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# MODELLING OF SURFACE CHARGE DECAY IN A SPHERICAL CAVITY WITHIN A SOLID DIELECTRIC MATERIAL USING FINITE ELEMENT ANALYSIS

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**Abstract**: The modelling of cavity surface charge decay through conduction along the cavity wall using the Finite Element Analysis (FEA) method is presented in this paper. A field-dependent cavity surface conductivity is proposed and the Phase-Resolved Partial Discharge (PRPD) patterns obtained from experimental measurements used to validate the simulation results generated using the model. A comparison between the simulation and measurement results has also been undertaken to verify the surface charge decay effect.

## 1. INTRODUCTION

The charges due to partial discharge (PD) that are accumulated on the cavity surface may decay with time through surface conduction along the cavity wall, resulting in charge recombination within the cavity [1]. This paper discusses the modelling of field-dependent cavity surface conductivity to determine the surface charge decay through conduction along the cavity wall, which depends on the magnitude and polarity of voltage across the cavity due to the applied field and voltage due to the cavity surface charges. A comparison between the simulation and measurement results is presented to support the proposed theory.

## 2. EXPERIMENTAL SETUP

Figure 1 shows the schematic diagram of the experimental setup that has been used in this work, which is based on the OMICRON mtronix PD system. It consists of a high voltage supply, a high voltage filter, a coupling capacitor $C_k$, a test object, a coupling device, a PD detector and a USB control which is connected to a personal computer (PC). The coupling device and the PD detector are used to detect and measure the apparent charge magnitude of the discharge signal from the test object. The output from the PD detector is connected to the USB control via fibre optic cables and the data is sent to the PC to display, store and analyse the PD events.

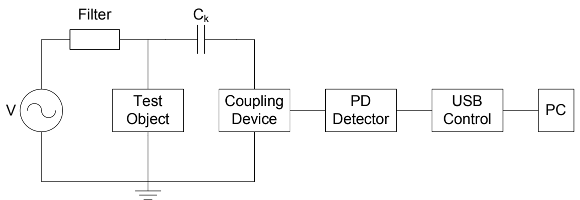


**Figure 1:** Schematic diagram of the experimental setup

Figure 2 shows the schematic diagram of the test object which consists of an artificial 1 mm diameter spherical void in the middle of dielectric material of 2.9 mm thick and 32 mm diameter. The material used is Araldite Rapid epoxy resin and its hardener, using a mixture ratio of 1:1. The whole test object is immersed in mineral oil to prevent surface discharge around the electrode and the material boundary. A 50 Hz, 20 kV sinusoidal voltage is applied to the test object.

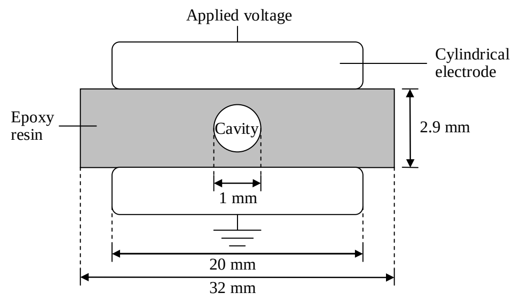


**Figure 2:** Schematic diagram of the test object

## 3. THE MODEL

Figure 3 shows details of the 2D model geometry of the test object that has been used in the experiment. The model consists of a homogenous dielectric material ($\varepsilon_r$=3.7) of 2.9 mm thickness and 10 mm diameter, a hemispherical cavity of 1 mm diameter due to the centre axis of symmetry and a cavity surface of 0.05 mm thickness to model the cavity surface charge decay through conduction along the cavity wall. The horizontal line in the cavity centre represents the area used to calculate the current during the PD process. A 50 Hz, 20 kV sinusoidal voltage is applied at the upper electrode while the lower electrode is always grounded.

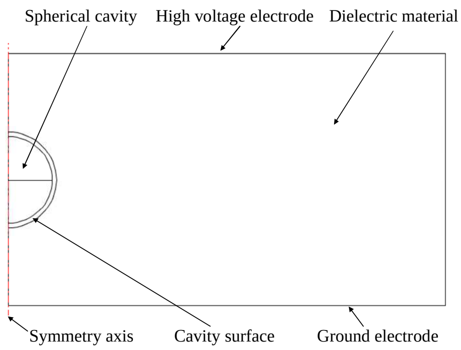


**Figure 3:** Complete 2D axial-symmetric model geometry

The cavity inception electric field is defined as [2-4]

$$E_{inc} = E_{lp} p \left[ 1 + \frac{B}{(2pr)^n} \right] \quad (1)$$

where $E_{lp}$, B and n are parameters of ionization processes characterization in the gas, p is the pressure in the cavity and r is the radius of the cavity. For air, $E_{lp}$ = 24.2 $VPa^{-1}m^{-1}$, n = ½ and B = 8.6 $Pa^{1/2}$ [2, 4]. The corresponding cavity inception voltage $U_{inc}$ can be obtained through the FEA model, where $U_{inc}$ is the voltage across the cavity when inception field $E_{inc}$ is reached.

When the voltage across the cavity U exceeds the inception voltage, $U_{inc}$ there is a possibility that discharge occurs in the cavity providing that there is a free electron in the cavity to start an avalanche. The electron generation rate due to field emission from the cavity surface, $N_{est}$ is calculated by

$$N_{est} = N_{es} \exp \sqrt{|U / U_{inc}|} \quad (2)$$

where $N_{es}$ is the initial electron generation rate which depends on the number of detrappable charges from the cavity surface due to the occurrence of previous PD. Assuming that the initial electron generation rate at $U_{inc}$ is $N_{e0}$, if previous PD occurs at voltages higher than $U_{inc}$, there will be extra electrons available for the next PD. Thus, these extra electrons are modeled by the term $U_{PD}/U_{inc}$ where $U_{PD}$ is the voltage of PD occurrence. However, since trapped electrons might decay through diffusion from shallow traps into deeper traps of the material [5], an extra term is added and is represented by $\exp(-t/\tau_{mat})$, where t is the time elapsed since previous PD occurrence and $\tau_{mat}$ is the material time constant. Therefore, the initial electron generation rate, $N_{es}$ can be described as

$$N_{es} = N_{e0} (U_{PD} / U_{inc}) \exp(-t / \tau_{mat}) \quad (3)$$

In order to consider the statistical aspect of PD, a probability function is used to determine the occurrence of discharge. The probability that a discharge occurs within the cavity is calculated by

$$P = 1 - \exp(-N_{est} dt) \quad (4)$$

where dt is the time stepping interval. P is compared with a random number R that is between 0 and 1. Only if P is greater than R will a discharge occur.

Discharge activity is modelled dynamically as an increase of conductivity in the cavity [6]. The conductivity in the cavity is calculated by

$$\sigma = \begin{cases} \sigma_{max} \{1 - \exp[-(|U / U_{inc}| + |I / I_{crit}|)]\} & \text{when PD} \\ 0 & \text{no PD} \end{cases} \quad (5)$$

where $\sigma_{max}$ is the maximum conductivity during PD, I is current through the cavity, $I_{crit}$ is the critical current to start an electron avalanche, U is voltage across the cavity and $U_{inc}$ is the inception voltage. Discharge stops when the voltage across the cavity drops less than extinction voltage, $U_{ext}$.

## 4. CAVITY SURFACE CHARGE DECAY

The PD charges that have been deployed on the cavity surface may decay in time either through diffusion from shallow traps into deeper traps of the material or recombination of positive and negative charges through conduction along the cavity wall and in the cavity.

The decay rate of accumulated charges due to PD on the cavity surface through conduction along the cavity wall depends on the surface conductivity and the applied frequency [1, 2, 7, 8]. The cavity surface conductivity varies depending on the aging level [8-10]. The surface charge decay through conduction is possible because the amount of charges that can be deposited firmly in the material along the cavity surface and being trapped is time dependent. During discharge process, when the first layer of charges has arrived on the cavity surface and before being trapped, it tends to repel the oncoming next layer of charges that is arriving, increasing the landing time of the next layer of charges on the cavity surface. The repelled charges might remain free on the entire cavity surface for certain periods of time or be free to move along the cavity wall before being trapped in a surface state.

In previous work [2, 7, 11], the 'rabbit-ear' like pattern in a PRPD histogram for a measurement at a frequency of 50 Hz occurs due to the detrapping of electrons from a negative cavity surface charge, when the polarity of the electric field in the cavity changes between two consecutive PD events. These PD models assume an increase in the effective detrapping work function of the cavity surface and a decrease in proportionality factor of number of detrappable charge when polarity of the electric field in the cavity changes compared to when the polarity does not change. However, this phenomenon might also be due to PD charges that still remain on the cavity surface decaying through conduction along the cavity wall before the next PD occurrence and causing the electron surface emission to decrease when the polarity of the electric field in the cavity changes. This surface charge decay can be modelled using a field-dependent cavity surface conductivity which depends on the surface charge movement along the cavity wall through surface conduction. The movement of the charge depends on the magnitude and polarity of voltage across the cavity due to the applied field and due to the cavity surface charge.

Figure 4(a-b) shows the movement of PD free charges along the cavity surface in relation to the voltage polarity in the cavity. When voltage due to the applied field in the cavity, $U_0$ and voltage due to surface charge from previous PD, $U_s$ have opposite polarity to each other but the magnitude of $U_0$ is larger than $U_s$, voltage across the cavity, $U_{cav}$ has opposite polarity to $U_s$. This causes free charges due to previous PD that are accumulated on the cavity surface to tend to move towards the center of the upper and lower cavity surface at where they are deposited by PD due to the influence of the electric field in the cavity, as shown in

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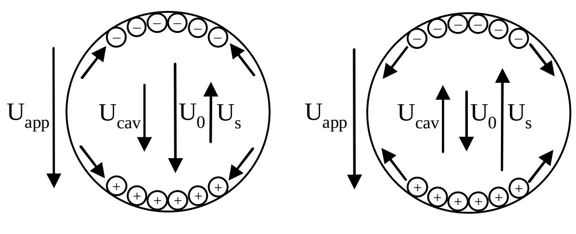
Figure 4(a). The charge mobility on the cavity surface becomes lower, thus the cavity surface conductivity becomes lower as well. When this happens, charge decay through conduction along the cavity wall is less likely to occur and the further charge decay is through diffusion into deeper traps of the material, where the decay rate depends on the material time constant [12]. Thus more free charges are still left on the cavity surface by the time the next PD is likely to occur, where these free charges enhance the magnitude of $U_{cav}$ when the polarity of the voltage across the cavity changes. This also explains why the initial electron generation rate, $N_{es}$ or the electron surface emission is larger when the polarity of the electric field in the cavity does not change between two consecutive PD events because more free charges still remain on the cavity surface.

When voltage due to the applied field in the cavity, $U_0$ and voltage due to the surface charge from previous PD, $U_s$ have opposite polarity to each other but the magnitude of $U_s$ is larger than $U_0$, or when both of them have similar polarity, the polarity of $U_{cav}$ becomes similar with $U_s$. This causes free charges due to previous PD that are accumulated on the cavity surface to tend to move towards the opposite direction from where they are deposited by the PD due to the influence of the electric field in the cavity, as shown in Figure 4(b). The charge mobility on the cavity surface becomes higher and the cavity surface conductivity becomes higher as well. The movement of positive and negative charges towards each other along the cavity wall causes charge recombination. Thus, free charges on the cavity surface decrease faster, reducing the magnitude of $U_s$ and the initial electron generation rate, $N_{es}$ is smaller by the time the next PD is likely to occur. Since this condition happens after the polarity of the electric field in the cavity changes between two consecutive PD events, the lower electron generation rate due to surface charge decay might be one of the reasons why the 'rabbit-ear' like pattern is obtained in experimental measurement results.




(a) (b)

**Figure 4:** The movement of PD free charges and electric potential direction (a) when $U_{cav}$ has the opposite polarity of $U_s$ and (b) when $U_{cav}$ has the same polarity of $U_s$ respectively

Thus, by accounting for surface charge decay through conduction along the cavity wall, the equation for initial electron generation rate, $N_{es}$ (3) needs to be modified. This is because the decay of surface charge affects the supply of charges for the next PD occurrence. Since voltage due to surface charge, $U_s$ is proportional to the amount of surface charge, the new equation for $N_{es}$ can be rewritten as

$$N_{es} = N_{e0}(U_{PD} / U_{inc})(U_s / U_{sPD}) \exp(-t / \tau_{mat}) \quad (6)$$

where $U_{sPD}$ is the voltage due to surface charge immediately after a PD occurrence. From (6), it can be seen that when surface charge decays through conduction, $U_s$ will decrease and in turn cause the initial electron generation rate, $N_{es}$ to decrease. However, $N_{es}$ cannot decrease to zero because electrons can also be supplied by charge injection from the electrode. Thus another control parameter is introduced into the simulation, this is the minimum electron generation rate, $N_{esmin}$.

The relationship between the cavity surface conductivity, $\sigma_s$ and the cavity surface time constant, $\tau_s$ can be derived by modelling the surface time constant as a RC decay time constant. Since the electric field in a spherical cavity is uniform, by assuming the cavity is a cylindrical cavity with radius r (the radius of the conducting surface) and distance between the upper and lower surface is d (the conducting surface in the direction of current flow), $\tau_s$ can be written as

$$\tau_s \cong RC = \frac{d}{2\pi r \sigma_s} \frac{\varepsilon_0 \pi r^2}{d} = \frac{\varepsilon_0 r}{2\sigma_s} \quad (7)$$

The parameters used in the simulation to reproduce the measurement results are detailed in Table I.

**Table 1:** Definition of parameters used in the simulation

| Definition | Value | Unit |
|---|---|---|
| Radius of cavity, r | 0.5 | mm |
| Thickness of insulation, D | 2.9 | mm |
| Applied voltage amplitude, $U_{app}$ | 20000 | V |
| Simulation number of cycles | 500 | |
| Time step during no PD, dt | 1/360f | s |
| Time step during PD | 1 | ns |
| Insulation relative permittivity, $\varepsilon_{rins}$ | 3.7 | |
| Cavity surface relative permittivity, $\varepsilon_{rsurf}$ | 3.7 | |
| Cavity relative permittivity, $\varepsilon_{rch}$ | 1 | |
| Insulation conductivity, $\sigma_{ins}$ | $1\times10^{-18}$ | S/m |
| Initial cavity surface conductivity, $\sigma_s$ | $1\times10^{-18}$ | S/m |
| Cavity conductivity at no PD, $\sigma_0$ | 0 | S/m |
| Maximum cavity conductivity at PD, $\sigma_{max}$ | $1\times10^{-2}$ | S/m |
| Surface conductivity for charge decay, $\sigma_{decay}$ | $8\times10^{-9}$ | S/m |
| Cavity inception voltage, $U_{inc}$ | 4.51 | kV |
| PD extinction voltage, $U_{ext}$ | 3.00 | kV |
| Critical current, $I_{crit}$ | 10 | mA |
| Material time constant, $\tau_{mat}$ | 4 | ms |
| Initial electron generation rate at $U_{inc}$, $N_{e0}$ | 2500 | N/s |
| Minimum electron generation rate, $N_{esmin}$ | 100 | N/s |

## 5. RESULTS

Figure 5(a-b) shows the simulated electric equipotential lines from the FEA model before and after the first PD in a spherical cavity within the dielectric material at 50 Hz of 20 kV sinusoidal voltage respectively. Initially without discharge, the electric field is higher in the cavity than the material due to the fact that the permittivity of air is lower than the

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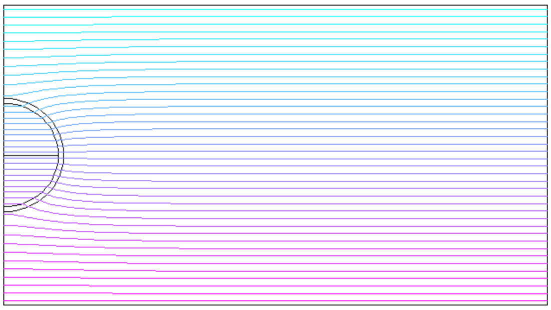
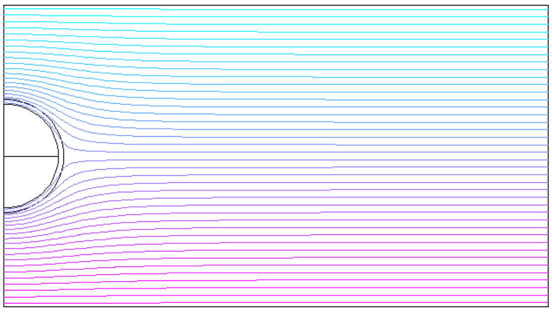
permittivity of the material. This is represented by the close-packed contour lines of electric potential in the cavity. The electric field in the whole spherical cavity is uniform in its direction and magnitude. There is no electric field due to surface charge before the first PD occurs within the cavity.

In Figure 5(b) the first PD occurs in the cavity. Discharge is assumed to affect the whole cavity. During discharge, the potential across the cavity decreases until it drops to less than the extinction voltage and discharge stops. The electric field in the cavity after discharge becomes very low due to charge movement across the cavity, which is represented by a lack of contour lines of electric potential within the cavity. However, the electric field on the upper and lower cavity surface is higher than the surrounding dielectric because charges from PD are accumulated on the cavity surface and trapped at surface state. This is represented by the closed-packed contour lines of electric potential on the upper and lower cavity surface. Those charges from the PD produce a field that is in the opposite direction of the applied field and causes a reduction of electric potential across the cavity.



(a)



(b)

**Figure 5:** Simulated electric equipotential lines of FEA model before (a) and after the first PD (b) at 50 Hz of 20 kV sinusoidal applied voltage

Figure 6 shows the plot of voltage across the cavity due to the applied voltage, $U_0$, voltage across the cavity, $U_{cav}$, voltage due to the surface charge, $U_s$ and the inception voltage, $U_{inc}$ against time for the first two complete cycles without considering the cavity surface charge decay through conduction along the cavity wall from the simulation of an applied 50 Hz, 20 kV sinusoidal voltage. In the absence of surface charge due to PD, the voltage across the cavity is equal to voltage due to the applied field in the cavity. After PD has occurred, the voltage across the cavity is modified by the presence of surface charge from PD. The voltage across the cavity, $U_{cav}$ in the presence of PD surface charge is equal to $U_0 + U_s$. When the surface charge decay through conduction along the cavity wall is not considered, where the cavity surface conductivity is fixed in the model throughout the simulation, $U_s$ remains constant until the next discharge occurrence, as shown in Figure 6 because the surface charge remains on the cavity surface.

When the surface charge decay through conduction along the cavity wall is considered in the model, the voltage plots become different, as detailed in Figure 7(a-c). Figure 7(a) shows the plot of $U_0$, $U_{cav}$, $U_{inc}$ and $U_s$, Figure 7(b) shows the field-dependent cavity surface conductivity plot, $S_s$ and Figure 7(c) shows the discharge pulse plot from the simulation of a 50 Hz, 20 kV sinusoidal applied voltage respectively. The voltage plots are influenced by the field-dependent cavity surface conductivity in the FEA model. In region A, when the polarity of $U_{cav}$ is opposite to $U_s$, the surface charge does not decay through conduction along the cavity wall. Thus $U_{cav}$ remains constant until the next PD occurrence. The initial cavity surface conductivity is set to a small value in the simulation. Thus the decrement of initial electron generation rate due to surface charge decay along the cavity wall does not occur. However, if the trapped surface charge decays through diffusion into deeper traps, the electric field in the cavity is unaffected but the electron generation rate decreases. This is because electrons in deeper traps are harder to detrap. With reference to Figure 7(a), PD happens almost immediately at point b and point c because the surface charge due to previous PD at point a does not decay through surface conduction and the time interval between them is small. Thus the initial electron generation rate is high when the inception voltage is exceeded at point b and point c.

In region B, when the polarity of $U_{cav}$ changes, the surface charge decay through conduction along the cavity wall starts to take place. This is because the polarity of $U_{cav}$ is similar with $U_s$. Thus the cavity surface conductivity in the model is set higher than its initial value, as shown in Figure 7(b). This causes $U_s$ to decrease and $U_{cav}$ in this region rises slower compared to the case when surface charge decay is not considered (Figure 6). The decay of surface charge through conduction causes the initial electron generation rate to decrease much faster, increasing the statistical time lag and PD is shifted forward in phase at point d. The magnitude of discharge pulse is also larger when PD happens at point d, as shown in Figure 7(c) because PD occurs at a higher level of voltage across the cavity than the inception voltage due to a longer statistical time lag. In region C, when the polarity of $U_{cav}$ is opposite to $U_s$ again, charges on the cavity surface due to previous PD do not decay through surface conduction. Thus the cavity surface conductivity is reset to its initial value. This region is a repetition of region A and this occurrence repeats in the following cycles (E, G).

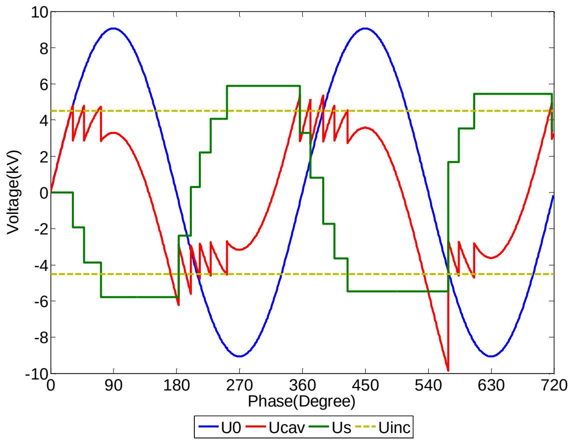
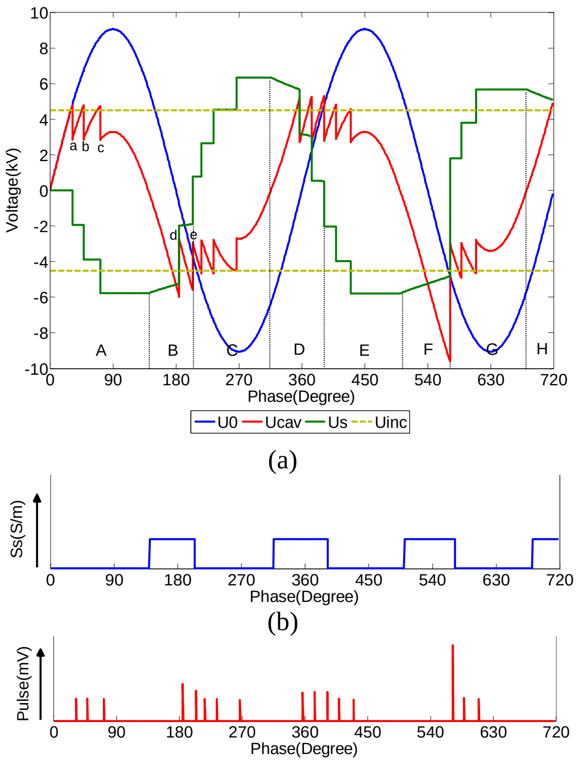



**Figure 6:** Plots of voltage without considering the surface charge decay through surface conduction. The surface conductivity is fixed throughout the simulation. Applied voltage: 50 Hz, 20 kV sinusoidal




(a)

(b)

(c)

**Figure 7:** Plots of (a) voltage by considering surface charge decay, (b) cavity surface conductivity and (c) discharge pulse against time for the first two complete cycles respectively. Applied voltage: 50 Hz, 20 kV sinusoidal

From the FEA model, when the cavity surface conductivity is higher than its initial value, the voltage across the cavity $U_{cav}$ is reduced. This explains the concept of surface charge decay through surface conduction. When the surface charge decays through conduction along the cavity wall, the electric field produced by the surface charge decreases and hence its electric potential decreases as well. When the polarity of voltage across the cavity is similar to the polarity of voltage due to cavity surface charges, the rising rate of $U_{cav}$ is slower compared to the case when the surface charge does not decay. Therefore, this is the reason why the surface conductivity in the simulation is set higher than its initial value when modeling the surface charge decay through conduction along the cavity wall.

The value of $\sigma_{decay}$ in Table 1 is assigned by simulating the PRPD patterns of the measurement data with various values of $\sigma_{decay}$. The cavity surface conductivity is changed to $\sigma_{decay}$ when the polarity of $U_{cav}$ is similar to $U_s$ provided that $U_s$ is not equal to zero and is reset to the initial conductivity value when the polarity of $U_{cav}$ becomes opposite to $U_s$ again or when $U_s$ is equal to zero.

Figure 9 and Figure 10(a-c) show the PRPD pattern of measurement result and the simulated PRPD patterns by using parameters in Table I with various values of cavity surface conductivity for a 50 Hz, 20 kV sinusoidal applied voltage respectively. It is seen from the simulation results, various values of surface conductivity yield different PRPD patterns by looking at the front slopes of the 'rabbit-ear' like pattern. This is because the value of cavity surface conductivity determines the decay rate of surface charge, where higher conductivity value causes faster surface charge decay rate and faster reduction rate of voltage due to surface charge in the cavity. The pattern in Figure 10(c) is the closest to the measurement results. Therefore, this validates the hypothesis that modelling of surface charge decay through conduction along the cavity wall by using field-dependent surface conductivity is acceptable.

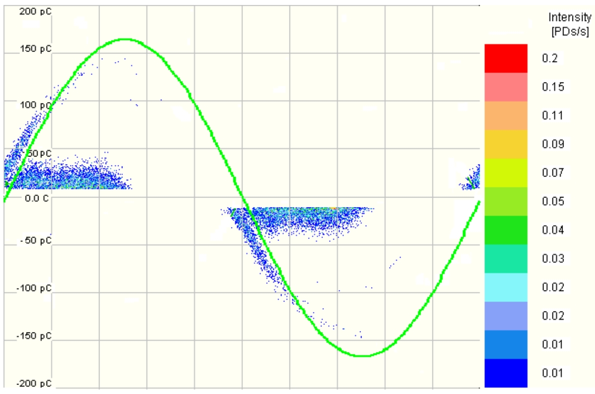
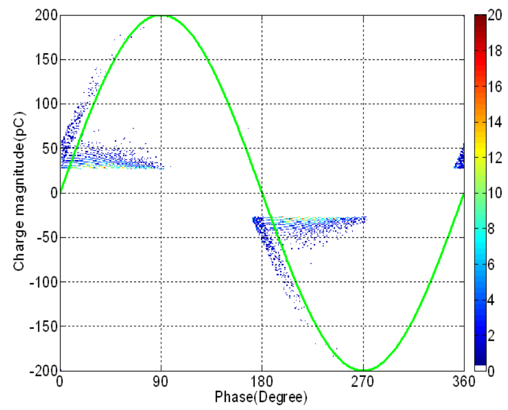
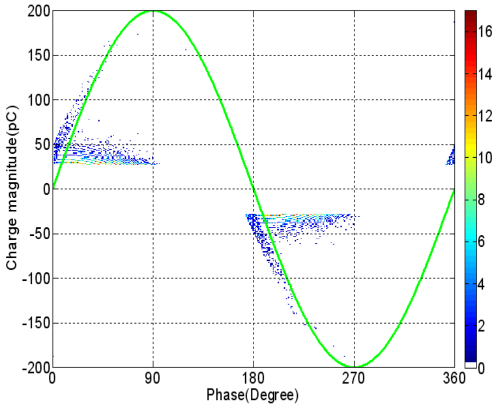


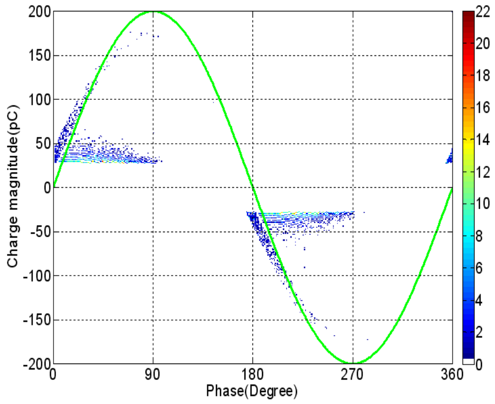

**Figure 9:** PRPD pattern of measurement result at 50 Hz, 20 kV sinusoidal




(a)




(b)




(c)

**Figure 10:** PRPD patterns of simulation with various values of cavity surface conductivity respectively; (a) $1x10^{-18}$, (b) $4x10^{-9}$ and (a) $8x10^{-9}$ S/m

## 6. CONCLUSION

When voltage across the cavity has the opposite polarity of voltage due surface charge, the accumulated charges on the cavity surface due to previous PD tend to concentrate at the centre of the cavity surface at where they are deposited. This ensures that the surface charge does not decay through conduction along the cavity wall and the initial electron generation rate is high when the next PD is likely to occur.

When voltage across the cavity has the same polarity of voltage due surface charge, the accumulated surface charges due to previous PD tend to move in the opposite direction from where they are deposited through conduction along the cavity wall and are subsequently reduced through recombination. This results in fewer charges left on the cavity surface and thus the initial electron generation rate is lower when the next PD is likely to occur.

The ‘rabbit-ear’ like pattern in the measurement result is obtained because surface charge decays through conduction along the cavity wall when the polarity of the voltage across the cavity changes between two consecutive discharge occurrences. The surface charge decay through conduction has been effectively modelled using an additional field-dependent cavity surface conductivity term.

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