x^4 \partial t} + (1+\frac{r}{R})\frac{\partial^4 e}{\partial x^4} - LK\frac{\partial^4 e}{\partial x^2 \partial t^2}$$
$$-(rC+\frac{L}{R})\frac{\partial^3 e}{\partial x^2 \partial t} + LC\frac{\partial^2 e}{\partial t^2} + LG\frac{\partial e}{\partial t} = 0 \qquad (1)$$

The above equation shows that the solution to the partial differential equation has a nonlinear relationship of parameter values and the distance x, where x is often referred to as the ratio of total length x/l of the winding.

By solving Equation (1) and also considering the quarter range cosine series the complete solution becomes:

$$e = E\frac{cosh(\beta x)}{cosh(\beta l)} + E\sum A_s e^{-\gamma_s t} cos(\omega_s t) cos(2k-1)\frac{\pi x}{2l} \qquad (2)$$

Where, A_s amplification factor, γ_s damping factor, ω_s harmonic frequency, k harmonic order, x distance from neutral line, l total length of transformer winding, β fixed distribution factor, E input voltage, e voltage level at distance x, and t the time vector.

Equation 2 has two terms a fixed distribution and a harmonic damping equation. The equation represents the high frequency oscillation at different points along the winding.

2.3. General Solution for the Fixed Distribution Equation

The fixed distribution equation can be written in general form as shown in Equation 3.

$$F_x(s) = A.\frac{A_P \cosh(\alpha x) + \sinh(\alpha x)}{A_P \cosh(\alpha l) + \sinh(\alpha l)} \qquad (2)$$

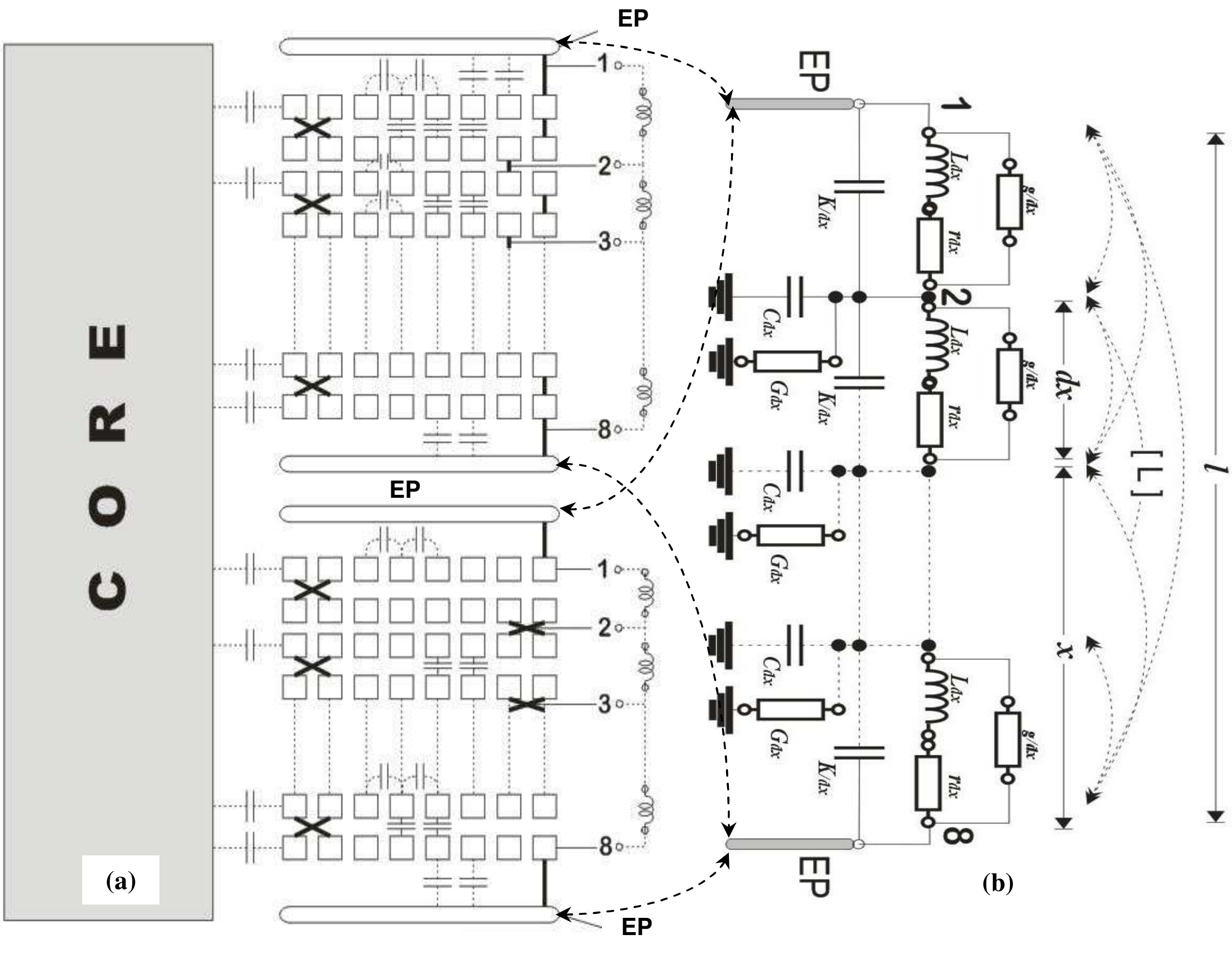

Figure 1: (a) Crossection view of transformer winding and, (b) Its equivalent lumped parameter model

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where A is a constant, A_P is the corresponding equivalent impedance coefficient at the end windings. Figure 2 shows the variation of fixed distribution curve with different parameter values of A_P and α (with A is fixed). In the case of real transformers, the only readily available terminals for measurement are at the bushing tap point and neutral to earth point. The bushing tap point can be modelled as a capacitance between the bushing core bar and earth.

2.4. The Solution in frequency domain.

The focus of the study is to model the intermediary winding response. The use of a frequency domain estimation was selected to determine values of the unknown parameters. The performance of the system is easily interpreted using the frequency domain, which covers all possible sets of transient signals, and therefore is suitable for modelling PD propagation or transformer winding deformation.

Therefore by considering only the available terminals, the neutral and the bushing, the frequency response between these two terminals may be measured and used as the basis for the estimation of the intermediary winding responses. Therefore, the following equation is derived to define any intermediary winding response at different x in frequency domain:

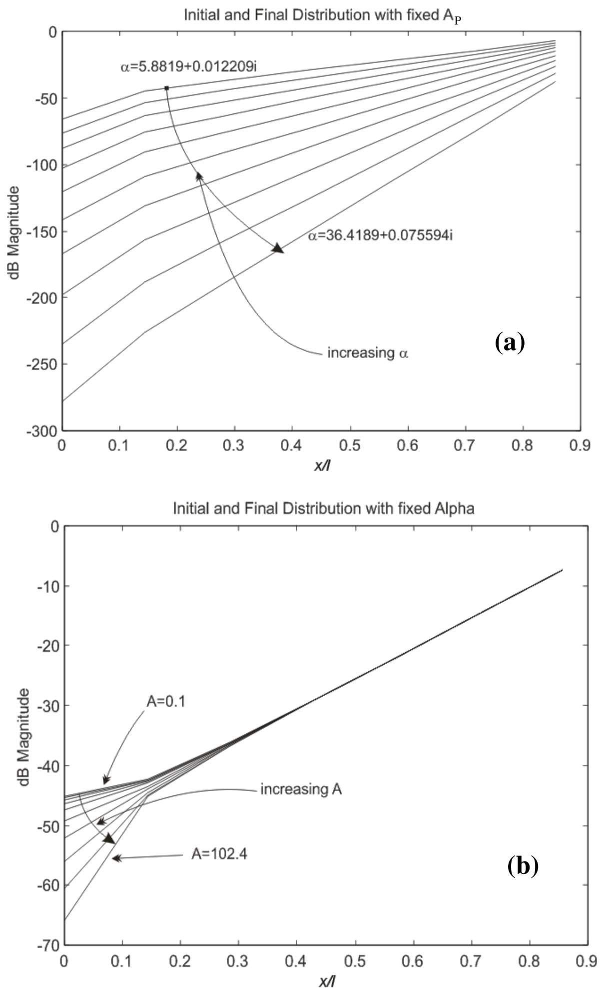

Figure 2: Transformer Distribution Curve (a) Fixed A_P (b) Fixed α

$$H_x(s) = F_x(s)\left[1 + G_n A_m A_c\right] \qquad (4)$$

where $H_x(s)$ is the dB level at any distance x along the transformer windings, $F_x(s)$ is the fixed distribution equation in (3), G_n is the constant gain, A_m amplification factor function with respect to k (harmonic order), and A_c is the quarter range cosine series function with respect to to k and x.

3. FREQUENCY RESPONSE MEASUREMENT

Frequency response measurements can be used as an alternative impulse response measurements [4]. The range of frequency under an ideal impulse test is infinite therefore it is of interest to obtain a frequency response measurement over the largest possible frequency range.

The frequency range of the measurement is dependent on the measuring equipment. The measurement was carried out using a network analyser, Agilent 4395A. It has a bandwidth frequency starting from 100Hz to 500MHz range with a sample bandwidth of 30Hz.

The measurement circuit is shown in Figure 3(a). Due to limitations of the measurement device, the data was taken many times and resampled to further refine the complete frequency response which contains a total 1400 sample points over range 100Hz to 500MHz.

3.1. Winding Frequency Response

The winding frequency response was measured for all 7 sections where the terminals of the winding are accessible for measurement. The experiment setup immitates a real transformer winding in operation, for which the winding is not accessible from the outside. Hence it can be assumed that the frequency response data obtained are representative of a transformer winding. Figure 4(a) and 4(b) show the corresponding frequency response and phase response for an increasing number of winding sections.

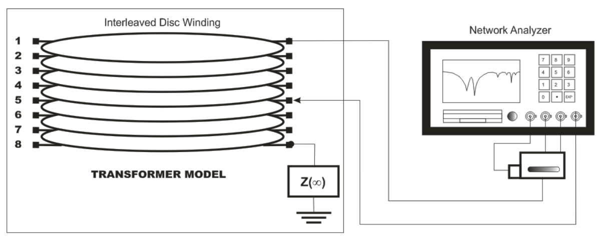

Figure 3: Frequency Response Measurement Circuit diagram.

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3.2. Transformer Winding Distribution from Frequency Response Measurement

Figure 2 shows the theoretical gain along the transformer winding by only considering the fixed distribution equation. Whereas Figure 5 shows the gain of winding sections at different frequencies noted by vertical straight lines. The gain of the winding frequency response of the first section is represented by the ‘black’ curve and the ‘blue’ curve represents the whole 7 Sections. Figure 6 shows the distribution level in dB versus the length of the transformer winding derived from Figure 5.

4. NONLINEAR LEAST SQUARE TECHNIQUE

The nonlinear least squares estimation technique is used to model a nonlinear set of unknown parameters a_k, k = 1,2, …, M to a predicted model with a known equation. The model predicts a functional relationship between the measured independent and dependant variables,

$$y(x) = y(x; a_1 ... a_M) \tag{5}$$

where the dependence on the parameters is indicated explicitly on the right hand side. The objective of the technique is to minimize Equation (6)

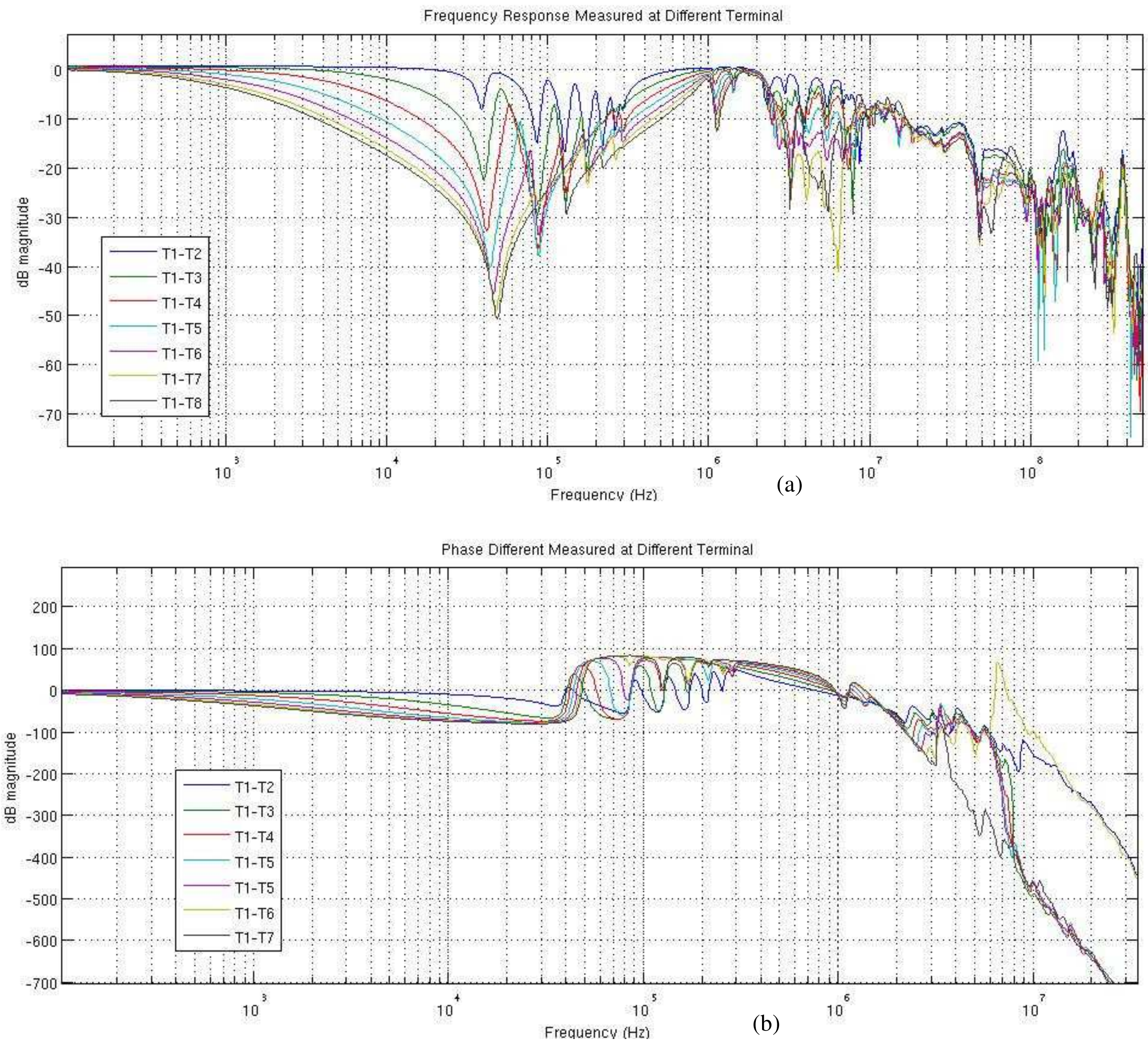

Figure 4: (a) The Frequency Response (b) The Phase Response

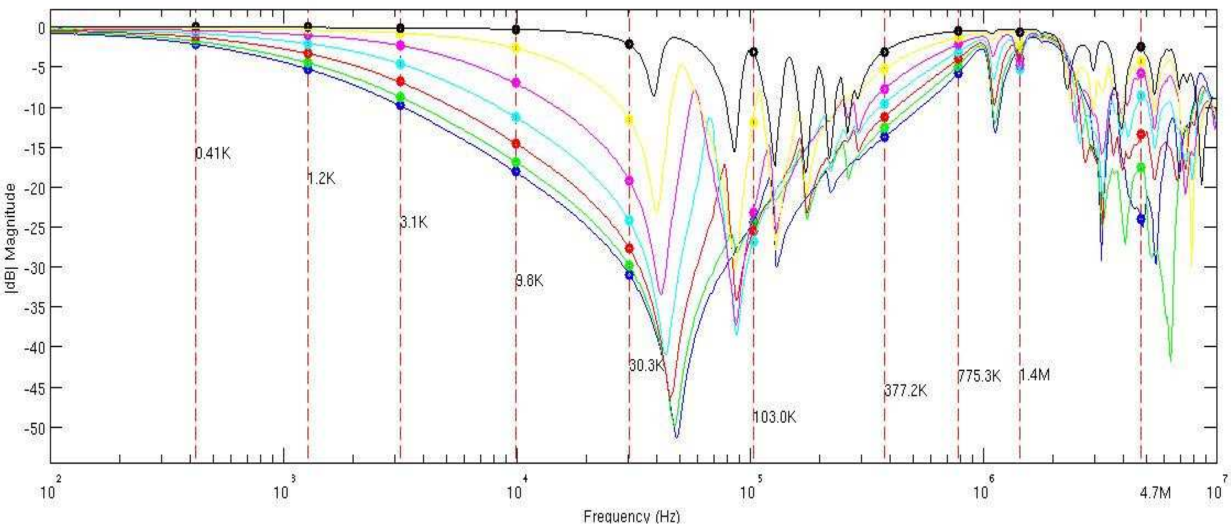

Figure 5: Frequency Response Measurement and corresponding gain for winding sections at specific frequencies.

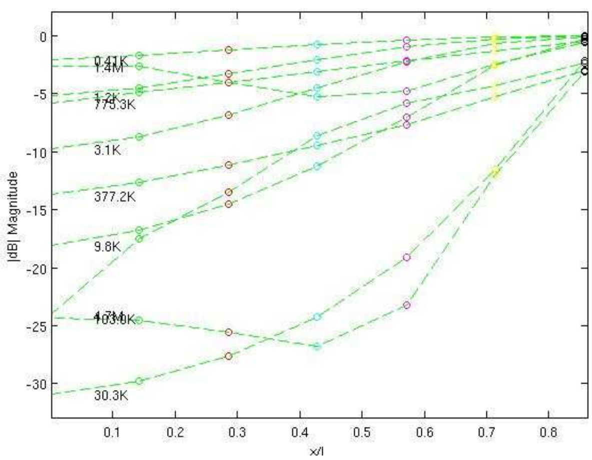

Figure 6: Gain Level Distribution at different Frequency extracted from Figure 5 starting from 0.41kHz to 4.7MHz.

$$\sum_{i=1}^{N}[y_i - y(x_i;a_1 \cdots a_M)]^2 \qquad (6)$$

4.1. Chi-Square Function

By using the chi-square function, the minimization of the model can be explained as follows:

The model to be fitted is

$$y = y(x;a_1 \cdots a_M) \qquad (7)$$

and the χ^2 function is

$$\chi^2(a_1 \cdots a_M) = \sum_{i=1}^{N} \frac{y_i - y(x_i;a_1 \cdots a_M)}{\sigma_i}^2 \qquad (8)$$

the calculation of the gradient of the χ^2 with respect to $a_{k\ldots}\ a_M$ has components:

$$\frac{\partial \chi^2}{\partial a_k} = -2\sum_{i=1}^{N} \frac{[y_i - y(x_i;a_1 \cdots a_M)]}{\sigma_i} \frac{\partial y(x_i;a_1 \cdots a_M)}{\partial a_k} \qquad (9)$$

and by taking the second derivative:

$$\frac{\partial^2 \chi^2}{\partial a_k \partial a_l} = 2\sum_{i=1}^{N} \frac{1}{\sigma_i^2}\left[\frac{\partial y(x_i;a_1 \cdots a_M)}{\partial a_k}\frac{\partial y(x_i;a_1 \cdots a_M)}{\partial a_l} - [y_i - y(x_i;a_1 \cdots a_M)]\frac{\partial^2 y(x_i;a_1 \cdots a_M)}{\partial a_l \partial a_k}\right. \qquad (10)$$

Rewriting terms in equation (9), results as follows:

$$\beta_k = -\frac{1}{2}\frac{\partial \chi^2}{\partial a_k} \qquad \alpha_{kl} = \frac{1}{2}\frac{\partial^2 \chi^2}{\partial a_k \partial a_l} \qquad (11)$$

A set of linear equations can be constructed to model the fitting technique;

$$\sum_{i=1}^{N} \alpha_{kl}\delta_{al} = \beta_k \qquad (12)$$

From which the minimization process is possible to estimate for the value of parameter a_k by using equation:

$$a_{\min} = a_{cur} + \delta_{al}[-\nabla\chi^2(a_{cur})] \qquad (13)$$

4.2. The Levenberg-Marquardt method

The Levenberg-Marquardt method makes the minimization technique more robust and efficient through the use of a fast gradient descent technique.

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The technique introduces a variable learning rate. If λ is defined as a constant learning rate such that:

$$\delta_{al} = \frac{1}{\lambda \alpha_{ll}} \beta_l \quad \text{or} \quad \lambda \alpha_{ll} \delta_{al} = \beta_l \tag{14}$$

by letting the new diagonal parameter to be:

$$\alpha'_{jj} = \alpha_{jj}(1+\lambda) \tag{15}$$

$$\alpha'_{jk} = \alpha_{jk} \qquad j \neq k \tag{16}$$

and replacing the above definition into equation (12) a new linear equation will result as follows:

$$\sum_{i=1}^{N} \alpha'_{kl}\, \delta_{al} = \beta_k \tag{17}$$

4.3. The Fitting Results

Figures 7 to 10 show Fitting Results of the transformer modeling and parameter estimation in frequency domain. The overall performance results in an average error of less than 0.02dB rms. Note that during the initial iterative procedure the performance gradient was unstable due to the sudden jump of the constant value of λ which was quickly recovered by the power of fitting algorithm (Figure 8b, 9b, 10b).

For the fitting process, Equation 4 is the model of equation to be fitted with unknown parameter values G_n, k, A and A_P. Equation 4 is used for this estimation as it has simplified unknown parameters and can be further simplified to have linear relationship that easily modelled using the fitting process. The unknown parameters matrix in this case is $[a_1\ a_2\ a_3\ a_4] = [G_n\ k\ A\ A_P]$. In Figure 7 to 10 the parameter values obtained for each frequency are defined.

5. CONCLUSION

There are two main factors that need to be considered that influence the accuracy of the technique proposed. First is the choice of theoretical model to be used which consequently determines the model accuracy of the fitting process. The possibility of fitting failure will occur if the initial model does not adequately represent the winding frequency response measurement.

The second factor, is the numerical solution of the fitting technique. Since there exists a nonlinear relationship of parameters to the fitting model, the derivation of the first order derivative and the second order derivative may be difficult to achieve. Therefore, the authors have overcome this problem by using a finite difference technique.

The results have shown that it is possible to model a transformer winding at high frequencies. This facilitates the estimation of the behaviour of a transient signal generated within a high voltage transformer. Moreover, with the assistance of a robust estimation technique, the proposed method can be further developed to assist in predicting and locating any arbitrary partial discharge that may occur within a transformer winding.

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