Robust Optimization Utilizing the Second-order Design Sensitivity Information

Dong-Hun Kim[1], Giwoo Jeung[1], Dong-Wook Kim[1], Heung-Geun Kim[1], David A. Lowther[2], Jan K. Sykulski[3]
[1]School of Electrical Eng. and Computer Sci., Kyungpook National University, Daegu 702-701, Korea
dh29kim@ee.knu.ac.kr
[2]ECE Department, McGill University, Montreal, Quebec, H3A 2A7, Canada
[3]School of ECS, University of Southampton, Southampton, SO17 1BJ, UK

***Abstract* — This paper presents an effective methodology for robust optimization of electromagnetic devices. To achieve the goal, the method improves the robustness of the objective function by minimizing the second-order sensitivity information, called a gradient index and defined by a function of gradients of performance functions with respect to uncertain variables. The constraint feasibility is also enhanced by adding a gradient index corresponding to the constraint value. The validity of the proposed method is tested with the TEAM Workshop Problem 22.**

I. INTRODUCTION

Due to a growing demand for high-performance and high-reliability electromagnetic devices or equipment, attention has recently been focused on the robust optimization of products with the aim of minimizing the variation of the performance as a result of uncontrollable factors such as manufacturing errors, operating conditions, material properties, etc. Until now, most of the attempts which have been made used the Taguchi's robust design concept or Monte Carlo simulation based on the assumption that design parameters are random variables with a probability distribution [1]. However, implementation difficulties usually arise because it is not easy to acquire probability data of uncertain variables and also information about which parameter is dominant may not be available.

To overcome the aforementioned drawbacks, this paper proposes an effective methodology utilizing the second-order sensitivity information, defined as a 'gradient index' (GI), for the robust optimization of electromagnetic systems [2]. The basic concept of the method is to obtain robustness of the objective function by minimizing a GI value calculated from the gradients of performance functions with respect to uncertain variables. Simultaneously, the constraint feasibility is also considered by adding a term determined with a constraint value and a gradient index corresponding to the constraint. Consequently, the method needs neither statistical information on design variations nor calculation of the performance reliability while it is searching for a robust optimal solution.

II. ROBUST OPTIMAL DESIGN USING A GRADIENT INDEX

The TEAM benchmark problem 22 is concerned with the design optimization of a superconducting magnetic energy storage system (SMES) as depicted in Fig. 1. In order to simplify the design problem, a constraint of the current quench condition on the superconductivity magnet is not considered here. A typical optimization problem for minimizing an objective function subject to a set of constraints is expressed as

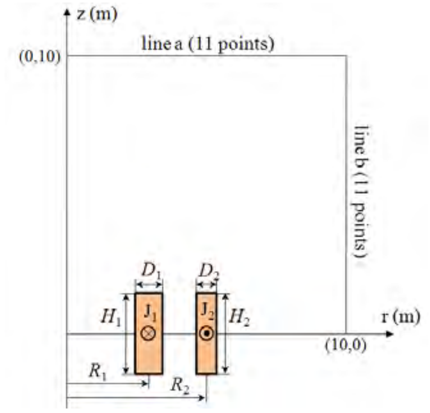

Fig. 1. Configuration of the SMES device with 8 design variables

$$\begin{aligned} &\text{Minimize} && f(\mathbf{x}) = \sum_{i=1}^{21} |B_{stray,i}(\mathbf{x})|^2 \\ &\text{subject to} && g_1(\mathbf{x}) = \left(\frac{E(\mathbf{x})-E_o}{0.05\times E_o}\right)^2 - 1 \le 0 \\ & && g_2 = (R_1 - R_2) + \frac{1}{2}(D_1 + D_2) \le 0 \\ & && \mathbf{x}_L \le \mathbf{x} \le \mathbf{x}_U \end{aligned} \tag{1}$$

where $\mathbf{x}$ is a design variable vector, $B_{stray,i}$ the stray field values calculated at the ith design iteration, E the stored magnetic energy and E_o the energy target value of 180 MJ. The values $\mathbf{x}_L$ and $\mathbf{x}_U$ denote the lower an upper bounds of the design variables, respectively.

To complement the above expressions, the proposed robust optimization for improving the robustness of the objective and the constraint functions is formulated as follows:

$$\begin{aligned} &\text{Minimize} && GI_f = \max_i |df(\mathbf{x})/du_i| \quad i = 1,2,\ldots 8 \\ &\text{subject to} && g_j(\mathbf{x}) + \Psi_j(g_j(\mathbf{x})) \le 0 \qquad j = 1,2 \\ & && f(\mathbf{x}) \cong M \end{aligned} \tag{2}$$

where GI_f is a gradient function of the objective function with respect to the uncertain variables u_i and M denotes the target value of the objective function. In order to enhance the robustness of the constraint feasibility, the term $\Psi_j(g_j)$ in (2) is added to each constraint

$$\Psi_j(g_j) = \begin{cases} 0 & g_j \langle \text{CT} \\ \dfrac{\kappa_j GI_{g_j}}{\text{CTMIN} - \text{CT}} & \text{CT} \langle g_j \langle \text{CTMIN} \\ \kappa_j GI_{g_j} & g_j \rangle \text{CTMIN} \end{cases} \tag{3}$$

$$GI_{g_j} = \max_i \left| dg_j(\mathbf{x})/du_i \right| \quad i = 1,2,\ldots 8$$

where GI_{gj} is a gradient function of the jth constraint function with respect to the uncertain variables. If the constraint is numerically critical (CT≤g_j≤CTMIN) or violated (g_j >CTMIN), a term proportional to GI_{gj} multiplied by a proper constant κ is added according to the value of the robustness of the constraint feasibility.

III. NUMERICAL IMPLEMENTATION

The proposed method has been implemented by combining the commercial finite element code MagNet [3] with a DOT optimizer [4] as shown in Fig. 2, where a modified feasible direction algorithm with the second-order sensitivity information by finite differencing is used. To obtain the GI values, the first-order sensitivity values are computed by the continuum design sensitivity analysis (CDSA).

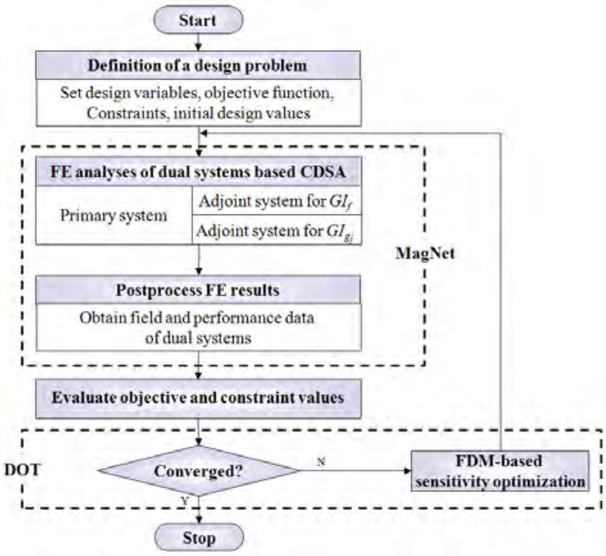

Fig. 2. Flow chart of the proposed robust optimization method

IV. RESULTS

The optimization problem for minimizing the stray fields of the SMES device of Fig. 1 is solved using two methods. The first is a deterministic method based on CDSA that does not take into account the effects of uncertain parameters; the second approach is the proposed robust optimization method. In this paper, all of the 8 design variables used in the deterministic method are selected to be the uncertain variables for the purpose of comparison between the two methods.

Starting with an initial design, the deterministic and the robust optima are presented in Table I. The stored energy values obtained from the two methods almost reach the target value of 180 MJ, but the robust optimum produces a better mean value of the stray fields than the deterministic algorithm. It is inferred that the deterministic optimal solution is trapped in one of the local minima near the constraint boundaries, while a better optimal solution is found by the robust optimization as the feasibility robustness of the constraints is improved. In Figs 3 and 4, the variations of the sensitivity values and the stray fields are compared between the deterministic and the robust optima, respectively.

TABLE I
DESIGN VARIABLES AND PERFORMANCE INDICATORS AT THE DETERMINISTIC AND ROBUST OPTIMA

Design variables	Unit	Lower bound	Initial values	CDSA optimum	Robust optimum	Upper bound
R1	mm	1000	2000	2108	1977	4000
D1	mm	100	500	412	404	800
H1	mm	200	1500	1504	1507	3600
R2	mm	1800	2500	2462	2348	5000
D2	mm	100	400	294	233	800
H2	mm	200	2000	1756	1871	3600
J1	A/mm^2	10	17	16.39	16.30	30
J2	A/mm^2	10	17	14.49	16.19	30
$B_{stray,mean}$	μT		23,055	153	34	
Energy	MJ		521	183	181	

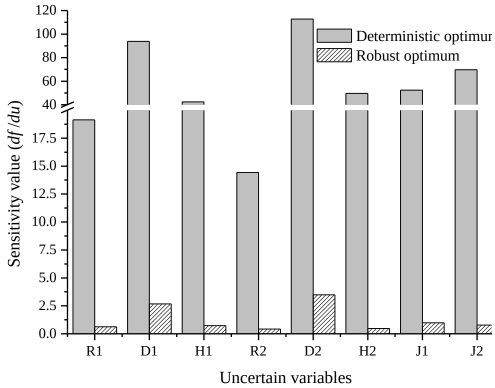

Fig. 3. Comparison of sensitivity values of uncertain variables

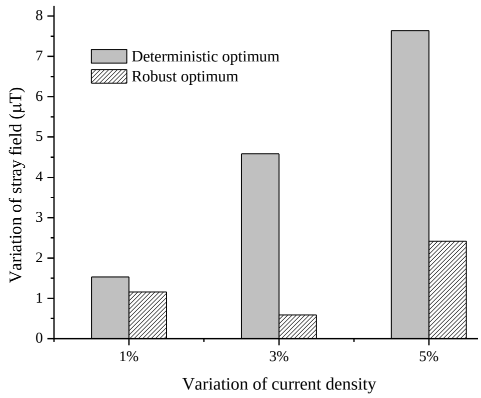

Fig. 4. Comparison of stray field variations when the current density changes

V. CONCLUSION

A robust optimization approach adopting the concept of a gradient index has been introduced in this paper. The results reveal that the proposed method offers high performance as well as robustness of the objective and the constraint functions.

VI. REFERENCES

[1] G. Steiner, A. Weber and C. Magele, "Managing uncertainties in electromagnetic design problems with robust optimization," *IEEE Transactions on Magnetics*, vol. 40, no. 2, pp. 1094-1099, 2004.

[2] J. S. Han and B. M. Kwak, "Robust optimization using a gradient index: MEMS applications," *Structural and Multidisciplinary Optimization*, vol. 27, no. 6, pp. 469-478, 2004.

[3] MagNet User's Manual, Infolytica Corporation, Quebec, Canada, 2008.

[4] DOT User Manual, Vanderplaats Research & Development, Inc., 2001.