Magnetic Damping of Levitated Liquid Droplets in AC and DC Field

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Abstract
The intense AC magnetic field required to produce levitation in terrestrial conditions, along with the buoyancy and thermo-capillary forces, results in turbulent convective flow within the droplet. The use of a homogenous DC magnetic field allows the convective flow to be damped. However the turbulence properties are affected at the same time, leading to a possibility that the effective turbulent damping is considerably reduced. The MHD modified K-Omega turbulence model allows the investigation of the effect of magnetic field on the turbulence. The model incorporates free surface deformation, the temperature dependent surface tension, turbulent momentum transport, electromagnetic and gravity forces. The model is adapted to incorporate a periodic laser heating at the top of the droplet, which have been used to measure the thermal conductivity of the material by calculating the phase lag between the frequency of the laser heating and the temperature response at the bottom. The numerical simulations show that with the gradual increase of the DC field the fluid flow within the droplet is initially increasing in intensity. Only after a certain threshold magnitude of the field the flow intensity starts to decrease. In order to achieve the flow conditions close to the 'laminar' a D.C. magnetic field >4 Tesla is required to measure the thermal conductivity accurately. The reduction in the AC field driven flow in the main body of the drop leads to a noticeable thermo-capillary convection at the edge of the droplet. The uniform vertical DC magnetic field does not stop a translational oscillation of the droplet along the field, which is caused by the variation in total levitation force due to the time-dependent surface deformation.

Introduction
A number of different methods have been developed, which allow the noncontact electromagnetic levitation of liquid metal droplets to be used to measure the properties of these highly reactive materials. Building on a technique for measuring heat capacity and total hemispherical emissivity used by Wunderlich & Fecht [1], Fukuyama et al. [2] developed a technique for measuring thermal conductivity. The technique involves applying a periodic laser heating to the top of an electromagnetically levitated droplet and measuring the temperature response at the bottom of the droplet. The thermal conductivity is determined by comparing the phase shift predicted by a semi-analytical model [3], with the phase shift determined by experiment [4]. The intense AC magnetic field required to produce levitation in terrestrial conditions, along with the buoyancy and thermo-capillary forces, results in turbulent convective flow within the droplet, which stops an accurate measurement of the thermal conductivity being made. The use of a homogenous DC magnetic field allows convective flow to be damped. However the turbulence properties are affected at the same time, leading to a possibility that the effective turbulent damping is considerably reduced. The effectiveness of the laminar flow damping as a function of the magnitude of the DC field is investigated by Tsukada et al. [3] using the finite difference numerical model for a fixed shape spherical droplet, where it is determined that the damping effect of static magnetic field > 5T are sufficient for accurate thermal conductivity measurements. In the present work different numerical modelling techniques are used to investigate the effects of turbulence and free surface deformation on the use of the period laser heating method of thermal conductivity measurement. The spectral collocation numerical model [5] has been used to solve the transient electromagnetic, fluid flow and thermodynamic equations. The MHD expanded k - ω turbulence model [6] is applied to investigate the turbulence damping effect due to the high magnetic field. Alternatively the problem for a fixed shape droplet are solved by the use of the COMSOL code, giving additional validation to the results and supplementing with detailed electromagnetic field representation. Both models have been adapted to incorporate the periodic laser heating at the top of the droplet, which is used to measure the thermal conductivity of the material.

Description of numerical models
We seek the numerical solution of the turbulent momentum and heat transfer equations for an incompressible fluid:

$$\partial_t \mathbf{v} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\rho^{-1}\nabla p + \nabla\cdot(\nu_e(\nabla\mathbf{v} + \nabla\mathbf{v}^T)) + \rho^{-1}\mathbf{f} + \mathbf{g}\,, \tag{1}$$

$$\nabla\cdot\mathbf{v} = 0\,, \tag{2}$$

$$C_p(\partial_t T + \mathbf{v}\cdot\nabla T) = \nabla\cdot(C_p\alpha_e\nabla T) + \rho^{-1}|\mathbf{J}|^2/\sigma\,, \tag{3}$$

where **v** is the velocity vector, p - the pressure, ρ - the density, ν_e is the effective viscosity computed from the 2-equation k-ω model), **f** is the electromagnetic force, **g** - the gravity vector, T - the temperature, α_e - the effective thermal diffusivity (related to ν_e), C_p - the specific heat, and $\mathbf{J}^2/\sigma$ is the Joule heat. We are considering the flow representation for an axisymmetric fluid droplet in the spherical co-ordinates with the detailed model representation given in previous publications [5]. The k-ω turbulence model including the magnetic field damping is given as in [6]. The boundary conditions at the external free surface are stated for the hydrodynamic stress tensor $\mathbf{\Pi}$ component projections on the surface, and the interface position **R**(t) is moving continuously with the calculated material velocity **v**(t):

$$\Pi_{nn} = \gamma K, \quad \Pi_{n\tau} = \gamma_T \nabla T \cdot \mathbf{e}_\tau, \quad \mathbf{e_n} \cdot \mathbf{v} = \mathbf{e_n} \cdot \partial_t \mathbf{R} \qquad (4)$$

where the subscripts n, τ correspond to projections onto $\mathbf{e}_n$ and $\mathbf{e}_\tau$ - the normal and tangent unit vectors at the free surface, $\gamma(T)$ is the temperature dependent surface tension coefficient, K - the local curvature. The boundary conditions for the turbulence variables are the absence of in/out flux at the free surface:

$$\partial_n k = 0, \quad \partial_n \omega = 0. \qquad (5)$$

The thermal boundary conditions are the radiation loss and the time modulated laser heating at the top part of the levitated droplet:

$$-\boldsymbol{n}.(-k\nabla T) = q_{laser} + \varepsilon\sigma(T_{amb}^4 - T^4), \qquad (6)$$

$$q_{laser} = \alpha 2P_0 / (\pi r_{laser}^2)(1 + \cos\omega_{laser} t)\exp(-2r^2 / r_{laser}^2)(\boldsymbol{n}.\boldsymbol{e}_{laser}),$$

where r_{laser} is the radius at which the laser intensity is less by a factor of e^2, ω_{laser} is the angular frequency of the beam, α is the absorptivity, the intensity of the laser beam at the centreline is related to the beam power P_0. In the experiment [3,4] a beam with P_0=9.56W, r_{laser} = 0.002 m, ω_{laser} = 0.1 is used.

Two types of mutually complementing numerical solutions (COMSOL and SPHINX) are obtained for the problem stated with the equations (1)-(6). The oscillating free surface solution is produced by the SPHINX code based on continuous transformation functions for the continuously varying shape [5]. A spectral-collocation method is used to solve the resulting equations, according to which the velocity vector components and pressure are represented as series of Chebyshev polynomials and Legendre functions [5].

Numerical modelling results

The problem for a fixed shape droplet can be solved by the use of the COMSOL code, giving additional validation to the results and supplementing with detailed electromagnetic field representation (Figure 1).

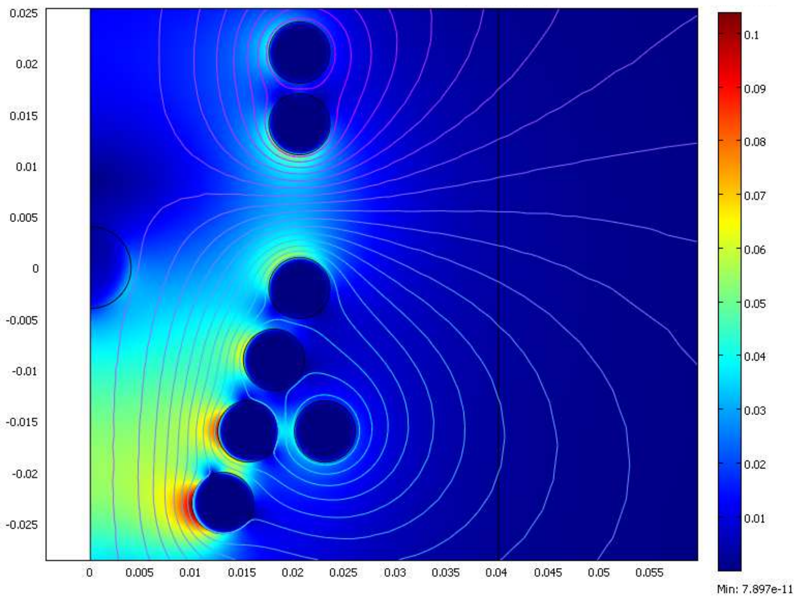

Fig 1: Magnetic field distribution (T): coil current 375 A (peak, 5 turns) and -375 A (peak, top 2 turns) at 200 kHz.

The finite element representation of the coil with a finite cross section gives a possibility to compare the solution to the often used filamentary approximation for the coil tube. The comparison for the total force acting on the spherical shape

droplet at various axial positions, as it happens in the experiment when the centre of mass of the droplet is moving in the vertical direction, gives evidence that to a certain accuracy the filamentary approach is sufficiently accurate to represent the effect of the coil. However, when inspecting the phase portrait of the rigid sphere oscillations in the field of this coil, we can conclude that a small deviation of the droplet in the vertical direction easily leads to an unstable situation when the droplet will fall out of a stable oscillation orbit. Moreover, this translational oscillation along the vertical axis leads to a change in the Joule losses within the droplet, affecting the time dependent response of the droplet temperature. When the laser heating is added, the modulation frequency for the laser should be chosen being in a different range to the transversal droplet oscillations.
The droplet in experimental conditions is subject to a noticeable surface deformation and an intense turbulent flow is developing in the droplet interior. These features are addressed by the use of the SPHINX code. In the absence of an additional DC magnetic field the computed flow and temperature distribution (without laser heating) are shown in the Figure 2. The rotational part of the time averaged AC electromagnetic force creates an intense internal flow of typical magnitude of about 0.1 m/s, which permits to estimate a typical Reynolds number of the order 10^4 . This flow generates a mildly turbulent flow where the effective turbulent viscosity and the thermal diffusion are enhanced by the action of the turbulence, see Figure 3 for the effective viscosity distribution.

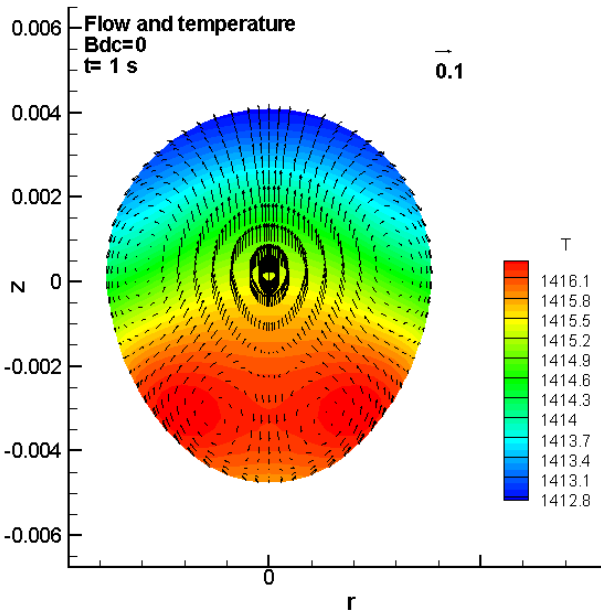

Fig. 2: Velocity and temperature without DC field.

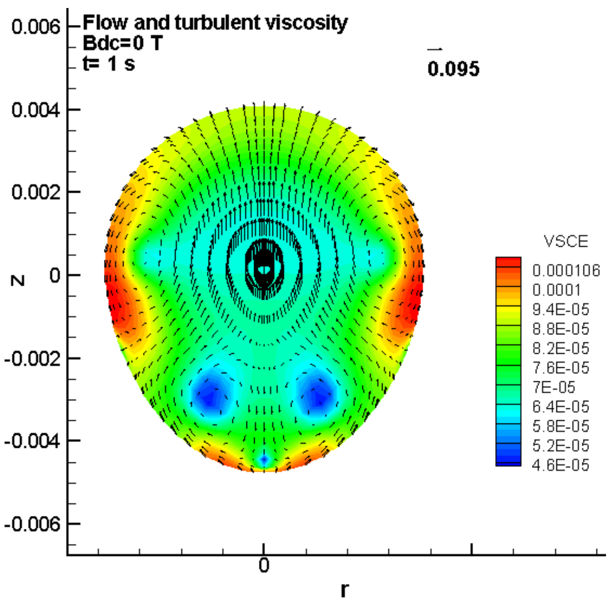

Fig. 3: Velocity and turbulent viscosity distribution without DC magnetic field.

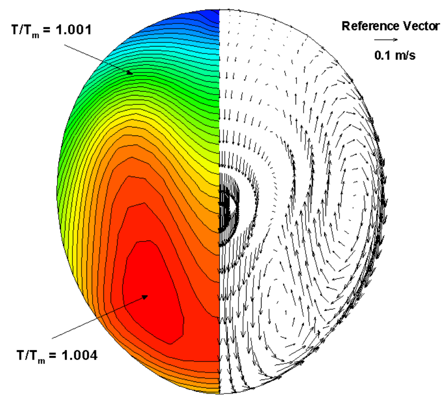

Fig. 4: Velocity and temperature with field B_{dc}= 1 T.

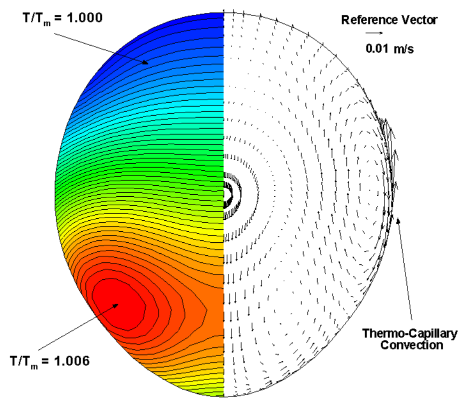

Fig. 5: Velocity and temperature with field B_{dc}= 5 T.

In an attempt to damp this turbulent diffusion and the electromagnetically driven convective flow, the authors of [3] proposed to use a uniform vertical DC magnetic field. They observed that a relatively large field of about 4 T is required

to bring the droplet to conditions close to the laminar flow and thermal diffusion. Our model permits to predict the flow behaviour when the DC field is being gradually increased. At a moderate B_z =1 T, the flow intensity actually starts to increase relative to the situation without the DC field, Figure 4. This is explained by the turbulence being damped by the magnetic field and the effective viscosity being reduced to almost a laminar value. But, at the same time, the large scale circulation flow grows to a typical value of about 0.2 m/s in the presence of the reduced viscosity. A further increase of the DC field leads to the gradual decrease in the average flow intensity, and the conditions approach the laminar viscosity and the heat transfer, as shown in the Figure 5. Comparing the Figures 2, 4 and 5 shows the effect of the high DC magnetic field damping. At the higher field values the thermocapillary effects start to be noticeable, as these are apparently not damped easily by the action of the DC magnetic field (Figure 5).
Finally the predictions of the modulated laser heating response as measured at the bottom of the droplet can be seen from the Figure 6, which is obtained for the conditions when the turbulent flow is either suppressed by the DC magnetic field or assuming that the flow is absent. Both SPHINX and COMSOL models were run with the laser frequency fixed at 0.1Hz to understand the dependency of the phase lag ϕ_s on the thermal conductivity k (W/mK). Fig. 6 shows the temperature response at the bottom of the droplet for various k over one cycle of the laser power modulation. The average temperature decreases when k is reduced, but the phase lag increases. When k=64 W/mK the phase lag is approximately 90°. Figure 7 shows the phase lag as a function of k computed with the numerical model. This gives a possibility to quantify the experimental observations and the relation between the phase lag and the thermal conductivity of the levitated liquid.

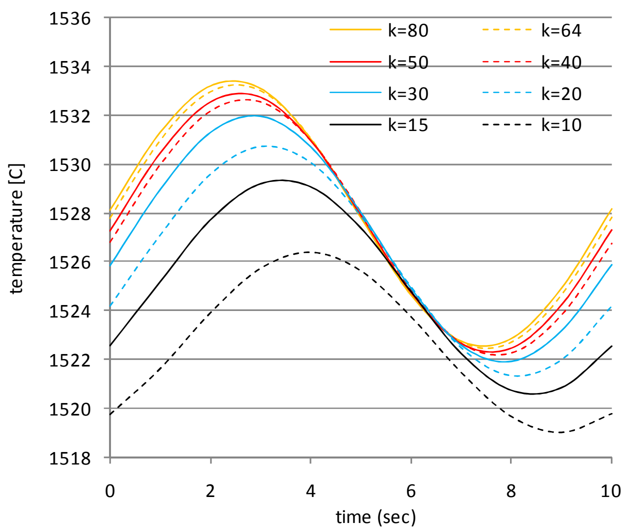

Fig. 6: The temperature modulation states at the droplet bottom as a function of time.

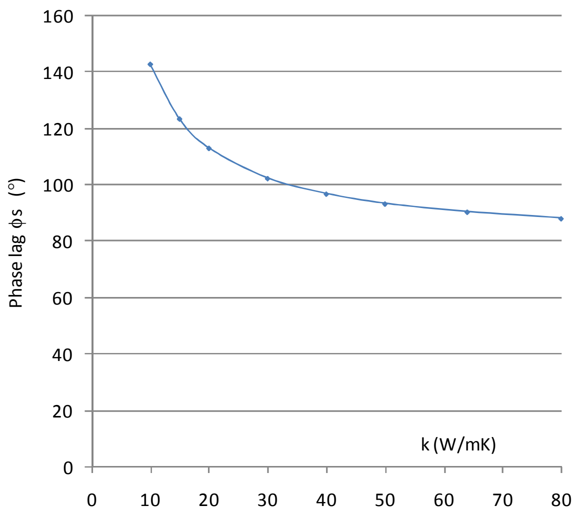

Fig. 7: Phase lag as function of thermal conductivity.

Conclusions

AC levitated liquid metal droplet is subject to large scale internal flow and increased transfer properties due to the small scale turbulence. Additional DC field damping is more effective for the small scale turbulence, and this could result in the increased large scale flow intensity. Sufficiently large DC magnetic field damps both the flow and turbulence, resulting in the conditions similar to laminar thermal diffusion. Thermal conductivity measurements are an example of possibilities for the material properties detection using the high DC magnetic field.

References

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