UNIVERSITY OF Southampton

School of Electronics and Computer Science

Modern design optimisation exploiting field simulation

Jan Sykulski

- **Design** is a process of searching for a device or structure which satisfies a set of requirements.
  - **It is an inverse problem**
    - The requirements may be expressed in terms of the physical sizes, the inputs and/or the outputs.
  - **Traditional design (‘trial and error’):**
    - Guess a solution
    - Build it and measure its performance
    - Modify the device to more nearly match the requirements
    - The modification is performed on the basis of simple models, design expertise and “know-how”.

- A design engineer has an appreciation of how a change in a particular parameter will affect the device performance.
  - In other words, he/she has a mental picture of how small changes in any parameter will affect each aspect of the desired performance
  - This is a concept of sensitivity…
- Alternately, if no experience or models exist, random variations can be tried, the performance measured and models developed…

Hierarchical (three-layer) structure

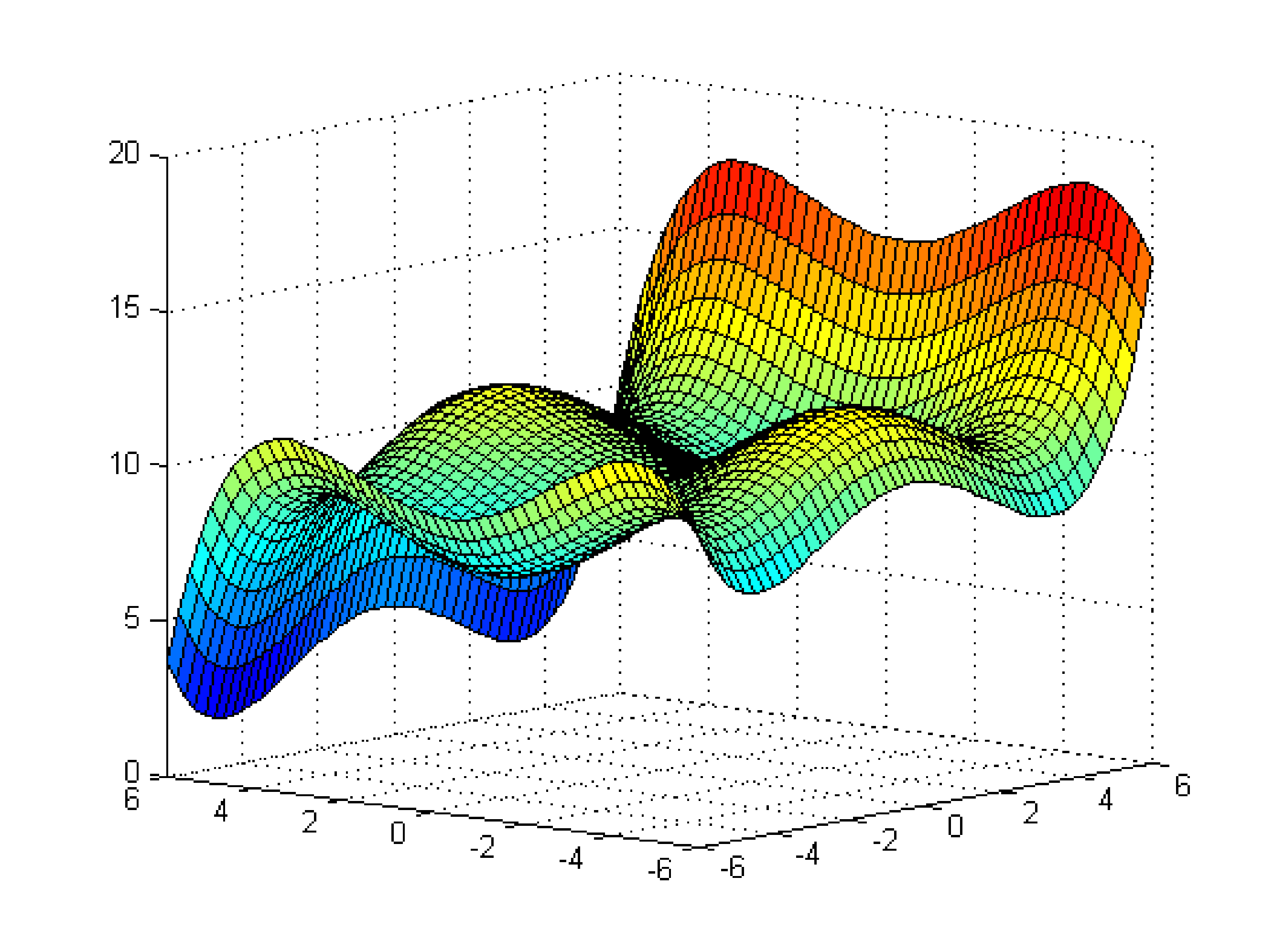
- Approximate solutions
  (e.g. equivalent circuits, semi-empirical, design sheets)
- Extensive optimisation
- Large design space

- 2D finite element, static or steady-state
- Constrained optimisation, coupling
- Medium design space

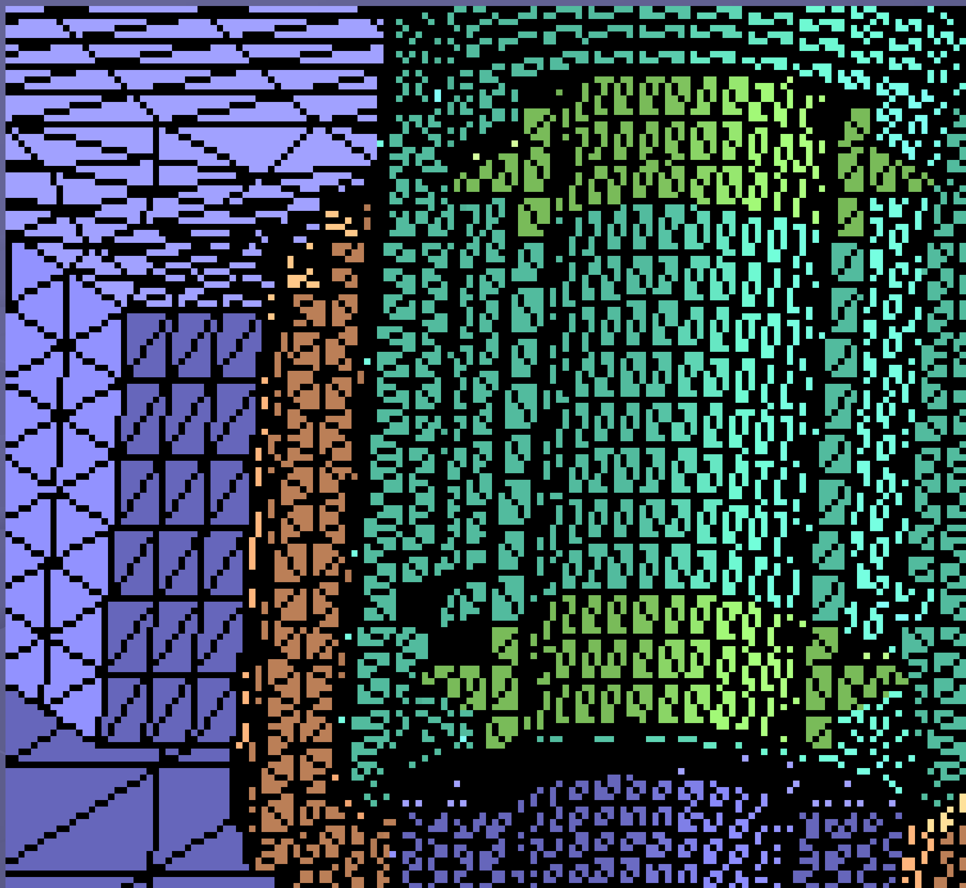
- 3D finite-element, transient
- Fine tuning of the design
- Small design space

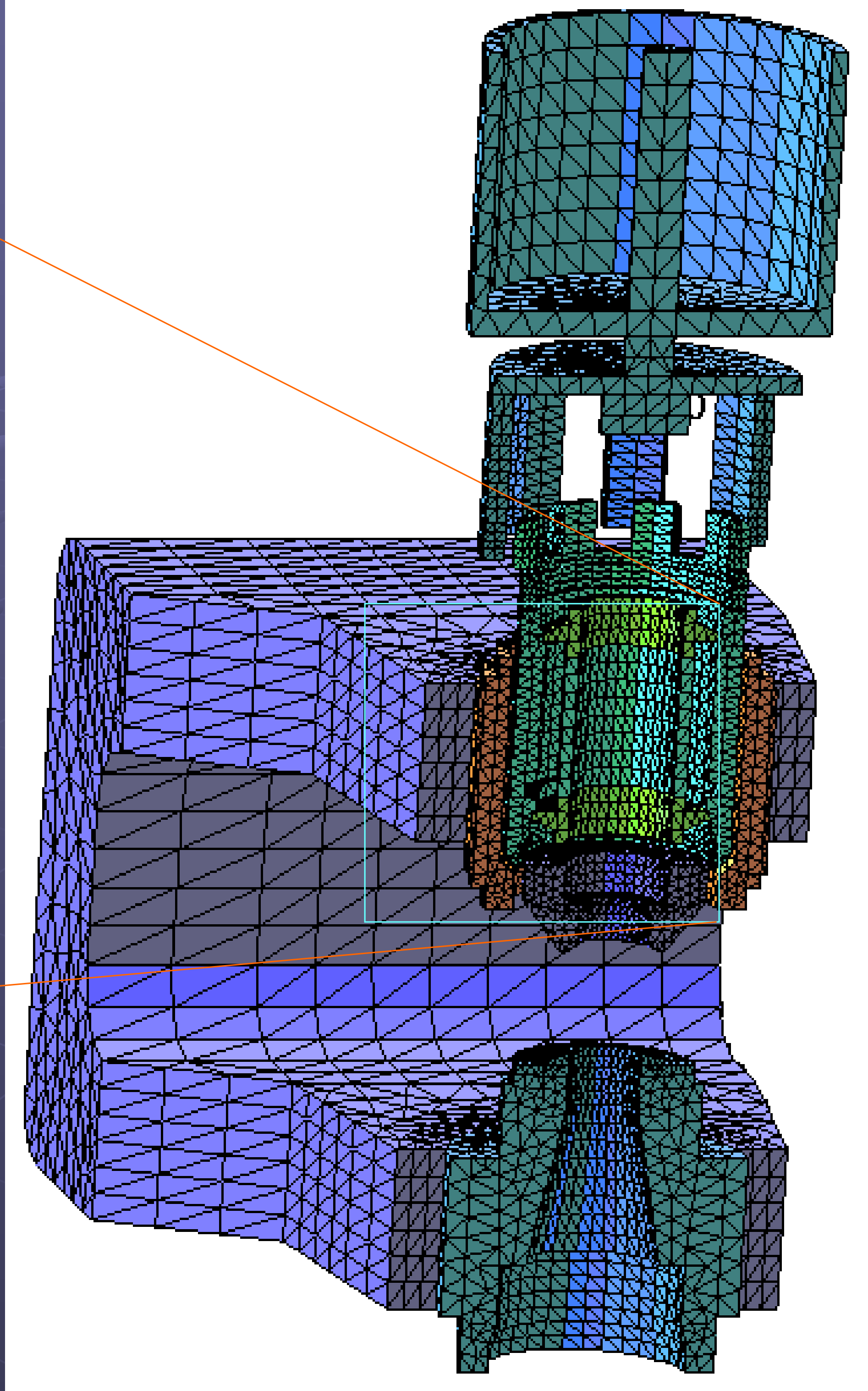
Knowledge base

Finite-element analysis



Finite-element analysis





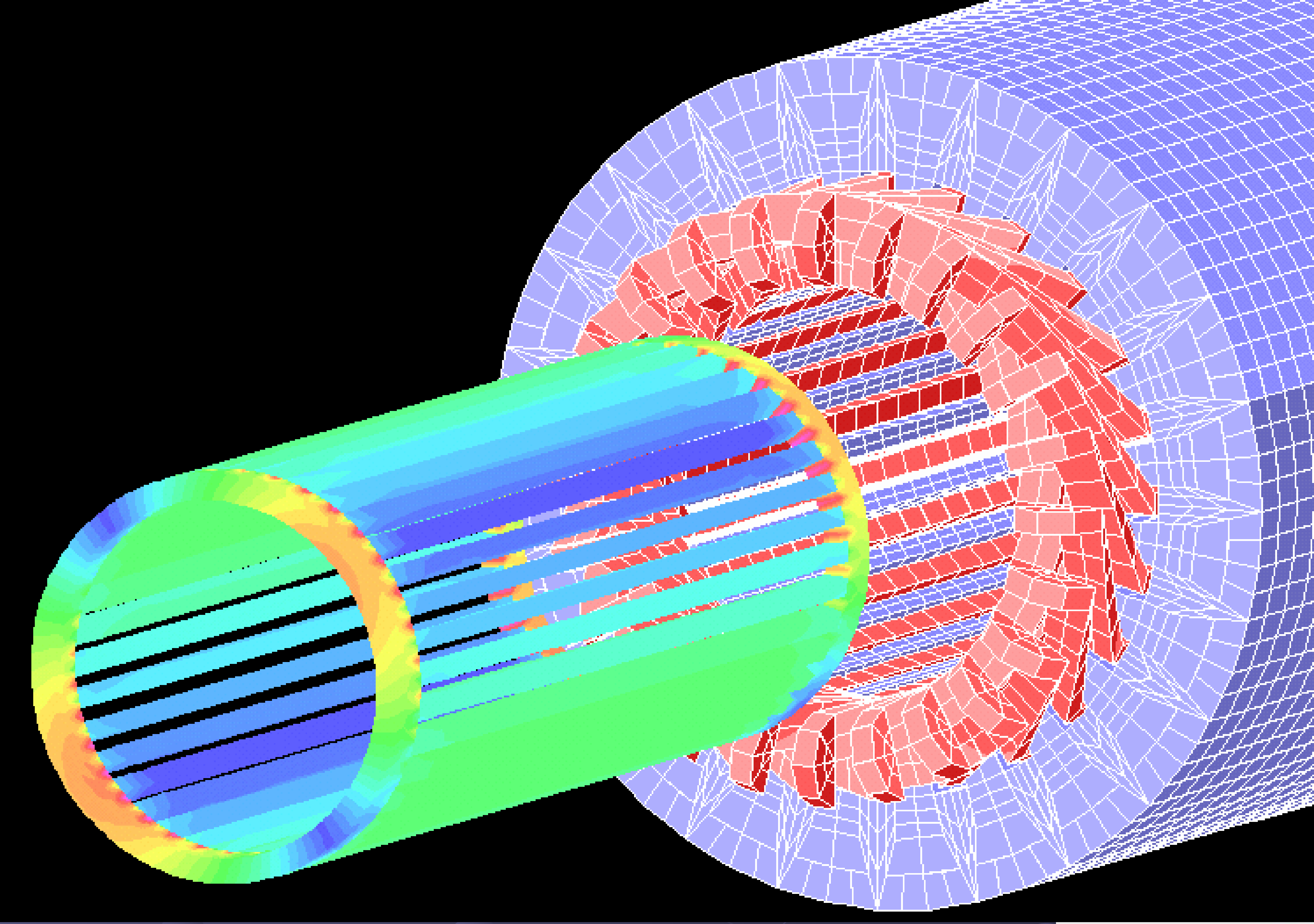
3D meshes

The state of the art

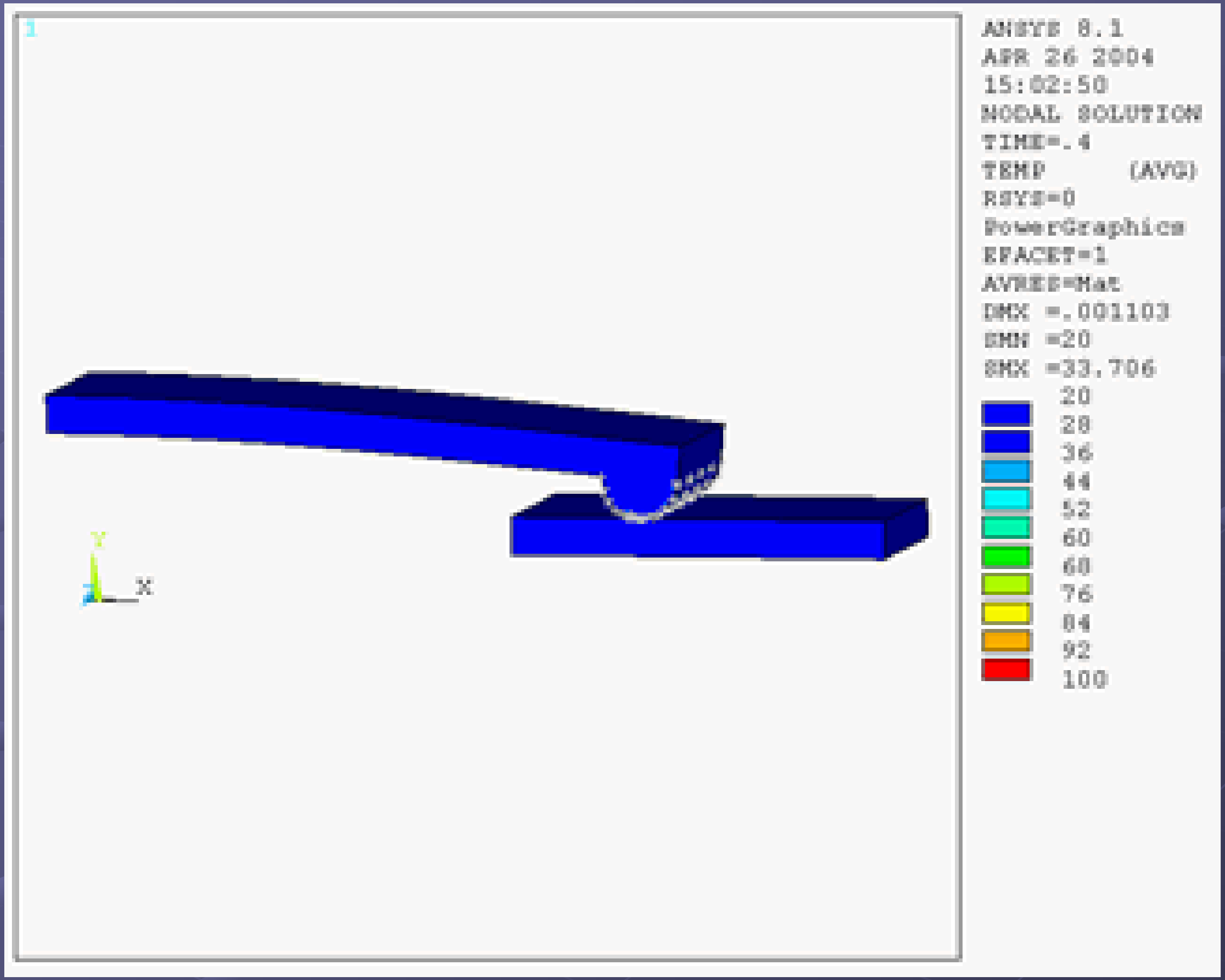
Contemporary software capable of solving

- 2D, axi-symmetric and 3D problems
- Eddy currents
- Non-linearity of materials
- Anisotropy and hysteresis
- Motion effects
- Static, steady-state and transient solutions
- Coupling to mechanical and thermal effects
- Connections to driving circuitry

Geometric modellers can handle most practical shapes



Multiphysics problems



The current and resultant Joule heating in an electric switch contact are modelled as the switch is actuated. Mechanical, thermal and current flow are modelled using direct coupled field elements.

Optimization techniques

Deterministic

- Always follows same path from same initial conditions
- Finds local minimum
- Fast: 5 to 100 evaluations

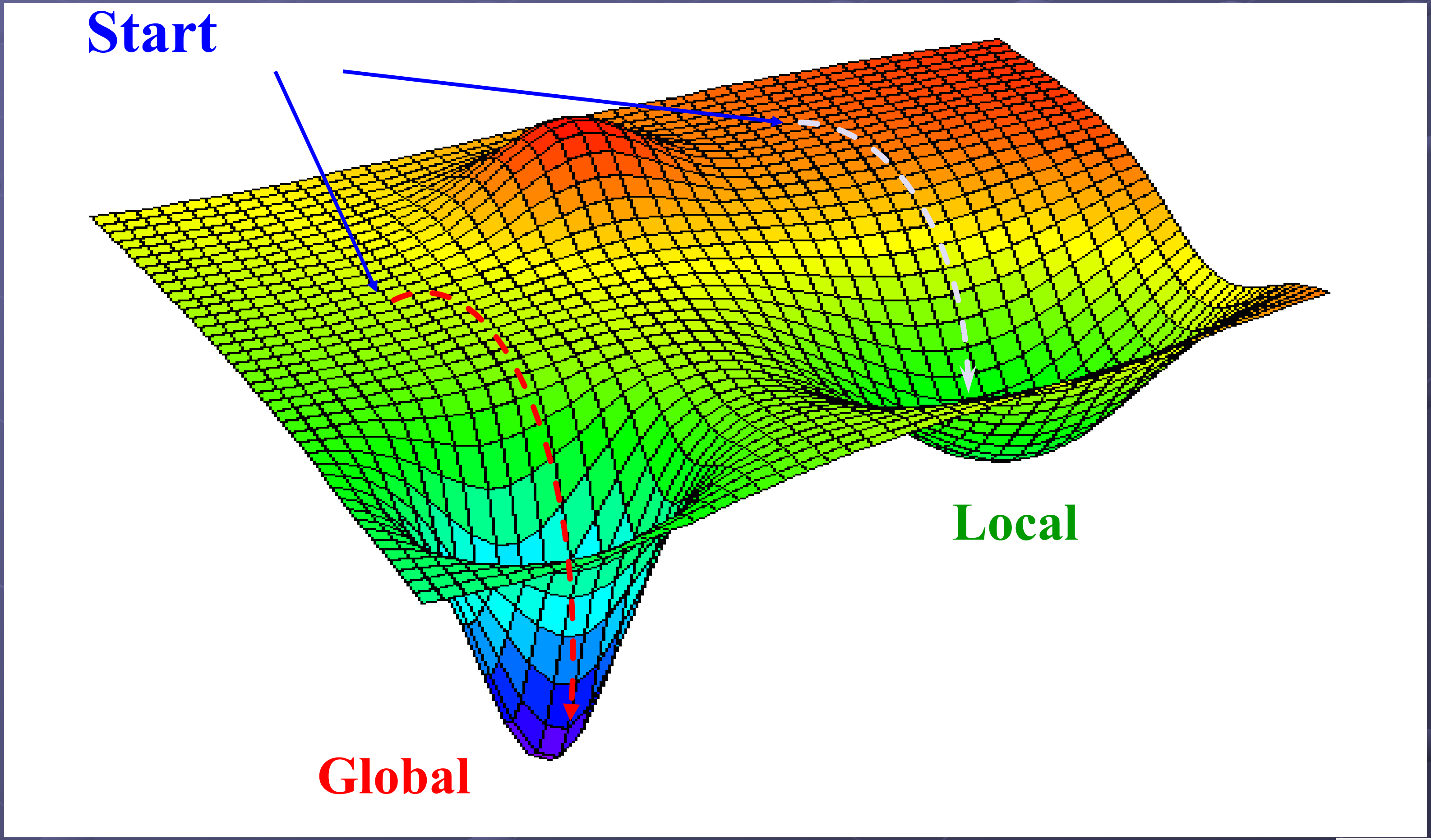
Stochastic

- Initial conditions do not determine path of optimization
- Attempt to find global minimum
- Slow: hundreds or thousands of evaluations

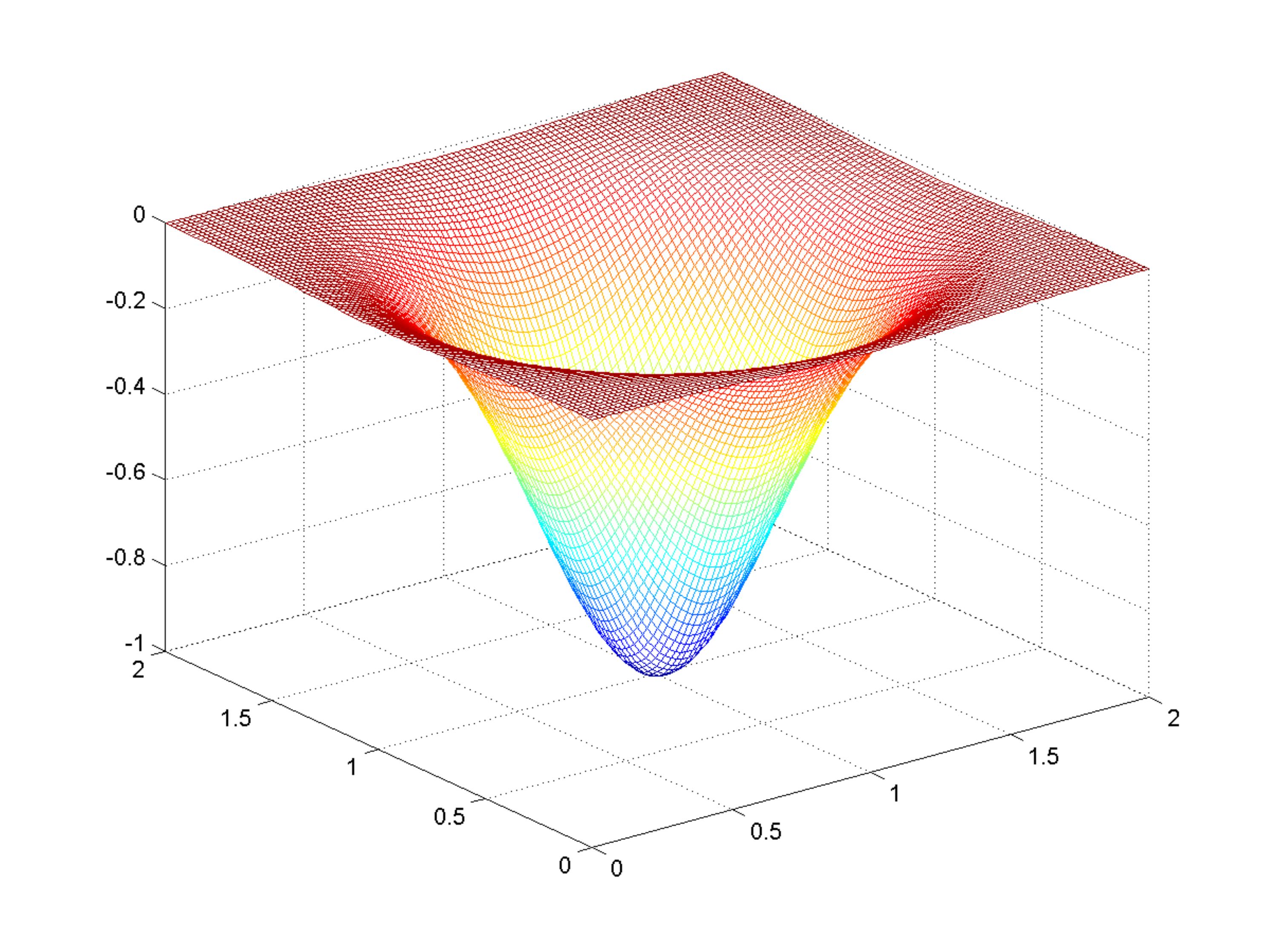
Optimization techniques

Deterministic

Stochastic



Optimization



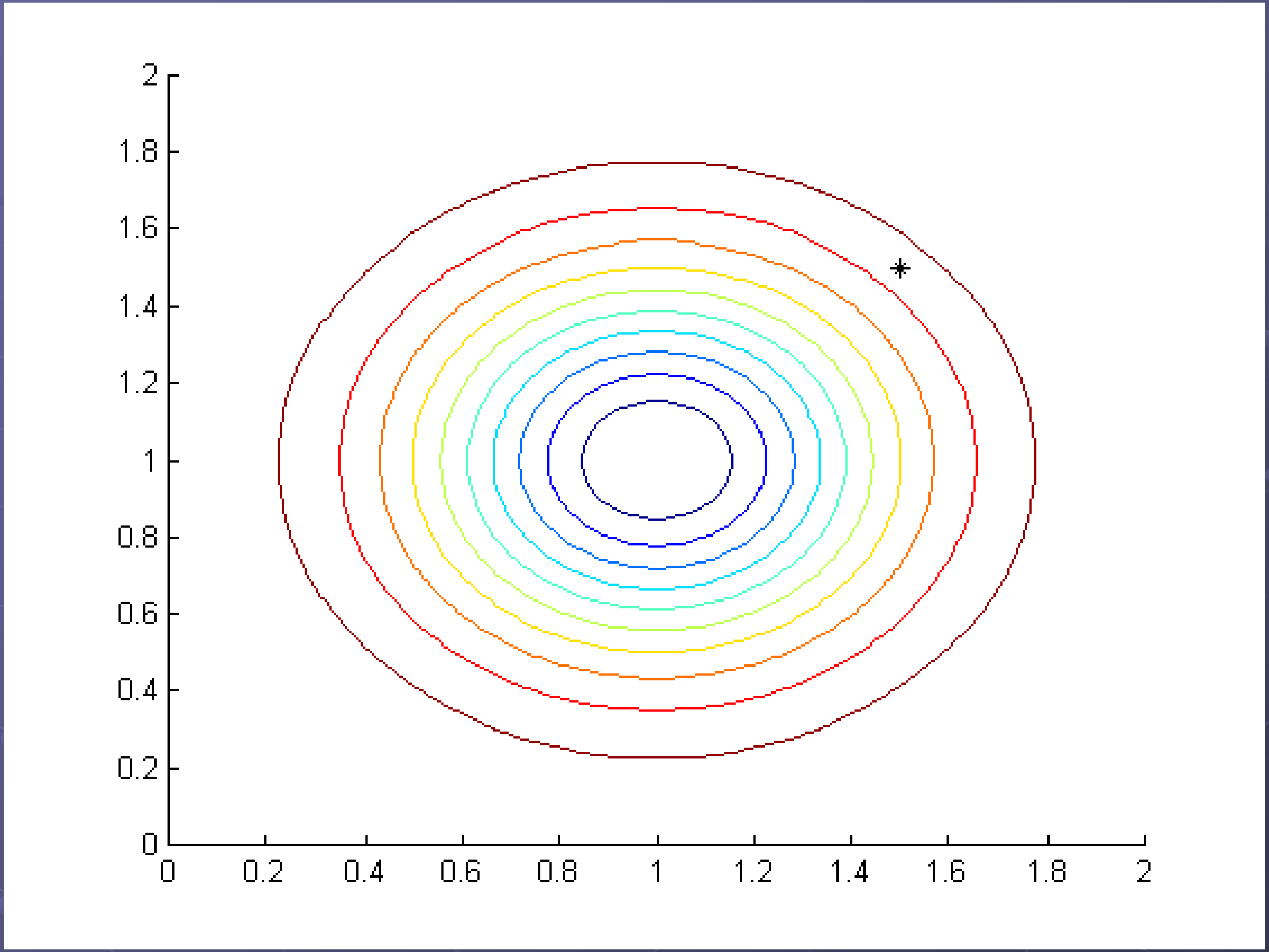
Single-minimum objective function

University of Southampton

International Symposium on Electromagnetic Fields in Mechatronics, Electrical and Electronic Engineering
10-12 September 2009, Arras France

Optimization

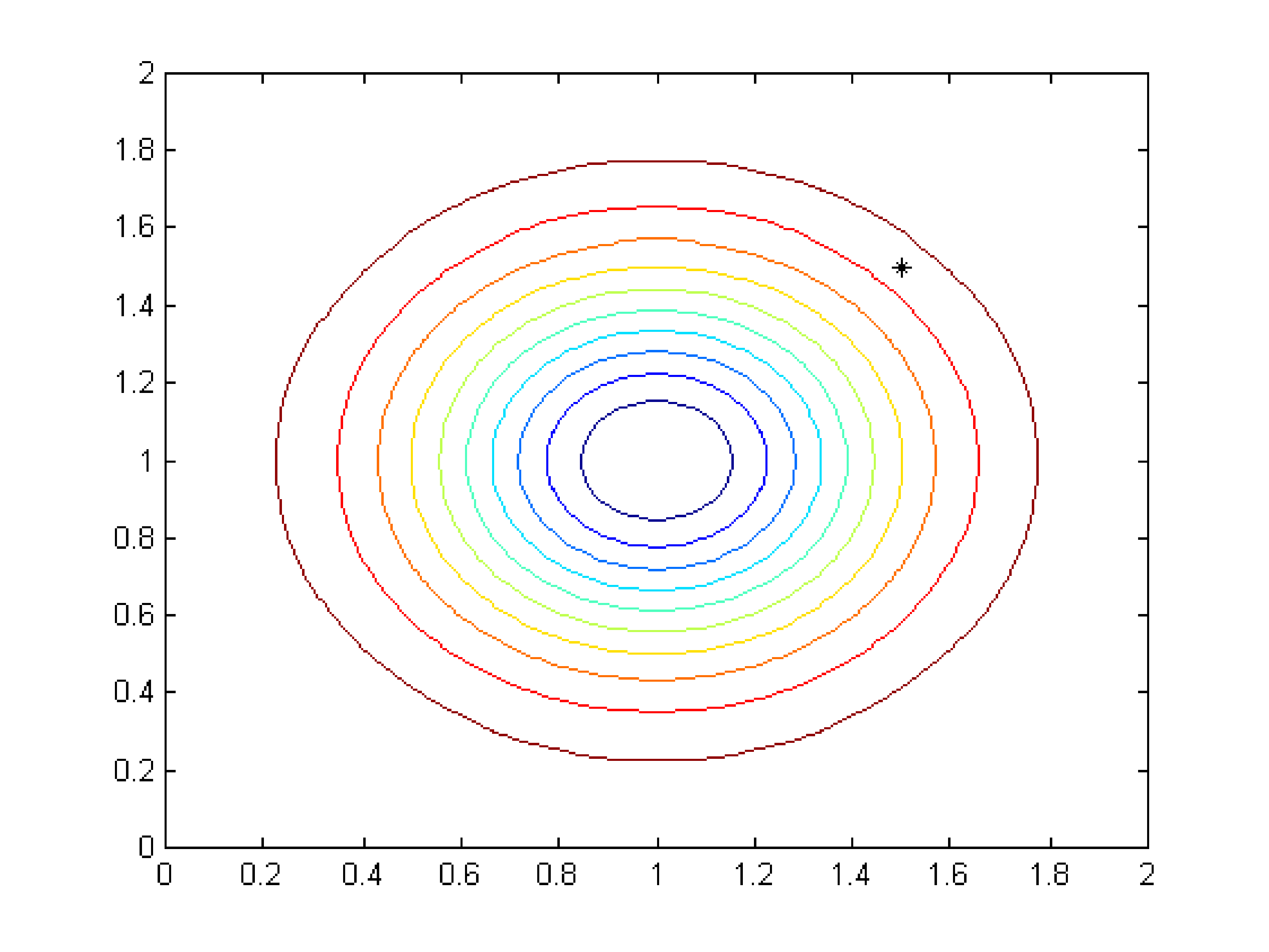
Deterministic algorithm



Single-minimum objective function

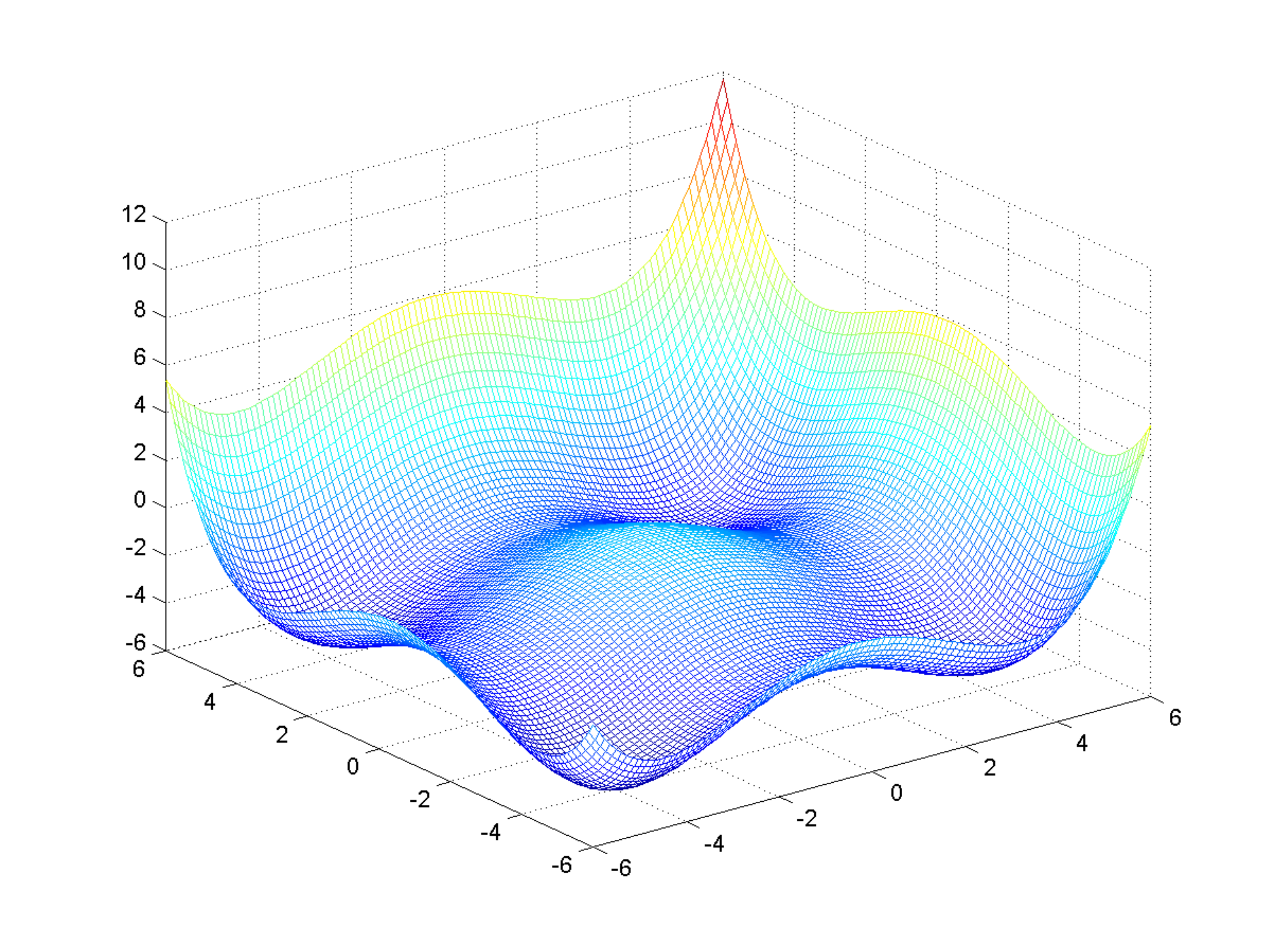
Optimization

Evolution strategy



Single-minimum objective function

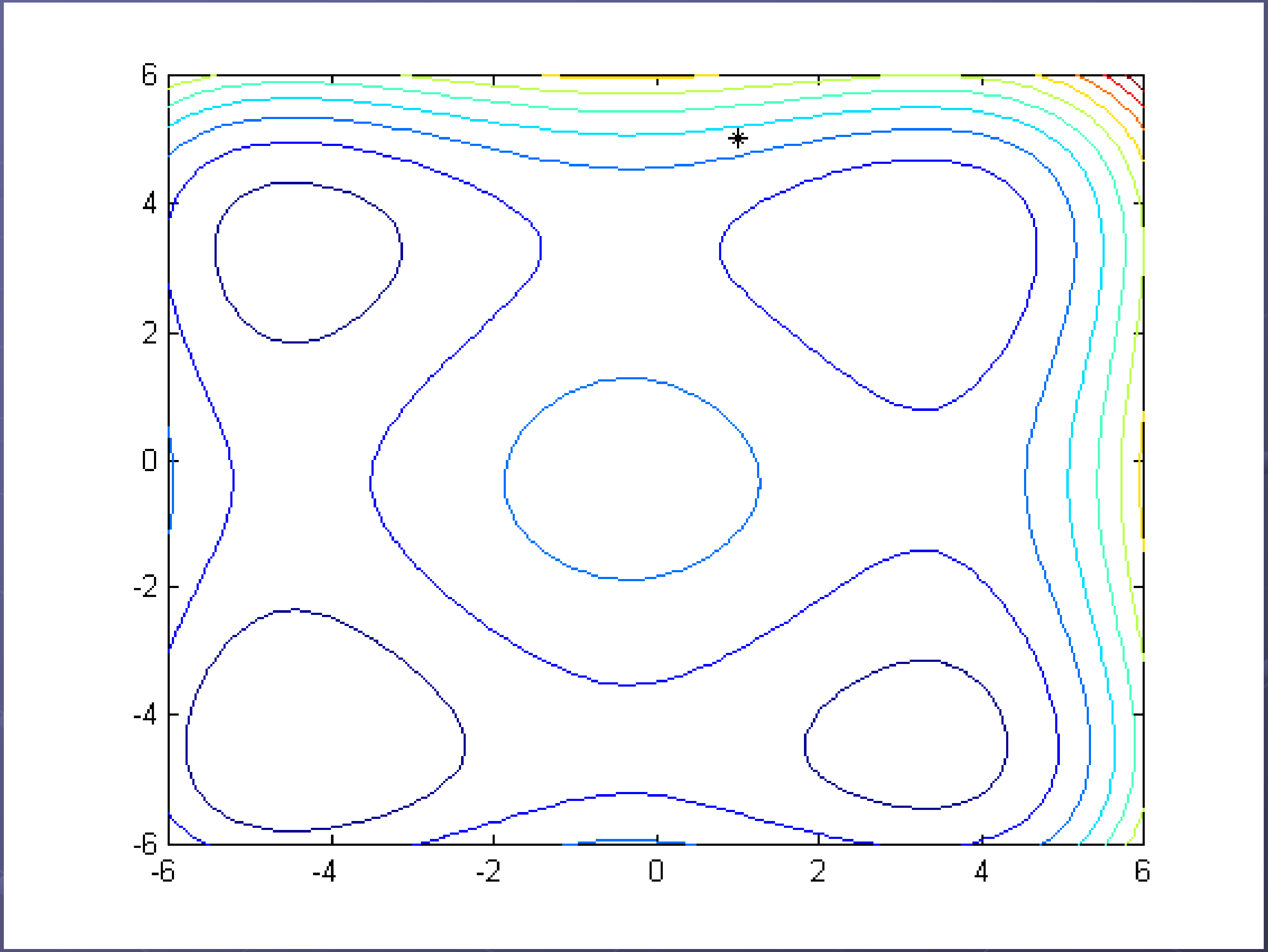
Optimization



Multiple-minima objective function

Optimization

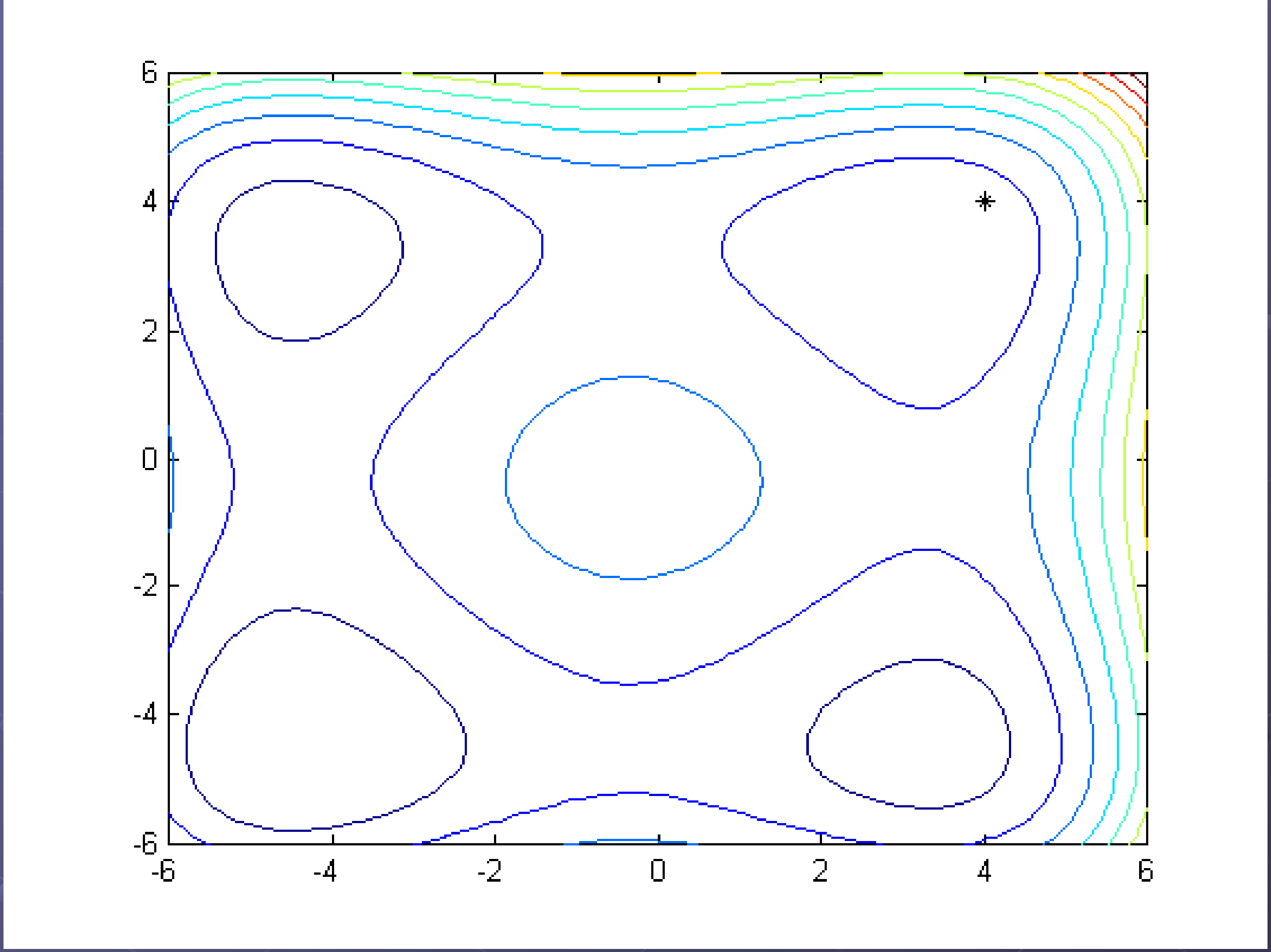
Deterministic algorithm



Multiple-minima objective function

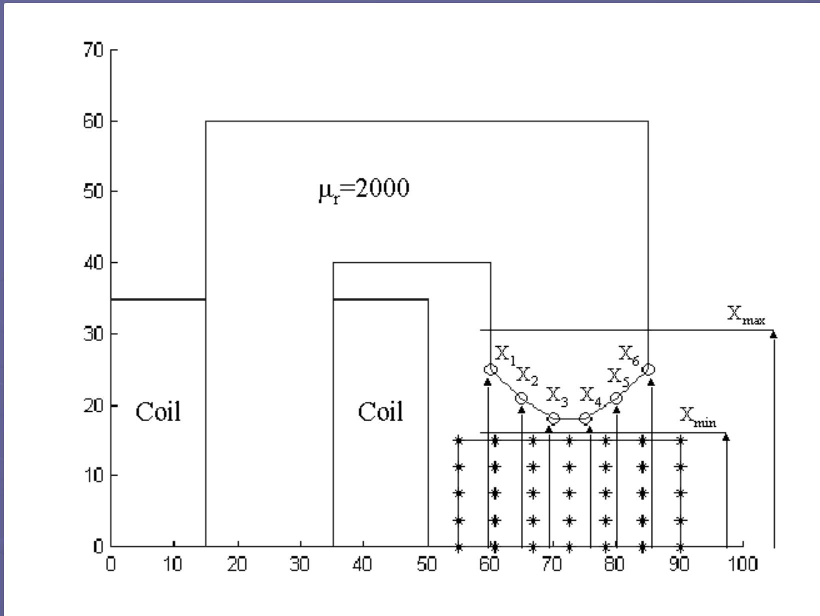
Optimization

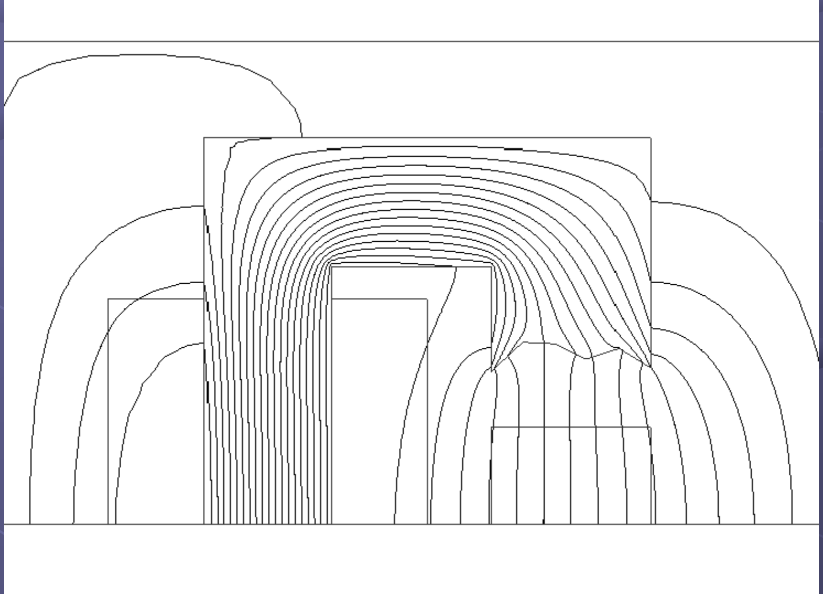
Evolution strategy



Multiple-minima objective function

C-core shaped magnet





$$F_C = \max_{i=1,35} \left| B_0 - B_i \right| \left(B_0 \right)^{-1}$$

Method	Starting	Optimum	n
DE1	9 random	0.0803	720
DE2	13 random	0.0704	881
ES	0.7532 / 0.4344 / 0.6411	0.0642	450
GBA	0.7532	0.0855	188
ES/DE/MQ	0.7532	0.0718	118

DE – Differential Evolution

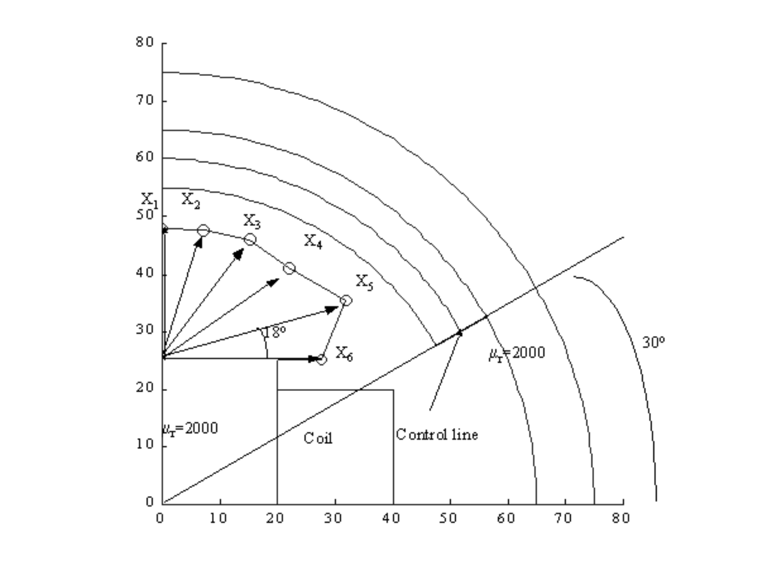
ES – Evolution Strategy

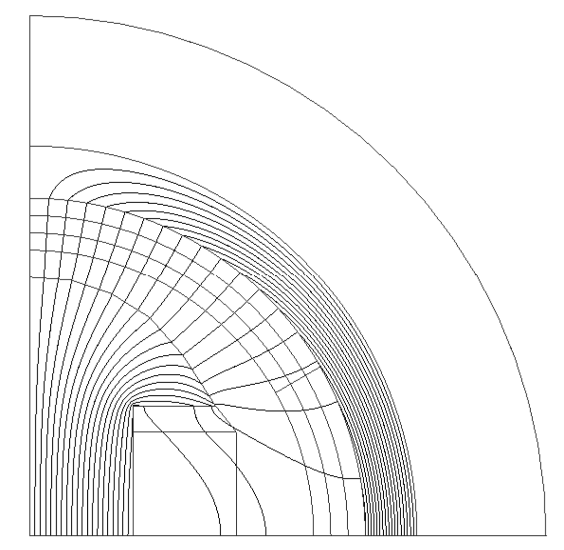
GBA – Gradient Based Method

Magnetiser

$$f = \sum_{k=1}^{59} \left(B_{desired,k} - B_{calculated,k} \right)^2$$

$$B_{desired,k} = B_{max} \sin\left(90^{o} - k\right) \qquad 1 \le k \le 59$$





Method	Starting	Optimum	n
DE1	11 random	1.235E-5	987
DE2	11 random	5.423E-5	1035
ES	1.457E-3	1.187E-5	433
ES	9.486E-2	1.318E-4	351
GBA	1.457E-3	1.238E-4	41
GBA	9.486E-2	2.433E-4	281
ES/DE/MQ	1.457E-3	1.961E-5	234
ES/DE/MQ	9.486E-2	2.125E-5	206
NF/GA/SQP		6.570E-5	189

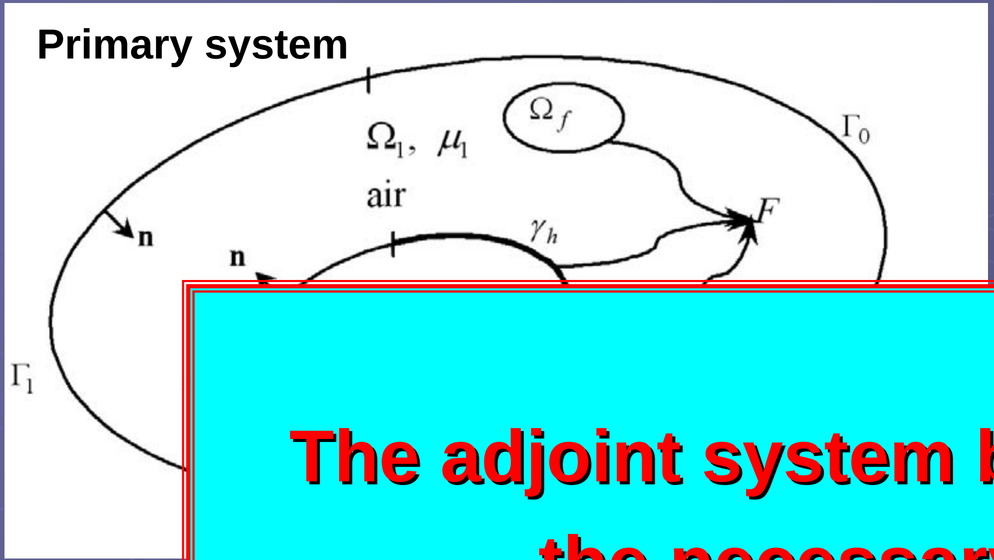
Unconstrained optimisation

Method	Optimum	N
ES/DE/MQ	1.58E-5	246
NF/GA/SQP	4.65E-5	155

Constrained optimisation

NF – Neuro-Fuzzy modelling
GA – Genetic Algorithm
SQP – Sequential Quadratic Programming

Applying Continuum Design Sensitivity Analysis



$$\int_{\Omega_1+\Omega_2} \upsilon\nabla\times\mathbf{A}\cdot\nabla\times\boldsymbol{\lambda}\,d\Omega + \int_{\Gamma_0+\Gamma_1+\gamma} \mathbf{n}\times(\upsilon\nabla\times\mathbf{A})\cdot\boldsymbol{\lambda}\,d\Gamma =$$

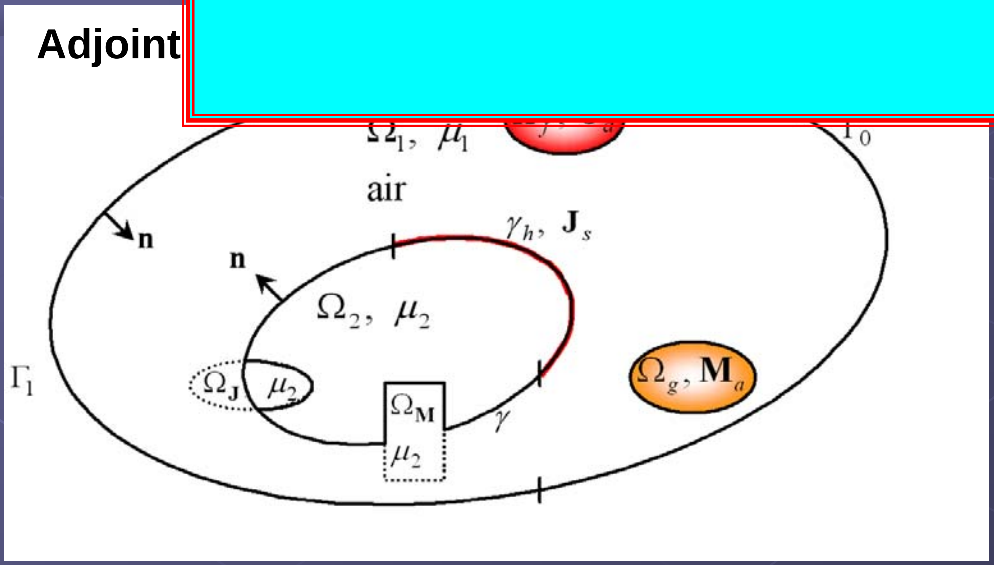
$$\int_{\Omega_1+\Omega_2} [\mathbf{J}_p\cdot\boldsymbol{\lambda} + \mathbf{M}_p\cdot\nabla\times\boldsymbol{\lambda}]\,d\Omega \qquad \text{for all } \boldsymbol{\lambda}\in\Phi$$

(h_1 =∂h/∂A, f_1=∂f/∂A, g_1≡∂g/∂H)

$$\ldots\nabla\times\boldsymbol{\lambda})\cdot\dot{\mathbf{A}}\,d\Gamma +$$

$$\text{for all } \dot{\mathbf{A}}\in\Phi$$

The adjoint system by itself satisfies all the necessary conditions to be solved with a standard EM package

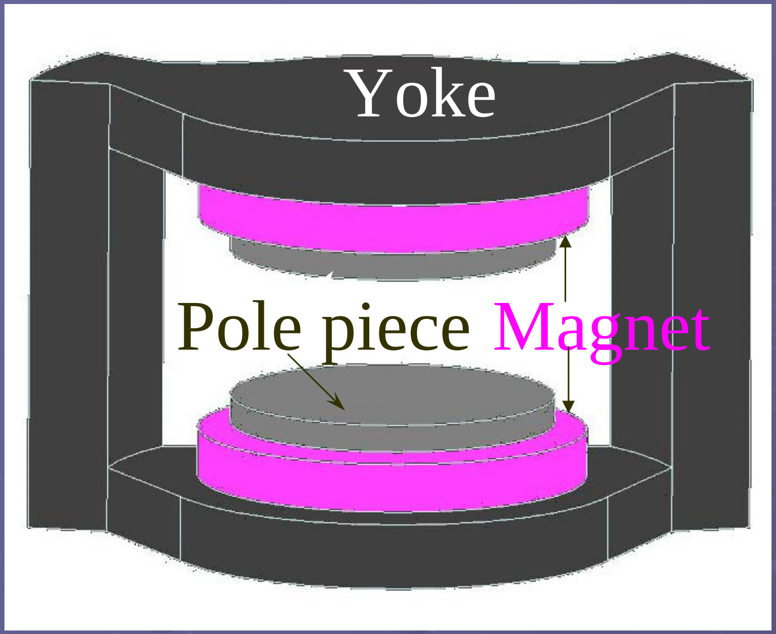


...stem

$\mathbf{f}_1 = \partial f/\partial \mathbf{A}_1$	$\mathbf{J}_a = [\partial f/\partial A_x, \partial f/\partial A_y, \partial f/\partial A_z]$
$\mathbf{g}_1 = \partial g/\partial \mathbf{H}_1$	$\mathbf{M}_a = [\partial g/\partial H_x, \partial g/\partial H_y, \partial g/\partial H_z]$
$\mathbf{h}_1 = \partial h/\partial \mathbf{A}_1$	$\mathbf{J}_s = [\partial h/\partial A_x, \partial h/\partial A_y, \partial h/\partial A_z]$

The units of the pseudo sources coincide precisely with those of the real sources.

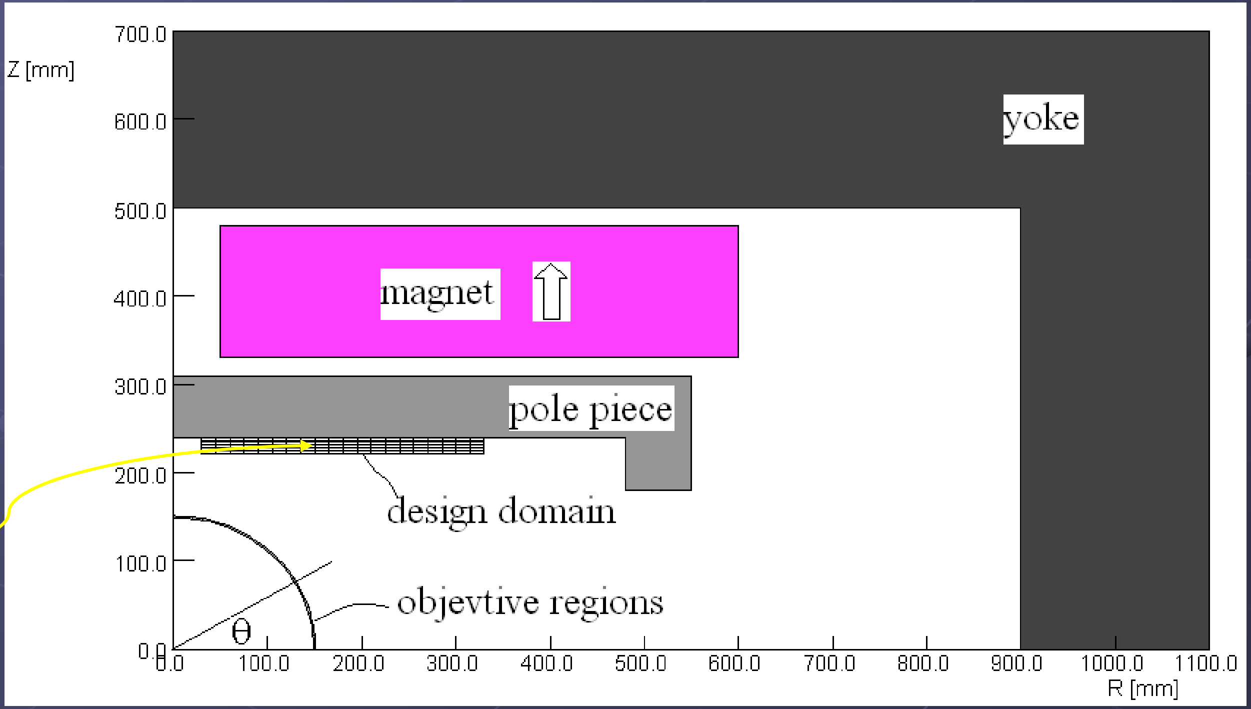
Optimized shimming magnet distribution of MRI system



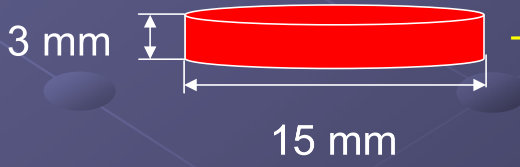
Objective function:

$$F = \sum_{i=1}^{45} (B_{zi} - B_{zo})^2, \quad \mathbf{M}(x,y) = \mathbf{M}_s(P)p^n$$

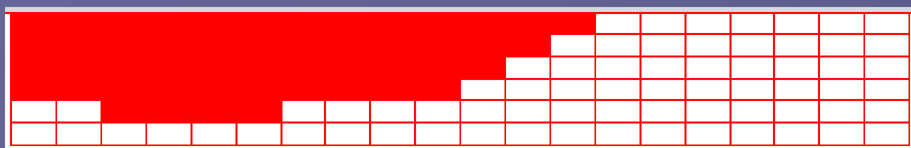
Simplified axi-symmetric model



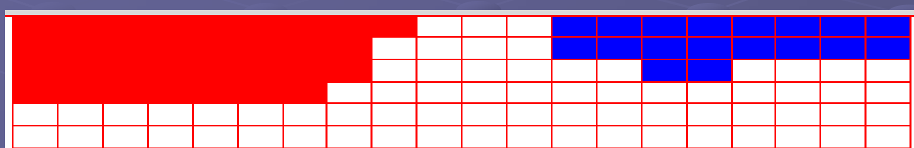
Shimming magnet (Br=0.222 T)



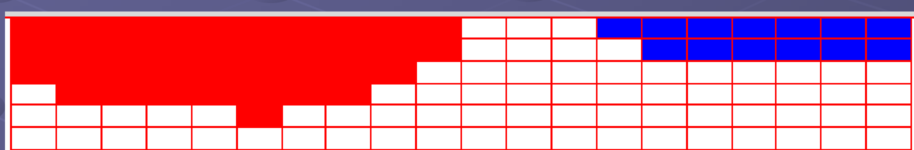
Changes of shimming magnet distribution during optimisation



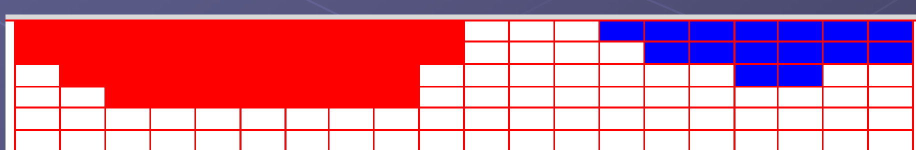
1 iteration



3 iterations

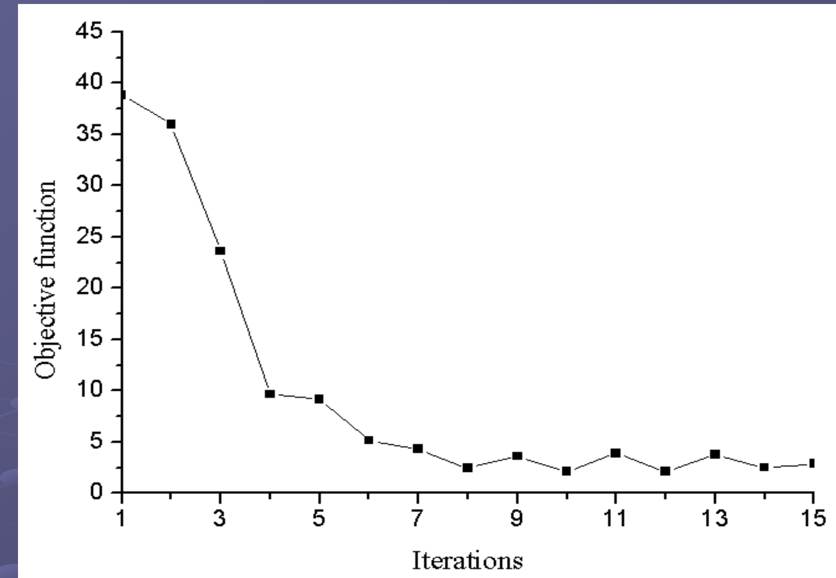


3 iterations

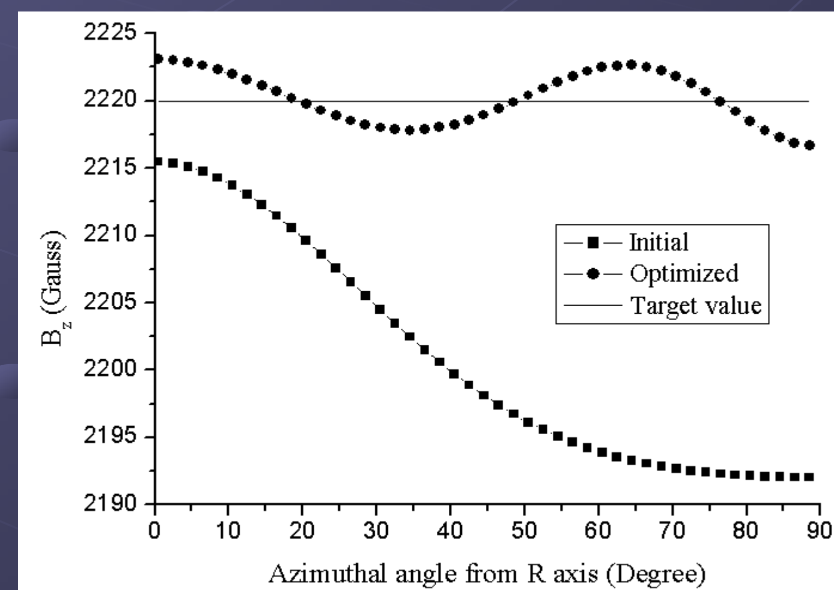


10 iterations

+Ms -Ms



Convergence

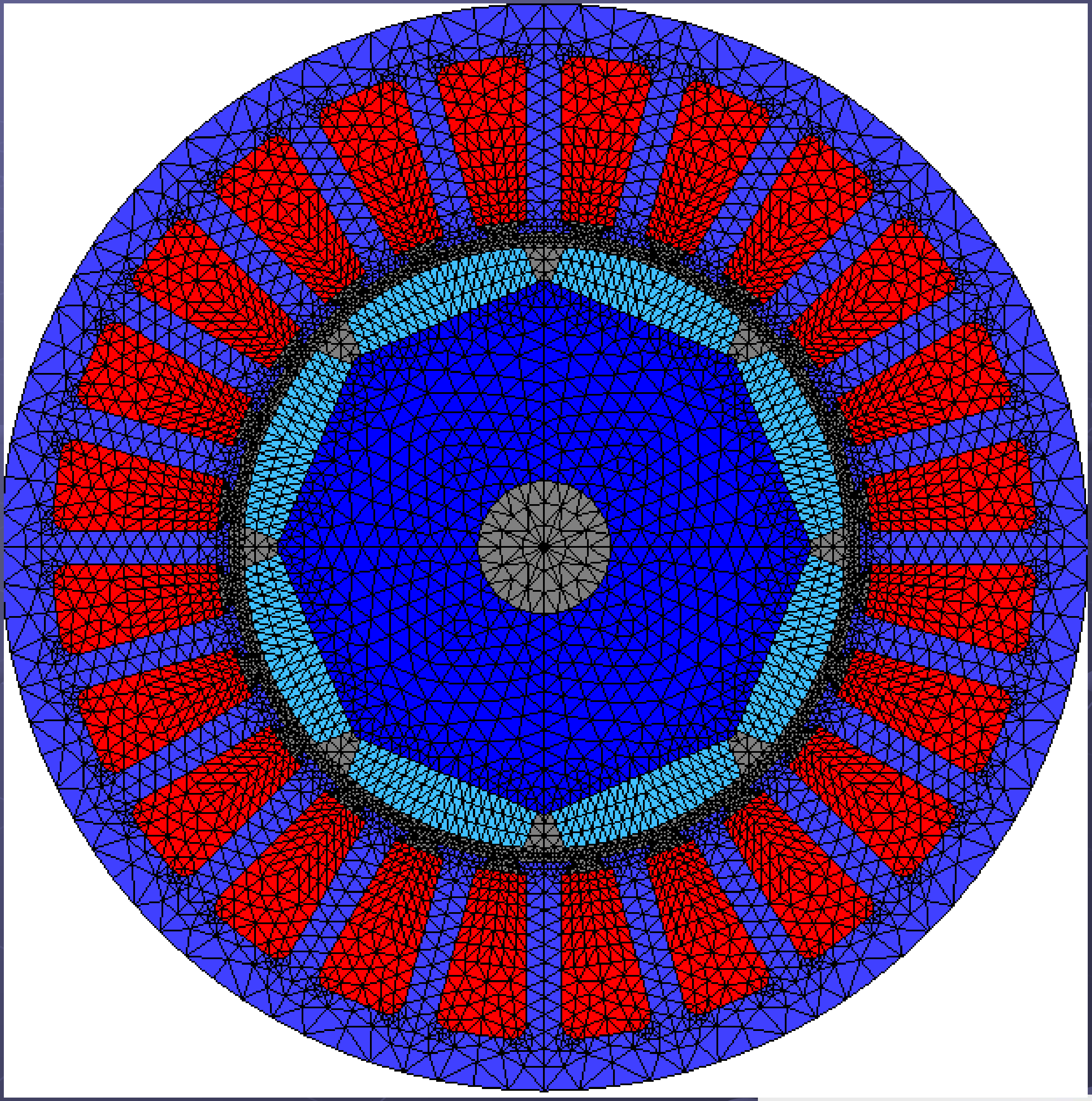


Flux distributions

Optimization

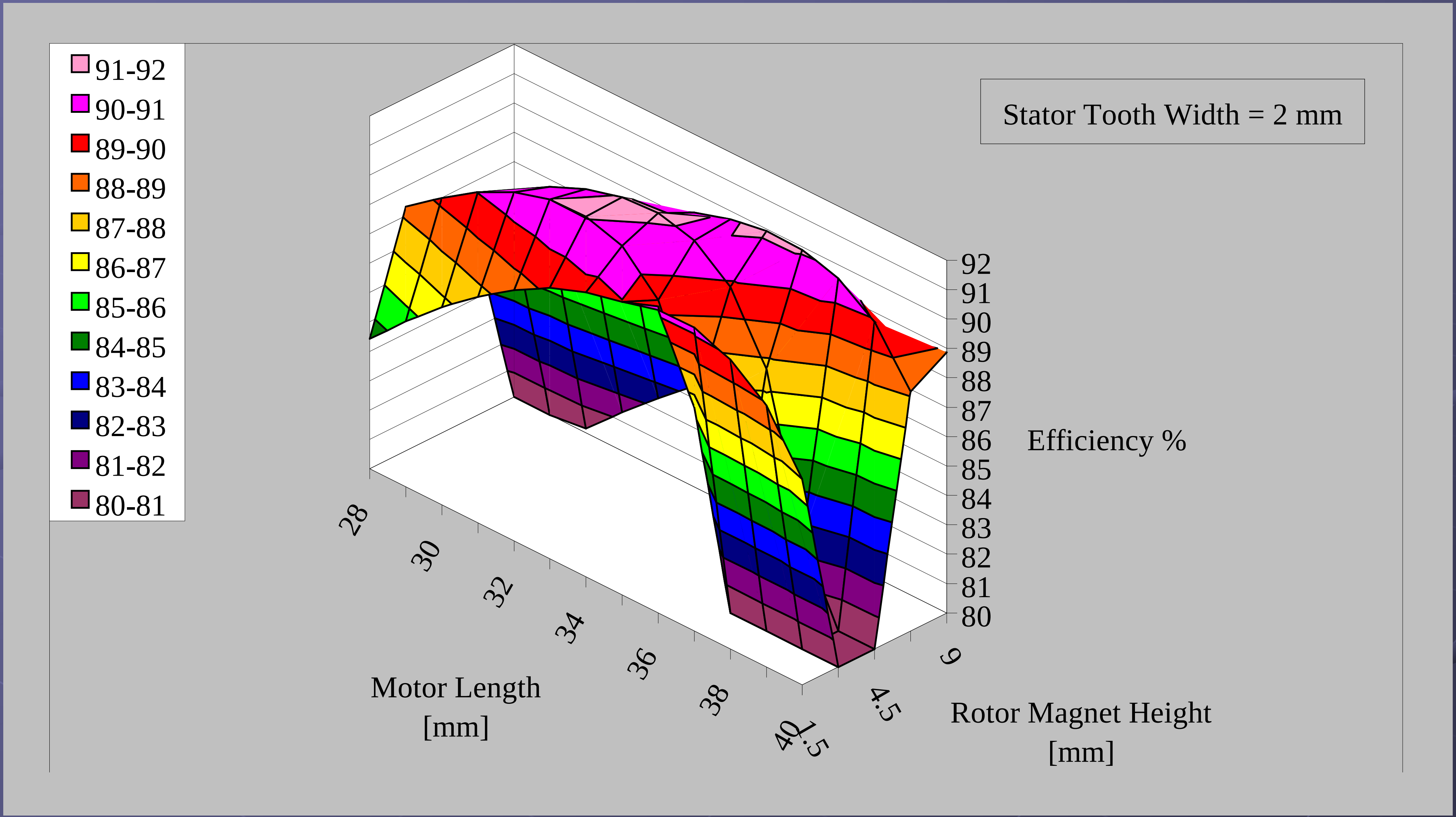
Brushless permanent magnet motor

BAE Systems



Optimization

Brushless permanent magnet motor



Brushless PM motor optimisation response surface (when varying three design parameters)

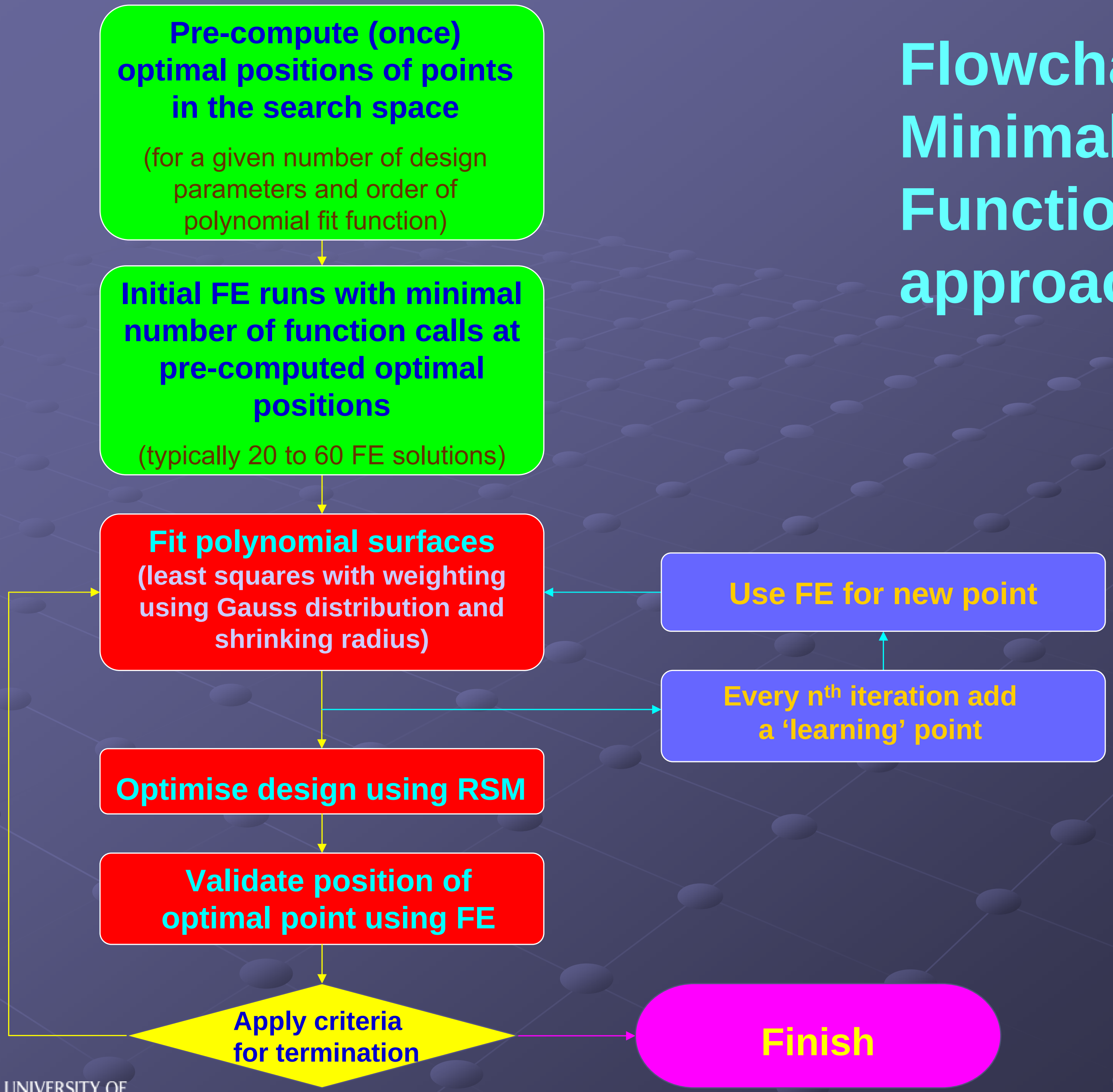
Optimization

Brushless permanent magnet motor

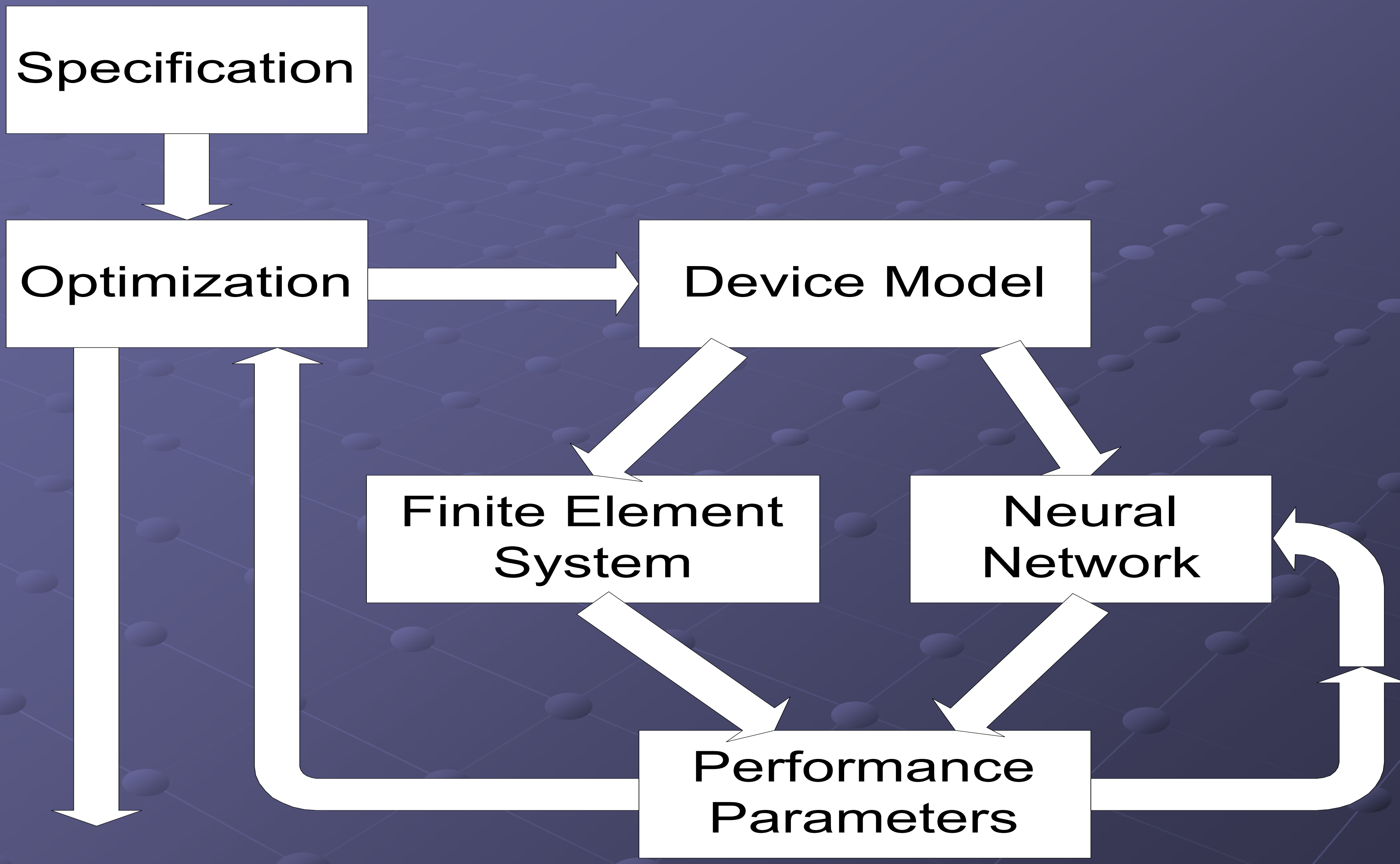
Order / Variables	0	1	2	3	4	5	6
1	1	2	3	4	5	6	7
2	1	3	6	10	15	21	28
3	1	4	10	20	35	56	84
4	1	5	15	35	70	126	210
5	1	6	21	56	126	252	462
6	1	7	28	84	210	462	924
7	1	8	36	120	330	792	1716
8	1	9	45	165	495	1287	3003
9	1	10	55	220	715	2002	5005
10	1	11	66	286	1001	3003	8008

The number of necessary function calls for RSM (Response Surface Methodology)

Flowchart of the Minimal Function Calls approach



Design process using on-line Neural Network

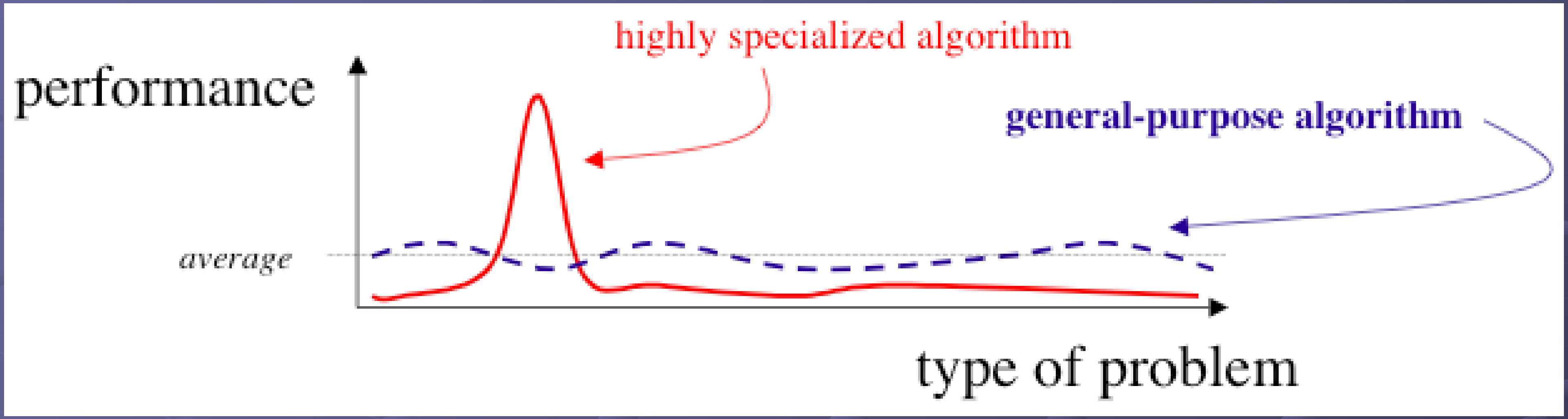


All this was late 1990s stuff !

2000

Early 21st century has brought lots of new developments

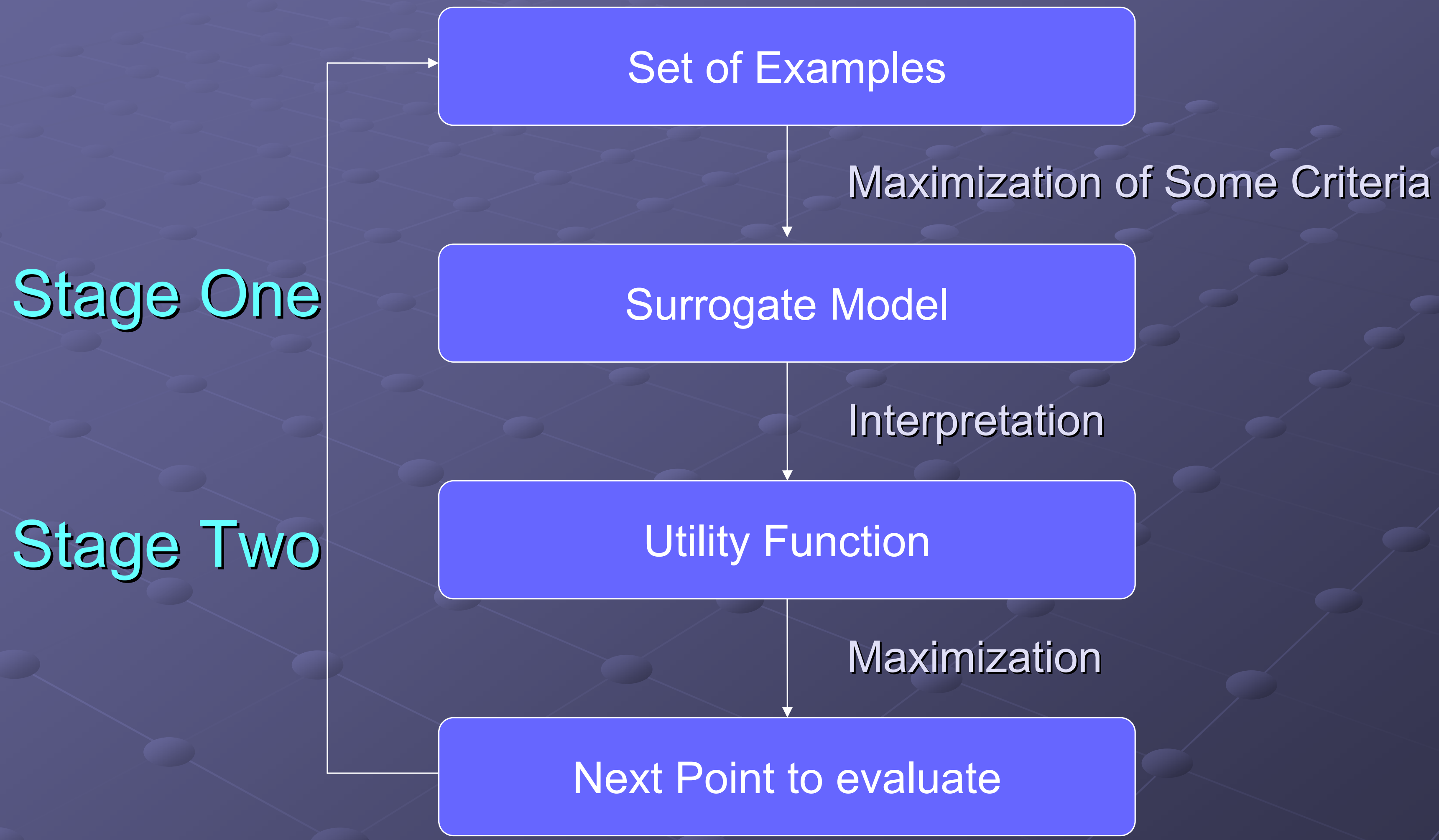
No-free-lunch theorem



An illustration of the no-free-lunch theorem, showing the performance of a highly specialized algorithm (red) and a general-purpose one (blue) on different problems.

Note that both algorithms perform *on average* equally well.

Optimization (Two Stage)



Constructing a surrogate model

- We want the surrogate model to approximate f
- Need to choose a set of basis functions φ

$$\sum_{k=1}^{m} a_k \pi_k(x) + \sum_{j=1}^{n} b_j \varphi(x - x_j)$$

- Set up fitting criteria to determine the a_k and b_k
- Find the parameters which minimize this criteria
- Making a surrogate model is itself an optimization problem

Basis functions

- $\|z\|$ linear interpolation
- $\|z\|^2 \log(\|z\|)$ thin plate spline
- $\sqrt{\|z\|^2 + \gamma^2}$ multiquadrics

Basis functions

- $\|z\|$ — linear interpolation
- $\|z\|^2 \log(\|z\|)$ — thin plate spline
- $\sqrt{\|z\|^2 + \gamma^2}$ — multiquadrics
- $\exp(-\sum_{l=1}^{d} \theta_l |z_l|^{p_l})$ — kriging

Single-Objective Optimization Problems (SOOP)

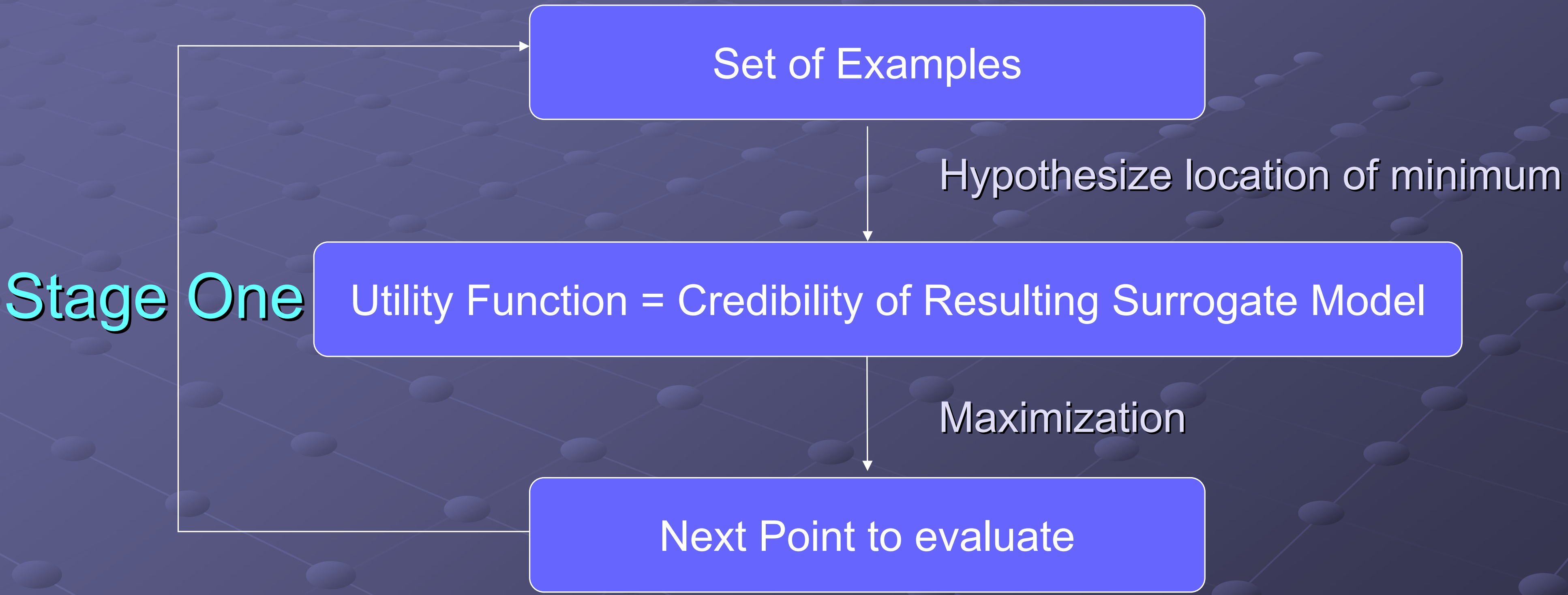
Using kriging surrogate models in SOOP evaluate the design vectors to maximize

- the probability of improvement (POI)
- the expectation value of the improvement (EI)
- the generalized expected improvement (GEI)
- the weighted expected improvement (WEI)
- the credibility of a hypothesis (CH) about the location of the minimum (also known as the one-stage approach)
- the 'minimizer entropy' (ME) criterion

A delicate balance between **exploration** and **exploitation** is controlled through 'cooling' schemes

Possible to select multiple design vectors for evaluation at each iteration

One-stage optimization



Multi-Objective Optimization Problems (MOOP)

- Non-scalarizing methods

  Each objective function is considered individually

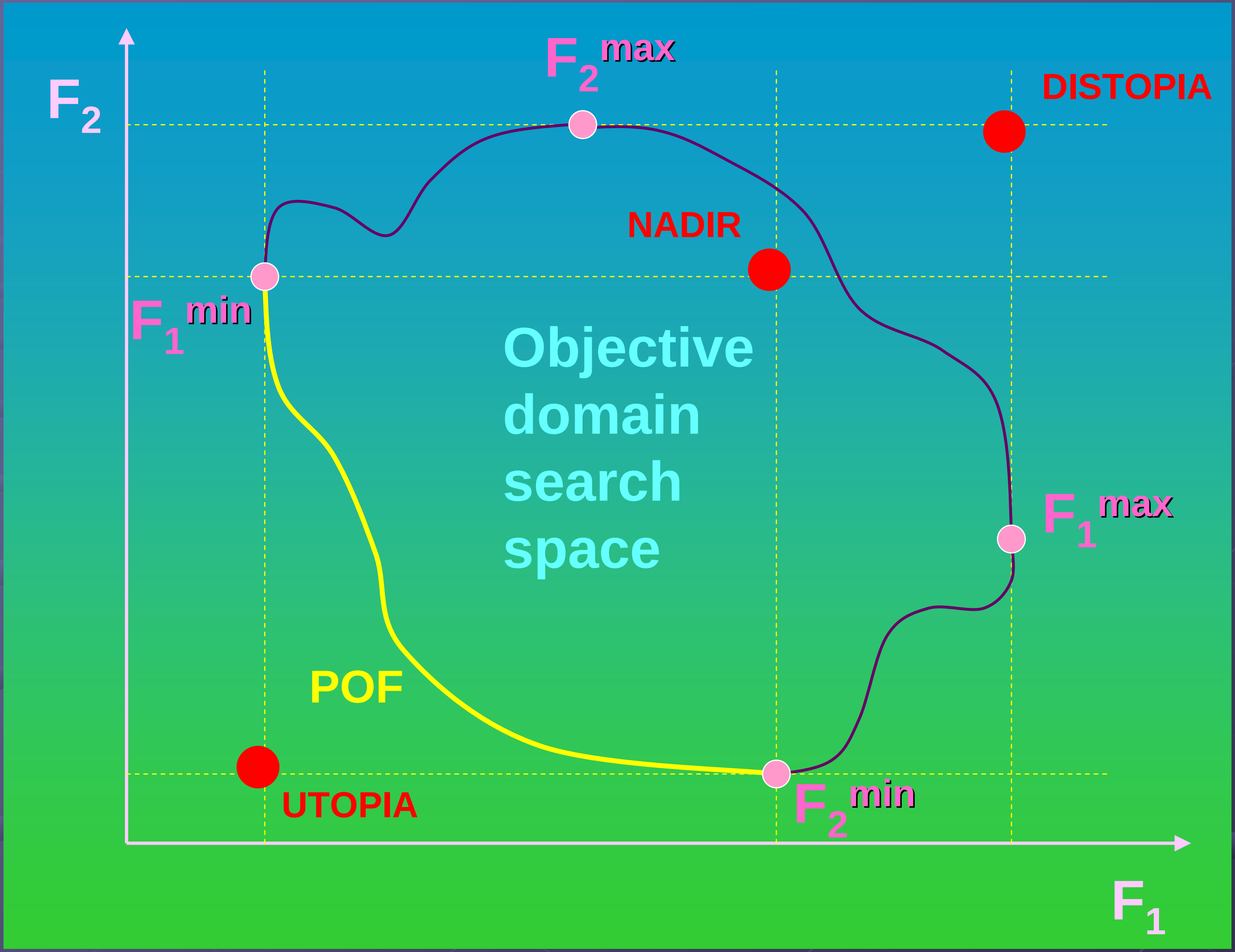
- Scalarizing methods

  Convert MOOP to SOOP and solve using a SOOP algorithm

  Available methods for converting MOOP into SOOP:

  - ε-constraint (ε-C)
  - weighting method (W)
  - weighted metrics (WM) (including the Tchebycheff metric) method
  - achievement scalarizing function approach (AF)
  - lexicographic ordering approach (LO)
  - value function method (VF)

Pareto Optimisation



POF – Pareto Optimal Front

Novel algorithms

Example 1

Scalarized One-Stage Algorithm using the 'credibility of hypotheses' function

- a Latin Hypercube experimental design is initially carried out
- objectives of the MOOP are normalized to be within the range [0,1]
- objectives are combined using the augmented Tchebycheff function
- independent optimization searches are launched (so the algorithm may be easily parallelized)
- the algorithm has a fixed number of iterations

The algorithm was tested on a difficult 2 dimensional test function, known as VLMOP2:

$$\begin{aligned}
\text{Minimize} \quad f_1(\mathbf{x}) &= 1 - \exp\Big(-\sum_{i=1}^{n}\big(x_i - \frac{1}{\sqrt{n}}\big)^2\Big) \\
\text{and} \quad f_2(\mathbf{x}) &= 1 - \exp\Big(-\sum_{i=1}^{n}\big(x_i + \frac{1}{\sqrt{n}}\big)^2\Big) \\
\text{with} \quad x_i &\in [-4, 4]
\end{aligned}$$

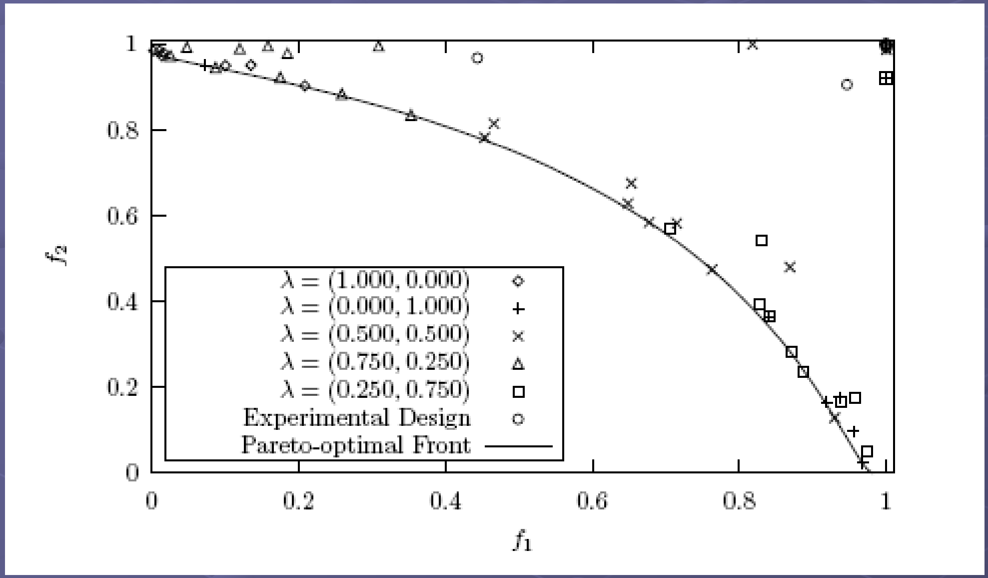
where n = 2.

5 different weighting vectors were used in the proposed algorithm, giving 60 iterations in total.

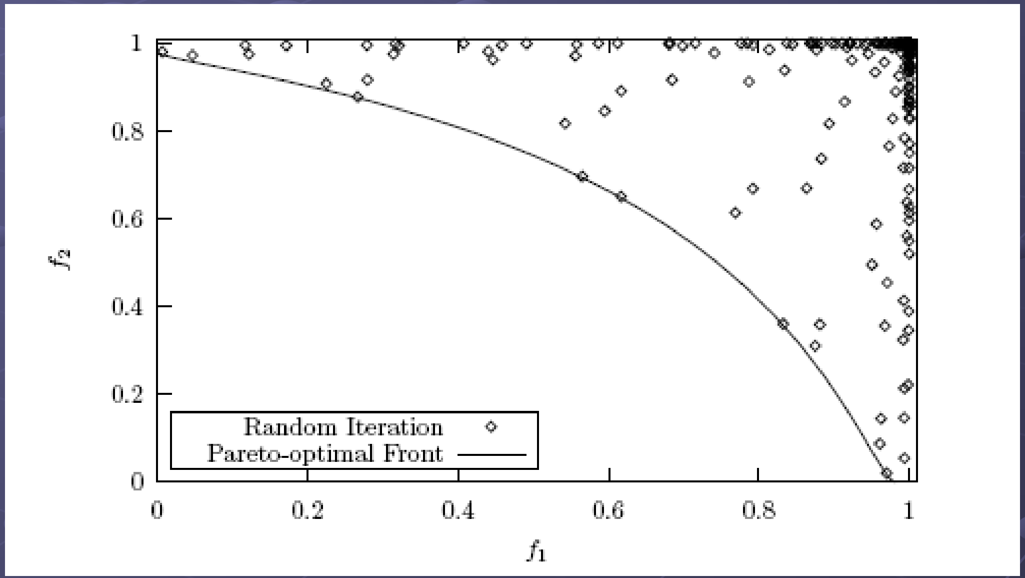
Novel algorithms

Example 1

Scalarized One-Stage Algorithm using the ‘credibility of hypotheses’ function



60 iterations of scalarizing one-stage algorithm



500 iterations of random search algorithm

Novel algorithms

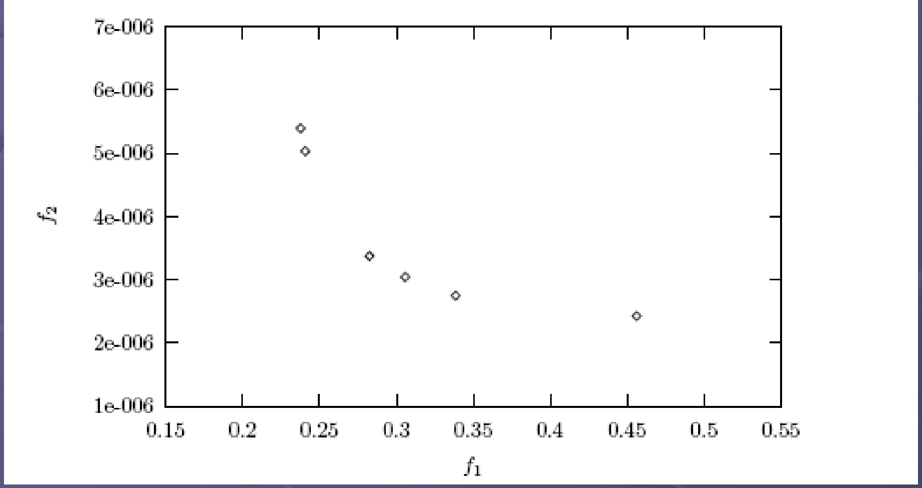
Example 2

Generalized ParEGO algorithm

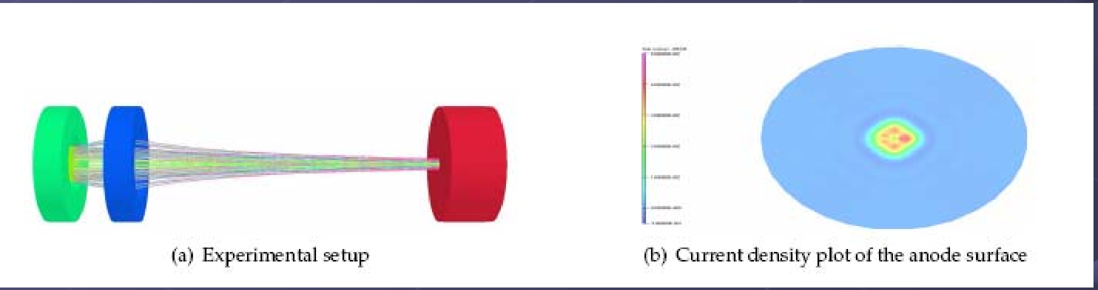
Uses:

- the probability of improvement (POI)
- the generalized expected improvement (GEI)
- the weighted expected improvement (WEI)

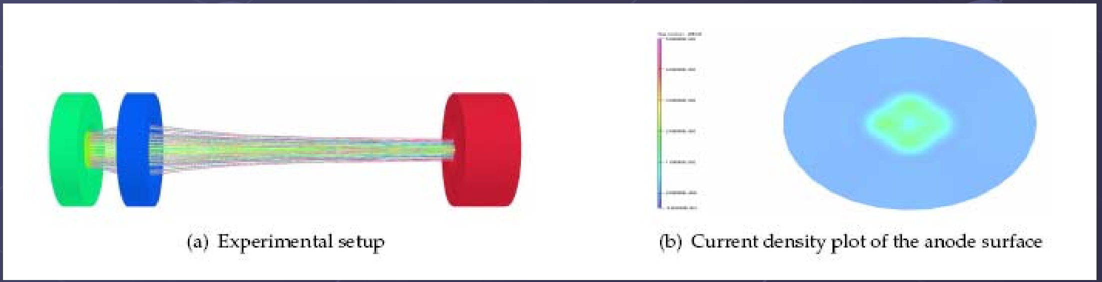
The algorithm was tested on an electromagnetic design problem. The voltage on, and position of, the focus electrode of an electron gun was varied so as to achieve two objectives: to focus the beam of electrons on the centre of the anode as much as possible, and to make the electrons hit the anode face as perpendicular as possible.



Pareto optimal front for electron gun problem



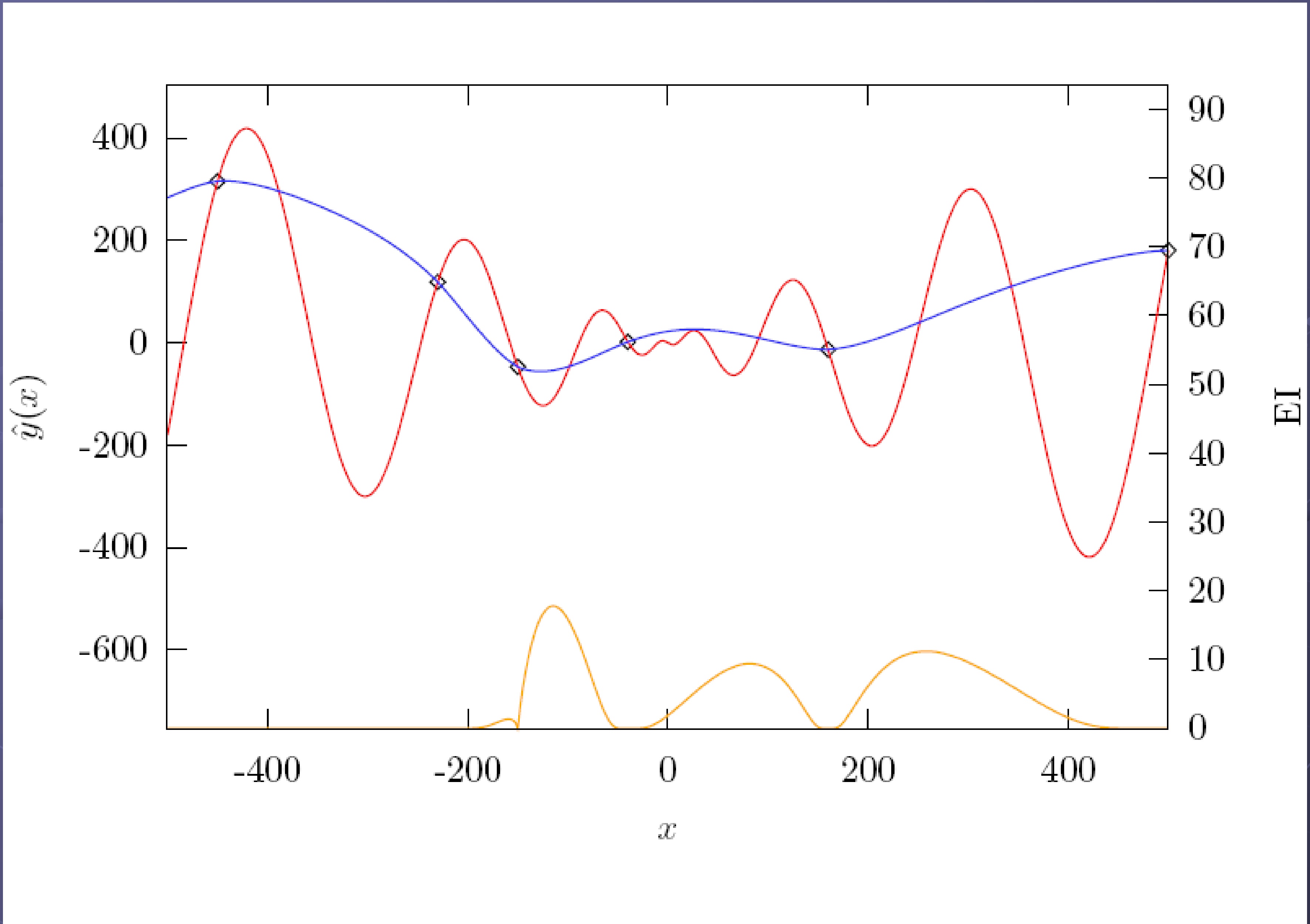
the extreme left Pareto point



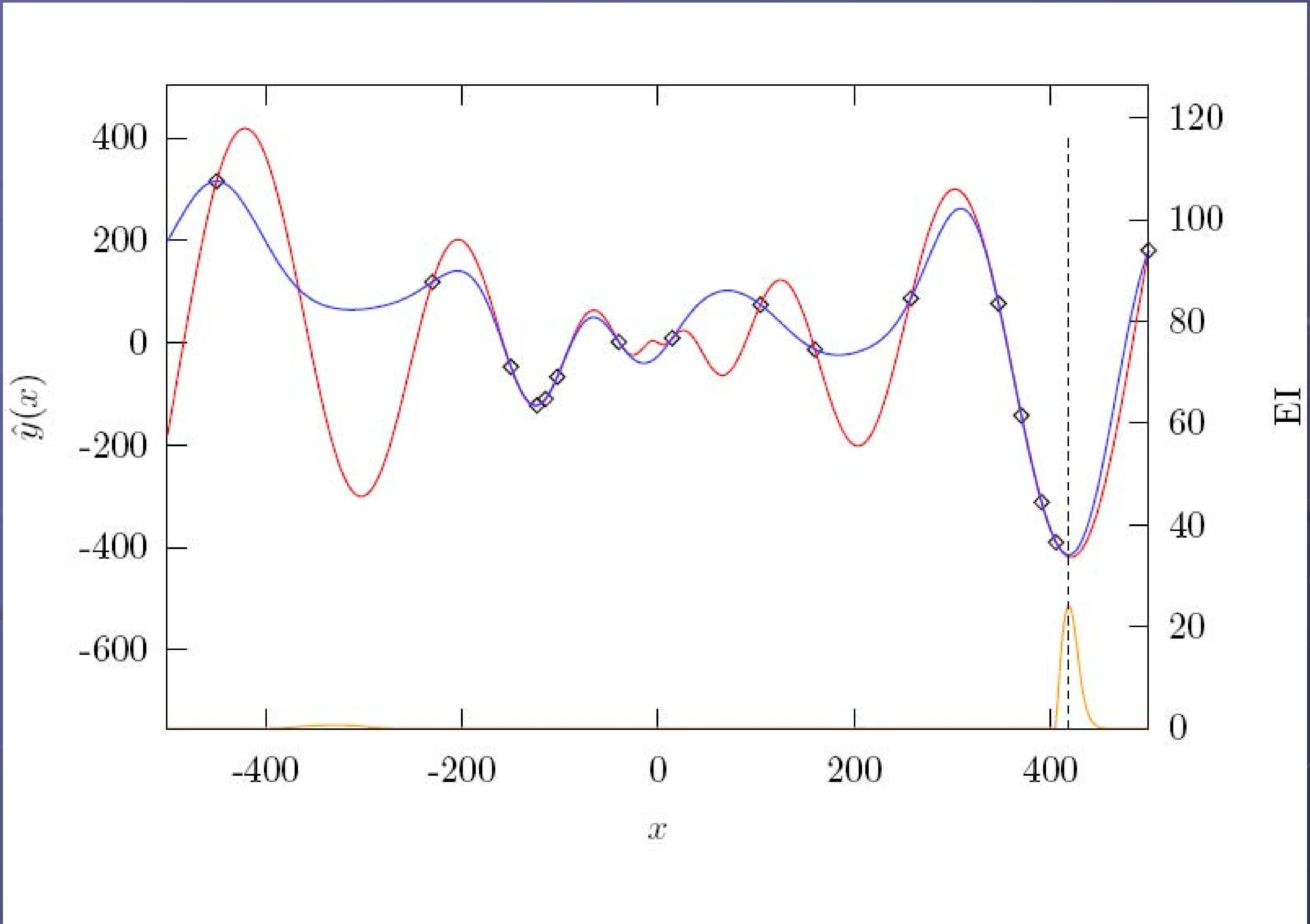
the extreme right Pareto point

Kriging: Example

- Maximize the expectation value of the improvement (EI)

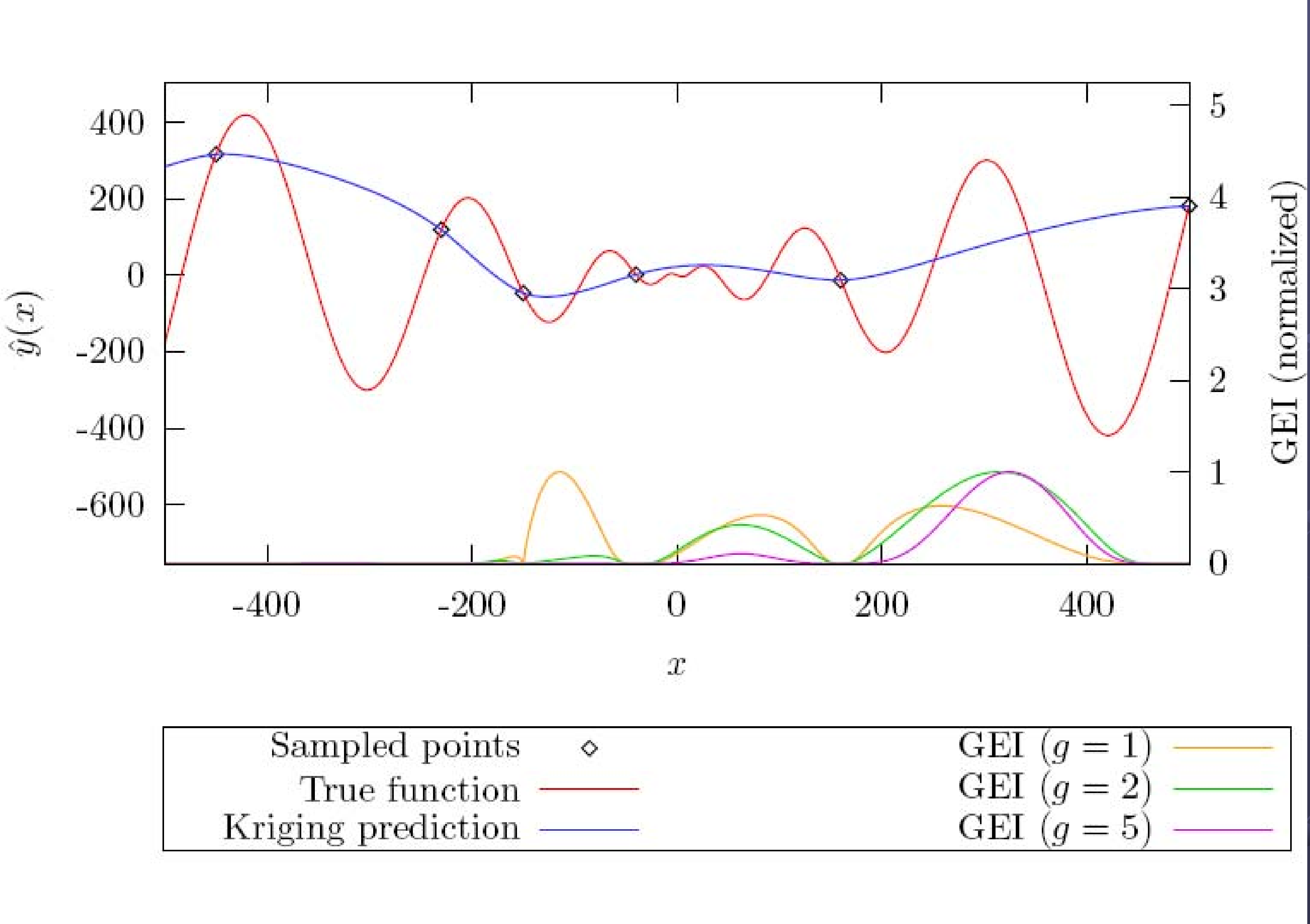


400
200
0
-200
-400
-600
$\hat{y}(x)$
90
80
70
60
50
40
30
20
10
0
EI
-400
-200
0
200
400
x


400
200
0
-200
-400
-600
$\hat{y}(x)$
120
100
80
60
40
20
0
EI
-400
-200
0
200
400
x

Kriging: Example

- Maximize the expectation value of the improvement (EI)
    - Generalize to “generalized expected improvement”
    - Generalize to “weighted expected improvement”



400
200
0
-200
-400
-600
$\hat{y}(x)$
5
4
3
2
1
0
GEI (normalized)
-400
-200
0
200
400
x
Sampled points
True function
Kriging prediction
GEI $(g = 1)$
GEI $(g = 2)$
GEI $(g = 5)$

Transformation of MOOPs to SOOPs

Family of Algorithms Available

Large Number of Selection Criteria from Single-Objective Optimization

x

Large Number of Methods for transforming a MOOP to a SOOP

Huge (Large x Large) Number of “Scalarizing” Multi-Objective Optimization Algorithms (made possible with kriging)

Selection Criteria	Scalarizing Method		
	ε-constraint	Weighting method	Weighted metric
Probability of Improvement			
Expected Improvement			
Weighted EI			
Generalized EI			
Credibility of Hypothesis			
Minimizer Entropy			

Selection Criteria	Scalarizing Method		
	ε-constraint	Weighting method	Weighted metric
Probability of Improvement			
Expected Improvement	Jones (1998)		
Weighted EI			
Generalized EI			
Credibility of Hypothesis			
Minimizer Entropy			

Selection Criteria	Scalarizing Method		
	ε-constraint	Weighting method	Weighted metric
Probability of Improvement			
Expected Improvement	Jones (1998)		Knowles (2006)
Weighted EI			
Generalized EI			
Credibility of Hypothesis			
Minimizer Entropy			

Selection Criteria	Scalarizing Method		
	ε-constraint	Weighting method	Weighted metric
Probability of Improvement			
Expected Improvement	Jones (1998)		Knowles (2006)
Weighted EI			
Generalized EI			Hawe and Sykulski (2007)
Credibility of Hypothesis			Hawe and Sykulski (2007)
Minimizer Entropy			

Concluding remarks

- Hierarchical design approach increasingly popular
- Field modelling (usually FEM based) important – both 2D and 3D

  but both computationally intensive
- Multi-objective optimization problems (MOOP) of particular interest
  - Kriging-assisted surrogate modelling
  - Pareto-optimisation
  - Design Sensitivity
- Choosing the ‘best’ optimisation algorithm for the task in hand

  is by itself an optimisation problem
- Optimisers increasingly available as part of commercial software

Thank you