Robust $\mathcal{H}_\infty$ Control for Model-Based Networked Control Systems with Uncertainties and Packet Dropouts

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Outline

Outline

Some Basics

An NCS is a control system in which the control loop is closed via a shared communication network.

The advantages:

- Low installation cost.
- Reducing system wiring.
- Easy maintenance.

The inherented problems:

- Packet dropout.
- Packet delay.
- Bandwidth constraint.

Existing Works

- Robust $\mathcal{H}_\infty$ control has been investigated for the NCS with delays [8,13–15].
- Most of existing works use fixed controller.
- In Model-based NCS, the network is only located between sensor and controller [16,17].

Outline

Our Novelty

- Study robust $\mathcal{H}_\infty$ control for NCS with packet dropouts.
- Consider the generic MB-NCS
  - the plant has time-varying norm-bounded parameter uncertainties;
  - packet dropouts occur in both the S/C and C/A channels.

Outline

Model Based NCS

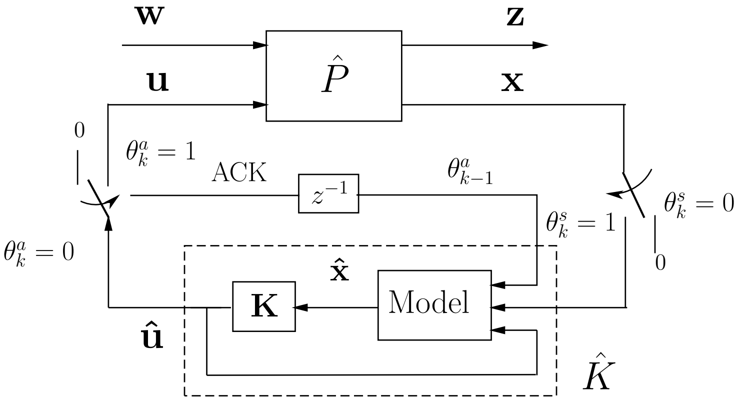


Figure: Networked control system $\hat{P}_K$.

Plant Description

The plant $\hat{P}$:

$$\begin{cases} \mathbf{x}(k+1) = [\mathbf{A} + \Delta\mathbf{A}(k)]\mathbf{x}(k) \\ \qquad + [\mathbf{B} + \Delta\mathbf{B}(k)]\mathbf{u}(k) + \mathbf{B}_w\mathbf{w}(k), \quad \forall k \in \mathbb{N}. \\ \mathbf{z}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k), \end{cases}$$

The time-varying parameter uncertainties satisfy:

$$[\Delta\mathbf{A}(k)\ \Delta\mathbf{B}(k)] = \mathbf{M}\ \mathbf{F}(k)\ [\mathbf{N}_a\ \mathbf{N}_b].$$

with $\mathbf{F}^{\mathrm{T}}(k)\mathbf{F}(k) \leq \mathbf{I}$.

Network Assumptions

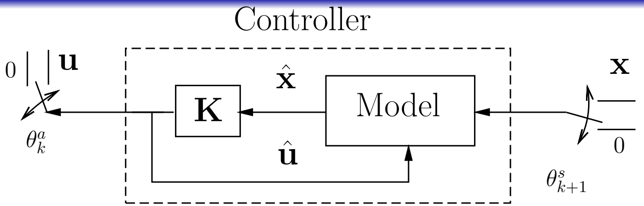
- Packet dropouts indicators:

$$\begin{cases} \theta^s_{k+1} \in \{0,1\}, & \text{in S/C channel;} \\ \theta^a_k \in \{0,1\}, & \text{in C/A channel.} \end{cases}$$

Then the system index $r_k = f(\theta^s_{k+1}, \theta^a_k) \in \mathcal{N} \triangleq \{1,2,3,4\}$.

- r_k is driven by Makov chain.
- TCP-like protocol.

Controller Mechanism



Controller is running as:

$$\begin{aligned} \hat{\mathbf{x}}(k+1) &= \theta_{k+1}^s \mathbf{x}(k+1) + (1-\theta_{k+1}^s)(\mathbf{A}\hat{\mathbf{x}}(k) + \mathbf{B}\mathbf{u}(k)) \\ &= \begin{cases} \mathbf{x}(k+1), & \theta_{k+1}^s = 1, \\ \mathbf{A}\hat{\mathbf{x}}(k) + \mathbf{B}\mathbf{u}(k), & \theta_{k+1}^s = 0. \end{cases} \end{aligned}$$

where $\mathbf{u}(k) = \theta_k^a \hat{\mathbf{u}}(k)$ with $\hat{\mathbf{u}}(k) = \mathbf{K}_{r_k}\hat{\mathbf{x}}(k)$, $r_k \in \mathcal{N}$.

State feedback gain matrices $\mathbf{K}_i$, $i \in \mathcal{N}$, but only $\mathbf{K}_3$ and $\mathbf{K}_4$ are needed, as $\theta_k^a = 0$ for $i = 1, 2$.

Outline

NCS Dynamics

The NCS $\hat{P}_K$ in the form of Markovian jump linear system:

$$\begin{bmatrix} \overline{\mathbf{x}}(k+1) \\ \mathbf{z}(k) \end{bmatrix} = \begin{bmatrix} \overline{\mathbf{A}}_{r_k}(k) & \overline{\mathbf{B}}_{r_k} \\ \overline{\mathbf{C}}_{r_k} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \overline{\mathbf{x}}(k) \\ \mathbf{w}(k) \end{bmatrix}, r_k \in \mathcal{N}$$

where $\mathbf{e}(k) = \mathbf{x}(k) - \hat{\mathbf{x}}(k)$, $\overline{\mathbf{x}}(k) \triangleq [\mathbf{x}^\mathsf{T}(k)\ \mathbf{e}^\mathsf{T}(k)]^\mathsf{T}$.

NCS Dynamics

$\overline{\mathbf{A}}_i(k) = \mathbf{\Phi}_i + \overline{\mathbf{M}}\,\overline{\mathbf{F}}(k)\mathbf{\Gamma}_i$, $i \in \mathcal{N}$, where

$$\mathbf{\Phi}_i = \begin{bmatrix} \mathbf{A} + \theta_k^a \mathbf{B}\mathbf{K}_i & -\theta_k^a \mathbf{B}\mathbf{K}_i \\ \mathbf{0} & (1-\theta_{k+1}^s)\mathbf{A} \end{bmatrix},$$

$$\mathbf{\Gamma}_i = \begin{bmatrix} \mathbf{N}_a + \theta_k^a \mathbf{N}_b \mathbf{K}_i & -\theta_k^a \mathbf{N}_b \mathbf{K}_i \\ (1-\theta_{k+1}^s)(\mathbf{N}_a + \theta_k^a \mathbf{N}_b \mathbf{K}_i) & -(1-\theta_{k+1}^s)\theta_k^a \mathbf{N}_b \mathbf{K}_i \end{bmatrix}.$$

$$\overline{\mathbf{M}} = \mathrm{diag}\{\mathbf{M}, \mathbf{M}\},\ \overline{\mathbf{F}}(k) = \mathrm{diag}\{\mathbf{F}(k), \mathbf{F}(k)\}.$$

Only $\mathbf{K}_3$ and $\mathbf{K}_4$ are needed, as $\theta_k^a = 0$ for $i = 1, 2$.

Outline

Robust stability

Theorem 1: The NCS $\hat{P}_K$ with $\mathbf{w}(k) \equiv \mathbf{0}$ and driven by the Markov chain $r_k \in \mathcal{N}$ is robustly stochastically stable if there exist scalars $\epsilon_i > 0$ and matrices $\mathbf{X}_i > 0$ for $i \in \mathcal{N}$ such that $\forall i \in \mathcal{N}$

$$\begin{bmatrix} -\mathbf{X}_i & \mathbf{X}_i \mathbf{\Phi}_i^T \mathbf{W}_i & \mathbf{X}_i \mathbf{\Gamma}_i^T \\ * & \epsilon_i \overline{\mathbf{W}}_i - \mathbf{X} & \mathbf{0} \\ * & * & -\epsilon_i \mathbf{I} \end{bmatrix} < 0,$$

where $\overline{\mathbf{M}}$, $\mathbf{\Phi}_i$ and $\mathbf{\Gamma}_i$ are given by the NCS dynamics, while

$$\mathbf{W}_i = \left[\sqrt{p_{i1}}\ \mathbf{I}\ \sqrt{p_{i2}}\ \mathbf{I}\ \sqrt{p_{i3}}\ \mathbf{I}\ \sqrt{p_{i4}}\ \mathbf{I} \right],$$

$$\overline{\mathbf{W}}_i = \mathbf{W}_i^T \overline{\mathbf{M}}\ \overline{\mathbf{M}}^T \mathbf{W}_i,$$

$$\mathbf{X} = \mathrm{diag}\{\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4\}.$$

Outline

Robust Stabilisation Control

Theorem 2: The NCS $\hat{P}_K$ with $\mathbf{w}(k) \equiv \mathbf{0}$ and driven by the Markov chain $r_k \in \mathcal{N}$ is robustly stochastically stable if there exist $\epsilon_i > 0$, $\mathbf{Q}_i > 0$ and $\mathbf{Y}_i$ for $i \in \mathcal{N}$ such that $\forall i \in \mathcal{N}$

$$\begin{bmatrix} -\tilde{\mathbf{Q}}_i & \tilde{\mathbf{\Phi}}_i^T \mathbf{W}_i & \tilde{\mathbf{\Gamma}}_i^T \\ * & \epsilon_i \overline{\mathbf{W}}_i - \tilde{\mathbf{Q}} & \mathbf{0} \\ * & * & -\epsilon_i \mathbf{I} \end{bmatrix} \triangleq \mathbf{\Theta}_i < 0,$$

where $\tilde{\mathbf{Q}}_i = \mathrm{diag}\{\mathbf{Q}_i, \mathbf{Q}_i\}$, $\tilde{\mathbf{Q}} = \mathrm{diag}\{\tilde{\mathbf{Q}}_1, \tilde{\mathbf{Q}}_2, \tilde{\mathbf{Q}}_3, \tilde{\mathbf{Q}}_4\}$,

$$\tilde{\mathbf{\Phi}}_i = \begin{bmatrix} \mathbf{A}\mathbf{Q}_i + \theta_k^a \mathbf{B}\mathbf{Y}_i & -\theta_k^a \mathbf{B}\mathbf{Y}_i \\ \mathbf{0} & (1-\theta_{k+1}^s)\mathbf{A}\mathbf{Q}_i \end{bmatrix},$$

$$\tilde{\mathbf{\Gamma}}_i = \begin{bmatrix} \mathbf{N}_a\mathbf{Q}_i + \theta_k^a \mathbf{N}_b\mathbf{Y}_i & -\theta_k^a \mathbf{N}_b\mathbf{Y}_i \\ (1-\theta_{k+1}^s)(\mathbf{N}_a\mathbf{Q}_i + \theta_k^a \mathbf{N}_b\mathbf{Y}_i) & -(1-\theta_{k+1}^s)\theta_k^a \mathbf{N}_b\mathbf{Y}_i \end{bmatrix}.$$

In this case, state feedback gain matrices $\mathbf{K}_i = \mathbf{Y}_i\mathbf{Q}_i^{-1}$, $i = 3, 4$.

Outline

Robust H_∞ Control

Theorem 3: Given a scalar $\gamma > 0$, the NCS $\hat{P}_K$ driven by the Markov chain is robustly stochastically stable with disturbance attenuation level γ, if there exist scalars $\epsilon_i > 0$, matrices $\mathbf{Q}_i > 0$ and $\mathbf{Y}_i$ for $i \in \mathcal{N}$ such that $\forall i \in \mathcal{N}$

$$\begin{bmatrix} -\tilde{\mathbf{Q}}_i & \mathbf{0} & \tilde{\mathbf{\Phi}}_i^T \mathbf{W}_i & \tilde{\mathbf{\Gamma}}_i^T & \tilde{\mathbf{C}}_i^T \\ * & -\gamma^2 \mathbf{I} & \overline{\mathbf{B}}_i^T \mathbf{W}_i & \mathbf{0} & \mathbf{0} \\ * & * & \epsilon_i \overline{\mathbf{W}}_i - \tilde{\mathbf{Q}} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\epsilon_i \mathbf{I} & \mathbf{0} \\ * & * & * & * & -\mathbf{I} \end{bmatrix} < 0,$$

where $\tilde{\mathbf{C}}_i = \begin{bmatrix} \mathbf{C}\mathbf{Q}_i + \theta_k^a \mathbf{D}\mathbf{Y}_i & -\theta_k^a \mathbf{D}\mathbf{Y}_i \end{bmatrix}$, $\mathbf{W}_i$, $\overline{\mathbf{W}}_i$, $\tilde{\mathbf{Q}}_i$, $\tilde{\mathbf{Q}}$, $\tilde{\mathbf{\Phi}}_i$ and $\tilde{\mathbf{\Gamma}}_i$ are given before.

In this case, state feedback gain matrices $\mathbf{K}_i = \mathbf{Y}_i \mathbf{Q}_i^{-1}$, $i = 3, 4$.

Outline

Plant and Network

- Unstable uncertain NCS of $\mathbf{x}(t) \in \mathbb{R}^3$, $\mathbf{u}(t) \in \mathbb{R}^2$, $\mathbf{z}(t) \in \mathbb{R}$ and $\mathbf{w}(t) \in \mathbb{R}$, with

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0.4 & 0.6 & 0.2 \\ 1 & 0.2 & -1.1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0.5 & 1 \\ 0.5 & 0.2 \\ 1 & 0.4 \end{bmatrix}, \mathbf{B}_w = \begin{bmatrix} 0.1 \\ 0.1 \\ -0.2 \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} 0.1 \\ -0.1 \\ 0.2 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0.2 & 0.3 & 0.3 \end{bmatrix}, \mathbf{D} = \begin{bmatrix} 0.7 & 0.9 \end{bmatrix},$$

$$\mathbf{N}_a = \begin{bmatrix} 0.5 & 0.2 & 0.3 \end{bmatrix}, \mathbf{N}_b = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}.$$

- Markov chain r_k with transition probability matrix

$$\boldsymbol{\Upsilon} = \begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.6 \\ 0.1 & 0.2 & 0.1 & 0.6 \\ 0.1 & 0.1 & 0.2 & 0.6 \\ 0 & 0.1 & 0.1 & 0.8 \end{bmatrix}.$$

Outline

H_∞ Control Solution

- Give disturbance attenuation level $\gamma = 0.45$.
- According to **Theorem 3**, we can obtain ϵ_i and $\mathbf{Q}_i$, $1 \leq i \leq 4$, as well as $\mathbf{Y}_3$ and $\mathbf{Y}_4$.
- Thus, derive state feedback gain matrices

$$\mathbf{K}_3 = \begin{bmatrix} 0.0004 & 0.0170 & 0.0331 \\ -0.0707 & -0.0964 & -0.1062 \end{bmatrix},$$

$$\mathbf{K}_4 = \begin{bmatrix} -0.5959 & -0.1417 & 0.4485 \\ 0.3396 & -0.3326 & -0.5625 \end{bmatrix},$$

as the solution of robust H_∞ control problem.

Conclusions

We have studied a generic class of model-based NCSs, where

- the plant has time-varying norm-bounded uncertainties;
- both the sensor-to-controller and controller-to-actuator channels experience random packet dropouts.

We have derived sufficient conditions, in the form of LMIs, for

- guaranteeing the robust stochastic stability;
- synthesising the stochastic stabilisation controller;
- designing the H_∞ controller.